

BEAM PHYSICS LIMITATIONS FOR DAMPING OF INSTABILITIES IN CIRCULAR ACCELERATORS*

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Abstract

The paper considers a beam interaction with a feedback system and major limitations on the beam damping rate. In particular, it discusses limitations on the system gain and damping rate, feedback system noise and its effect on the beam emittance growth, x - y coupling effect on damping, and suppression of high order modes.

CAUSALITY IN DAMPERS

Causality binds amplitude and phase for an amplifier or electric circuit. This relationship is described by Kramers-Kronig relations. However, there are no requirements of causality in beam-based feedbacks because a reduction of delay in a signal propagation from pickup to kicker may result in that the electric signal arrives to the kicker earlier than a particle bunch which produced this signal in the pickup, thus breaking causality. That allows one to adjust the complex gain of the feedback to basically anything what may require. It can be also used for a frequency response correction of a power amplifier. At high frequencies it is done by analogue circuits. At lower frequencies digital filters represent more effective means.

To break the causality one needs to split the signal into few paths with different delays and frequency responses. Figure 1 presents an example of filter which, with use of 3 branches, makes $1/\sqrt{\omega}$ gain dependence over 4 orders of magnitude with reasonably good phase response. The filter can be described by the following expression:

$$G(\omega) = \sum_{k=1}^3 \frac{A_k e^{i\omega\tau_{1k}}}{(1+i\omega\tau_{2k})(1+i\omega\tau_{3k})} . \quad (1)$$

Such or similar filter may be used for damping rate reduction with frequency as described below.

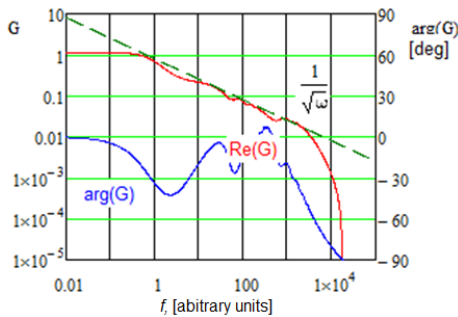


Figure 1: Amplitude and phase characteristics of $1/\sqrt{\omega}$ filter. Parameters of Eq. (1) are: $\tau_1=[0.1, 0.01, 0]$, $\tau_2=[1, 0.02, 4 \cdot 10^{-4}]$, $\tau_3=[0.02, 0.004, 8 \cdot 10^{-6}]$, $A=[1, 0.11, 0.03]$.

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A practical implementation of complex gain correction was carried out at Fermilab for gain correction of stochastic cooling systems during Tevatron Run II (see Section 7.2.3 in Ref [1]).

EMITTANCE GROWTH SUPPRESSION

An increase of beam energy of a hadron collider results in an increase of its size and a reduction of revolution frequency. It leads to a decrease of frequencies of lowest betatron sidebands. Considering that the spectral density of noise, which excites betatron motion, increases fast with frequency decrease one obtains that this increase may become dangerous when betatron frequency sidebands approach few kilohertz range. This effect was first observed at Tevatron where it prevented an operation at low value of fractional part of betatron tune [2]. Later experiments verified an existence of the problem (see Section 6.3.3. in Ref [1]). This effect was strongly manifested at the beginning of LHC commissioning where “transverse noise” resulted in fast emittance growth intermittent with emittance jumps called “hump” effect [3].

Major sources of emittance growth are fluctuations of bending field and quad displacements due to ground motion. The bending field fluctuations are excited by fluctuations of current in dipoles and may be also excited by mechanical oscillations of liners inside SC dipoles. Note that in the LHC the magnetic field of dipoles is “frozen” into the liners and their size oscillations, excited by acoustic noises, result in oscillations of magnetic field. Typical requirement to the bending field stability of $\Delta B/B \leq 10^{-9}$ is quite tight. Consequently, size fluctuations of sub-angstrom level may be dangerous.

The transverse emittance growth driven by transverse kicks is determined by their spectral density at the betatron sidebands [4]:

$$\left(\frac{d\varepsilon}{dt} \right)_0 = \frac{\omega_0^2}{4\pi} \sum_k \beta_k \sum_{n=-\infty}^{\infty} S_{\theta_k}((\nu - n)\omega_0) . \quad (2)$$

Here ω_0 is the circular revolution frequency, ν is betatron tune, and $S_{\theta_k}(\omega)$ is the spectral density of the angular kicks at the k -th location with beta-function of β_k . The spectral density is normalized as:

$$\overline{\Delta\theta^2} = \int_{-\infty}^{\infty} S(\omega) d\omega , \quad (3)$$

where $\sqrt{\overline{\Delta\theta^2}}$ is the rms value of the kicks. For the white noise, Eq. (2) is simplified to the well-known result: $(d\varepsilon/dt)_0 = (f_0/2) \sum_k \beta_k \overline{\Delta\theta_k^2}$, where $f_0 = \omega_0/2\pi$. Note that

the emittance growth happens only if there is a spread in particle betatron tunes. In its absence a bunch would be moving as whole without emittance growth. In the case of hadron collider of LHC size or more Eq. (2) puts severe limitations on acceptable value of noise spectral density.

A solution for the problem was first suggested in Refs. [4,5]. The idea is based on damping of betatron oscillations before betatron motion from a kick may decohere. It was shown that damping of betatron oscillations with damping rate (in amplitude) of $\lambda=f_0g/2$ is suppressed as:

$$\frac{d\varepsilon}{dt} \approx \frac{16\pi^2\overline{\Delta\nu^2}}{g^2+16\pi^2\overline{\Delta\nu^2}} \left[\left(\frac{d\varepsilon}{dt} \right)_0 + \frac{f_0g^2}{2\beta_p} \sigma_{bpm}^2 \right], \quad (4)$$

where the second addend accounts for the emittance growth excited by the feedback system (transverse damper) itself, σ_{bpm} describes the damper noise referenced to the rms accuracy of beam position measurements in the pickup, β_p is the beta-function in the pickup, g is the dimensionless gain of the damper, and $\sqrt{\overline{\Delta\nu^2}}$ is the rms spread of betatron tunes. For head-on collisions of round beams $\sqrt{\overline{\Delta\nu^2}} \approx 0.2\xi$, where ξ is the total linear tune shift due to beam-beam interactions. As can be seen from Eq. (4), if the gain is much larger than the tune spread, the gain increase does not increase the emittance growth related to the damper noise; while contribution due to external noise is suppressed as $1/g^2$. It yields that the gain should be sufficiently large so that the contribution of damper noise would dominate the emittance growth.

Note also that the betatron motion chromaticity is another source of beam decoherence. Therefore, it is desirable to have the gain larger than the synchrotron tune.

As it was already mentioned that the emittance growth due to LHC ‘‘hump’’ presented a serious challenge at the beginning of LHC commissioning. The problem was resolved by large gain increase in the LHC transverse dampers and a redistribution of gains inside their electronics which reduced the pickup noise [6]. Power supplies responsible for creation of the ‘‘hump’’ were found in about half year. For the LHC the rms pickup resolution is estimated to be in the range of 0.2 – 0.5 μm .

Spectral density of external noise decreases fast with frequency. Consequently, only external noise at the lowest betatron sidebands contributes to the emittance growth. Typically, the instability rate of multi-bunch instabilities also decreases with frequency. That enables a reduction of feedback gain with frequency increase. Let us consider how this reduction affects the emittance growth driven by the damper. The pickup signal from a collider bunch is quite large ($\gg 1$ V). Therefore, noise of a digital damper is typically determined by resolution of ADCs digitizing pickup signal, and, to good accuracy, the pickup noise can be considered as the white noise. For a ring with n_b uniformly distributed bunches ($n_b \gg 1$) we can consider that all noises are in the frequency band $[-f_0n_b/2, f_0n_b/2]$. The corresponding spectral density is $\sigma_{bpm}^2/(2\pi f_0n_b)$. Taking this into account we can rewrite Eq. (4) as follows:

$$\frac{d\varepsilon}{dt} \approx \sum_{n=-n_b/2}^{n_b} \frac{16\pi^2\overline{\Delta\nu^2}}{g_n^2+16\pi^2\overline{\Delta\nu^2}} \left[\left(\frac{d\varepsilon}{dt} \right)_n + \frac{f_0g_n^2}{2\beta_p} \frac{\sigma_{bpm}^2}{n_b} \right], \quad (5)$$

where g_n is the damping rate at the n -th betatron sideband, $(d\varepsilon/dt)_n = (\omega_0^2/4\pi) \sum_k \beta_k S_{\theta_k} ((\nu-n)\omega_0)$ is the contri-

bution to the emittance growth from the external noise at n -th betatron sideband, and we accounted that the external noise for higher harmonics is negligible (or it can be referenced to the main band). As one can see for $g_n^2 \gg 16\pi^2\overline{\Delta\nu^2}$ the contribution of damper noise to the emittance growth does not depend on gain distribution over frequency. However, if the gain at high frequencies can be reduced to be smaller than $\sqrt{16\pi^2\overline{\Delta\nu^2}}$; then accounting that the external noise is negligible at high frequencies one obtains that the effect of damper noise can be reduced resulting in a smaller emittance growth.

DIGITAL FILTERS

Modern dampers, including LHC dampers, are digital. That creates a possibility to use large number of previous turns in computation of each kick. That potentially could reduce the damper sensitivity to pickup noise. Let us consider a general damper where each turn correction is determined by weighted sum of previous beam positions:

$$\delta\theta_n = \frac{g_1}{\sqrt{\beta_p\beta_k}} \sum_{k=0}^{K-1} A_k (x_{n-k} + \delta x_{n-k}). \quad (6)$$

Here β_p and β_k are the beta-functions in the pickup and kicker, respectively, x_n and δx_n are the beam positions and their errors at turn n , and g_1 is the relative damper gain. A requirement to suppress sensitivity to the beam orbit offset in the pickup results in that $\sum_{k=0}^{K-1} A_k = 0$. We also assume that the gain g_1 is sufficiently small so that a perturbation theory could be used.

An introduction of complex variable,

$$z = \frac{x}{\sqrt{\beta}} - i \left(\sqrt{\beta}\theta + \alpha \frac{x}{\sqrt{\beta}} \right), \quad (7)$$

reduces the betatron motion to a rotation in the complex plane with betatron frequency ($z = e^{i\mu} z_0$). Substituting Eq. (7) into Eq. (6) and dropping non-resonant terms one obtains the damping rate:

$$g_d = -\frac{i}{2} g_1 e^{-i\mu_{pk}} \sum_{k=0}^{K-1} A_k e^{-i\mu_0 k}, \quad (8)$$

where $\mu_0/2\pi$ is the betatron tune, and μ_{pk} is the betatron phase advance between pickup and kicker [6].

Now we consider the emittance growth excited by noise of the BPM measurements. The same as above we assume that g is sufficiently small. Then, omitting (temporarily) the damping term (x_{n-k}) in Eq. (6) one obtains:

$$z_{n+1} = e^{i\mu_0} \left(z_n - i e^{-i\mu_{pk}} \frac{g_1}{\sqrt{\beta_p}} \sum_{k=0}^{K-1} A_k \delta x_{x-k} \right). \quad (9)$$

Each error, δx_n , makes K contributions to the sum thus multiplying the effect of this error. Let only a single measurement be erroneous. Then after K turns we obtain:

$$\begin{aligned} z_K &= e^{i\mu_0 K} z_0 - i e^{-i\mu_{pk}} \frac{g_1}{\sqrt{\beta_p}} \delta x_0 \sum_{k=0}^{K-1} A_k e^{i\mu_0(K-k)} \\ &= e^{i\mu_0 K} z_0 + \frac{2g_d}{\sqrt{\beta_p}} e^{i\mu_0 K} \delta x_0, \end{aligned} \quad (10)$$

where z_0 is the initial complex amplitude of the particle in the pickup, δx_0 is an error of position measurement, and at the end of transformations we used Eq. (8). Averaging over initial phase of the oscillations and kick amplitudes we obtain an increase of the emittance due to a single kick:

$$\delta \mathcal{E} = \frac{1}{2} (|z + \delta z|^2 - |z|^2) = \frac{1}{2} |\delta z|^2 = 2 |g_d|^2 \frac{\overline{\delta x^2}}{\beta_p}, \quad (11)$$

where $\overline{\delta x^2}$ is the squared rms error of single measurement. Taking into account that different kicks are statistically independent one obtains the emittance growth rate without its suppression by the damper:

$$\frac{d\mathcal{E}}{dt} = 2f_0 |g_d|^2 \frac{\overline{\delta x_{bpm}^2}}{\beta_p}. \quad (12)$$

Comparing this equation with Eq. (4) ($g_d \rightarrow g/2$) one can conclude that the digital filter does not help to reduce an effect of BPM noise on $d\mathcal{E}/dt$; but a usage of large number of turns in the damper increases the damper sensitivity to the betatron tune and, as it will be shown in the following section, reduces the maximum achievable gain.

LIMITATIONS ON THE GAIN

An increase of collider size results in increased sensitivity to external noise and, consequently, requires a higher damping rate. Here we consider limitations on the damping rate for a digital transverse damper. In the general case the turn-by-turn transformation referenced to the pickup location is:

$$\mathbf{x}_{n+1} = \mathbf{M}_{kp} \left(\mathbf{M}_{pk} \mathbf{x}_n + \mathbf{G} \sum_{k=0}^{K-1} A_k \mathbf{x}_{n-k} \right), \quad (13)$$

where $\mathbf{x}_n = (x, \theta_x)^T$ is the vector describing a location of the bunch center of gravity in the 2D phase space, \mathbf{M}_{kp} and \mathbf{M}_{pk} are the transfer matrices from kicker-to-pickup and pickup-to-kicker,

$$\mathbf{G} = \begin{bmatrix} 0 & 0 \\ 0 & g_1 \end{bmatrix} \quad (14)$$

and coefficients A_k are defined by Eq. (6). We look for a solution in the form $\mathbf{x}_{n+1} = \Lambda \mathbf{x}_n$ which results in an equation for the eigen-value Λ :

$$\left| \mathbf{M} - \Lambda \mathbf{I} + \mathbf{M}_{kp} \mathbf{G} \sum_{k=0}^{K-1} \frac{A_k}{\Lambda^k} \right| = 0, \quad (15)$$

where \mathbf{I} is the identity matrix, and we accounted that the ring transfer matrix is $\mathbf{M} = \mathbf{M}_{kp} \mathbf{M}_{pk}$. As one can see the number of roots of Eq. (15) is equal to $2K$. Consequently, for the filter order $K \leq 2$ this equation can be solved analytically, and a numerical solution is required otherwise. The damping rate is defined by following equation:

$$\lambda = f_0 \ln \left(\max(|\Lambda_n|) \right), \quad (16)$$

where $\max()$ chooses the largest modulo of the eigen-values of $|\Lambda_n|$. For small g_1 the perturbation theory solution yields the result presented in Eq. (8).

To demonstrate behaviour of damping for large gains we consider 3 cases: (1) one-turn system ($A_0 = 1$), (2) two-turn system with notch filter ($A_0 = -A_1 = 1$), and the LHC damper.

The solution for the one-turn system is straightforward. It has two roots:

$$\Lambda = c + \frac{g_1 s_{kp}}{2} \pm i \sqrt{1 - g_1 s_{pk} - \left(c + \frac{g_1 s_{kp}}{2} \right)^2}, \quad (17)$$

where μ_{kp} is the kicker-to-pickup phase advance, and $c = \cos(\mu_0)$, $s = \sin(\mu_0)$, $c_{kp} = \cos(\mu_{kp})$, $s_{pk} = \sin(\mu_{pk})$, $\mu_0 = 2\pi[\nu]$, and $[\nu]$ is the fractional part of betatron tune. Figure 2 shows a dependence of eigen-values modulo on the gain for $[\nu] = 0.42$ and $\nu_{pk} = 0.25$. As one can see an increase of gain results in modulo splitting for gain above:

$$g_m = \begin{cases} -\sin(\mu) / \sin^2(\mu_{kp} / 2), & \pi \leq \mu \leq 2\pi, \\ \sin(\mu) / \cos^2(\mu_{kp} / 2), & 0 \leq \mu \leq \pi, \end{cases} \quad (18)$$

where the maximum damping is achieved. Figure 3 presents a dependence of maximum of eigen-values modulo on the betatron tune for the optimal gain, g_m . As one can see damping is greatly decreased near half integer tunes. One turn damping is possible at tunes equal to 0.25 and 0.75. Note also that a properly designed system of two pickups and two kickers has not a dependence of g_m on the betatron tune and can damp oscillations in 1 turn.

The solution for the two-turn system has four roots. One of them is equal to zero and can be omitted. The three other are solutions of cubic equation. Its solution is straightforward and therefore is not presented here. Figure 4 shows a dependence of eigen-values modulo on the gain for the optimal pickup-to-kicker phase advance $\nu_{pk} = (1-\nu)$

/2. Behaviour is similar to the case of one turn system but now there are 3 non-trivial eigen-values. Figure 5 presents a dependence of maximum of eigen-values modulo on the betatron tune for the optimal gain. As one can see the damping disappears near integer resonances and, compared to the one-turn system, the maximum damping rate is significantly degraded.

An increase in number of turns decreases the maximum damping rate approximately as $1/K$, where K is the number of turns used in computation of corrections. Figure 6 presents the damping rate, defined by Eq. (16), for the LHC horizontal damper [6] where damper corrections are computed from the beam positions at 7 previous turns. That significantly reduces the maximum achievable damping rate and introduces strong dependence of damping rate on the machine tune as shown in Figure 7.

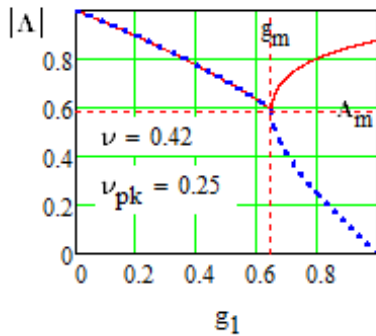


Figure 2: Dependence of eigen-values on the gain for one-turn system; $[\nu] = 0.42, \nu_{pk} = 0.25$.

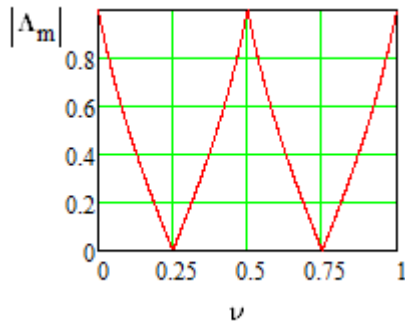


Figure 3: Dependence of maximum of eigen-values at the optimal gain on the betatron tune for the one-turn system.

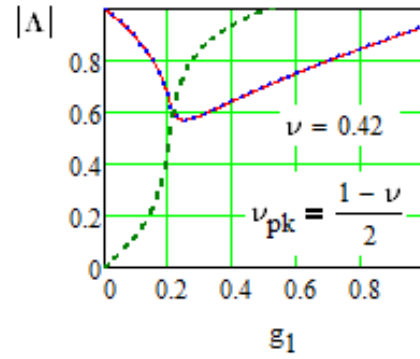


Figure 4: Dependence of eigen-values on the gain for two-turn system; $[\nu]=0.42, \nu_{pk}=0.25$.

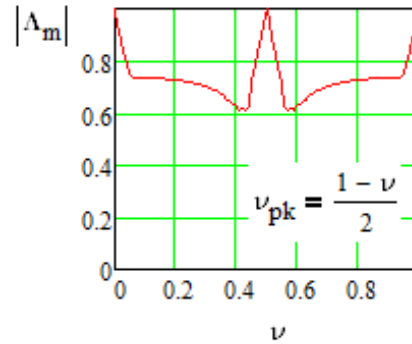


Figure 5: Dependence of maximum of eigen-values at the optimal gain on the betatron tune for two-turn system. The maximum damping gate: $\lambda / f_0 \approx -\ln(0.61) \approx 0.49$.

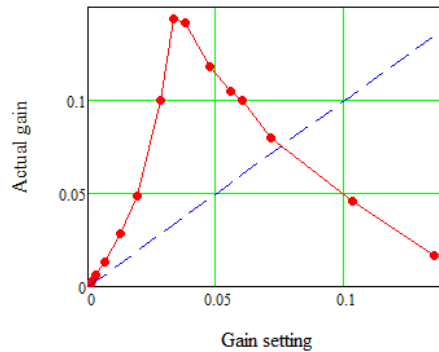


Figure 6: Dependence of damping rate on the gain setting for the LHC horizontal damper for beam 1.

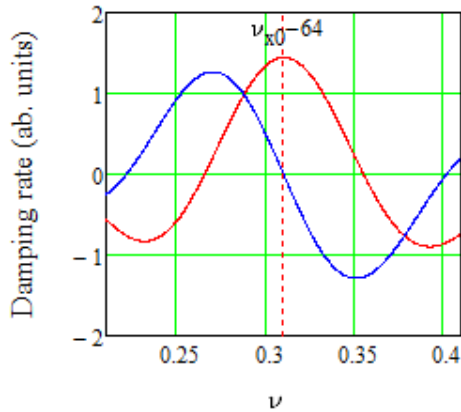


Figure 7: Dependence of real (red) and imaginary (blue) parts of damping rate on the machine tune for the LHC beam 1 horizontal damper.

ANALOG PRE-PROCESSING AND POST-PROCESSING

Analog pre-processing and post-processing in digital dampers may significantly affect an excitation of intra-bunch high order modes (HOMs) and, consequently, may limit the damping rate.

Typical signals from a strip-line pickup are shown in Figure 8. The signal consists of from the forward signal proportional to the bunch dipole moment (red line) and its reflection from the downstream end of the pickup (blue line). The total signal is shown by black curve. The way how this signal is modified before digitization and how digitization is done determines sensitivity of the damper to HOMs. Note also that a non-zero chromaticity changes the transverse offset along the bunch in the course of synchrotron motion making the signal of zero mode resembling signals of HOMs.

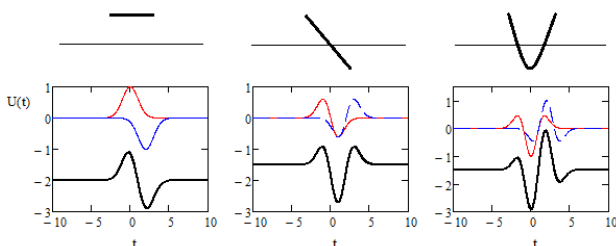


Figure 8: Typical pickup signals for the dipole (left) and higher head-tail modes. Top plots show changes of bunch transverse offset along the bunch.

The following analogue pre-processing methods applied before digitization are usually used:

- An integration which delivers the bunch center of gravity
- Mixing pickup signal with RF with subsequent low pass filtering making bell-shape form of a signal

- In the case of very short bunches an excitation of oscillator with subsequent digitization at slower sampling rate.

If the bunch length is much smaller than the bunch-to-bunch distance then, if special care is applied, this pre-processing may deliver the bunch center of gravity which is weakly sensitive to HOMs. To reduce sensitivity to the signal base line (voltage outside of signal waveform) the digitization before and at the bunch is used. It becomes close to impossible to avoid excessive sensitivity to HOMs if bunch length is comparable to the bunch-to-bunch distance which is typical for proton synchrotrons.

Normally, kicker power amplifiers do not amplify low frequencies. It makes an amplifier signal bipolar and, consequently, it makes kicks being bipolar. If bunch length is comparable to the bunch-to-bunch distance, then making uniform kick along the bunch becomes very challenging. Consequently, that makes it impossible to make uniform kick along the bunch and to avoid an excitation of HOMs.

Depending how signal of a HOM is pre-processed before the digitization and how kicker signal excites the same HOM (post-processing) the damper can amplify or damp this HOM. Note that these problems need to be addressed even if many digitization points are used in the pickup measurements and formation of kicker voltage.

EFFECTS OF X-Y COUPLING

Usually effects of x - y coupling do not play significant role in damping of instabilities. However, in the course of Tevatron Run II, it was observed that switching on a one-plane damper could introduce instability in another plane. The reason of such behaviour was strong x - y coupling which could not be completely compensated because of large uncontrolled skew-quad components in superconducting dipoles. Running dampers for both planes made the beam stable. In this section we consider how such problem can be analysed.

The analysis can be done similar to a single dimensional case described by Eq. (13) where 2D matrices have to be replaced by 4-D matrices; and the matrix \mathbf{G} has to be replaced by 4D matrix

$$\mathbf{G}_x = \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad \text{or} \quad \mathbf{G}_y = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} \quad (19)$$

for the horizontal and vertical dampers, respectively.

In majority of applications a perturbation theory solution is sufficient. In this case we can use a perturbation theory developed in Ref. [7]. It yields that if the perturbation changes the transfer matrix from \mathbf{M} to $\mathbf{M} + \Delta\mathbf{M}$ then the betatron tune shifts due to the perturbation are:

$$\Delta\nu_n = -\frac{1}{4\pi} \mathbf{v}_n^+ \mathbf{S} \Delta\mathbf{M} \mathbf{v}_n, \quad n=1,2. \quad (20)$$

Here \mathbf{S} is the unit symplectic matrix, and \mathbf{v}_n are the eigen-vectors of unperturbed motion. Two other eigen-vectors

(values) are complex conjugated to the first couple. That makes altogether 4 linearly independent eigen-vectors. Leaving only the first order terms in Eq. (13) one obtains:

$$\Delta \mathbf{M}_n^{x,y} = \mathbf{M}_{kp} \mathbf{G}_{x,y} \sum_{k=0}^{K-1} A_k \Lambda_n^{-k} \quad (21)$$

That results in for the horizontal damper:

$$\Delta v_n = -\frac{1}{4\pi} \left(\sum_{k=0}^{K-1} A_k \Lambda_n^{-k} \right) \mathbf{v}_n + \mathbf{S} \mathbf{M}_{kp} \mathbf{G}_x \mathbf{v}_n. \quad (22)$$

where Λ_n are corresponding eigen-values of unperturbed motion. For the vertical damper \mathbf{G}_x needs to be replaced by \mathbf{G}_y . Note that a knowledge of 4D optics is required to use Eq. (22).

DAMPER DIAGNOSTICS

The spectrum of pickup measurements has information about the betatron frequency and the phasing of the damper. Figure 9 shows the spectrum simulated for the LHC damper. Measurements resulted in similar behavior. One can see that the spectral density is suppressed at the betatron frequency. The LHC damper has two independent pickups. Therefore, the spectrum suppression at the betatron tune is about half. It would be close to 100% if only one pickup is used. The width of the gap determines the damping rate. Its asymmetry characterizes the damper phasing.

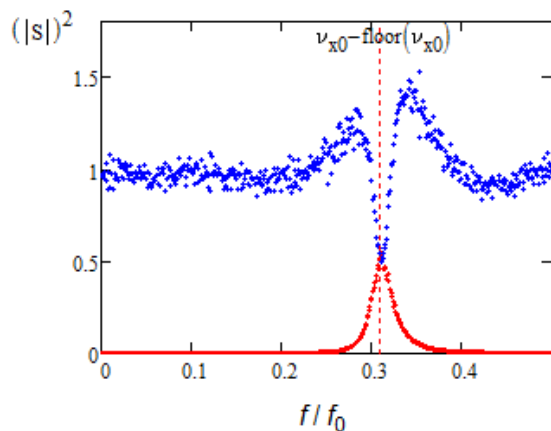


Figure 9: Spectral density of noise (blue) for two-BPM LHC damper; red dots – actual beam motion.

CONCLUSION

Next generation hadron colliders will have size significantly exceeding the LHC size. That will make them more susceptible to the external noise and will require dampers with damping time of few turns. Their maximum damping rate is limited by number of previous turns used for computation of each correction. It was shown that using large number of turns does not deliver any increase in damping efficiency but reduces the maximum achievable damping rate and makes damper more sensitive to the betatron tunes. Therefore, a usage of large number of turns is undesirable. It was also found that a reduction of system gain with frequency can be useful to reduce an effect of damper noise and, consequently, its heating power. Special care has to be applied to minimize an excitation of intrabunch HOMs which also can limit the damping rate.

REFERENCES

- [1] V. Lebedev and V. Shiltsev, Eds, *Accelerator Physics at the Tevatron Collider*. New York, NY, USA: Springer-Verlag, 2014.
- [2] P. Zhang, “A Study of Tunes Near Integer Values in Hadron Colliders”, Fermilab, Batavia, IL, USA, Rep. FERMILAB-FN-577, Dec. 1991.
- [3] G. Arduini *et al.*, “Hump: how did it impact the luminosity performance?”, in *Proc. 2010 Evian II Workshop on LHC Beam Operation*, Evian, France, 7-9 Dec. 2010.
- [4] V. Lebedev *et al.*, “Emittance growth due to noise and its suppression with feedback system in large hadron colliders”, *Particle Accelerators*, 1994, vol. 44, pp. 147-164.
- [5] V. Lebedev, “Computer simulation of the emittance growth due to noise in large hadron colliders”, *Particle Accelerators*, 1994, vol. 44, pp. 165-199.
- [6] W. Höfle *et al.*, “Suppression of Emittance growth by excited magnet noise with the transverse damper in LHC in simulation and experiment”, in *Proc. IPAC’11*, San Sebastian, Spain, Sep. 2011, paper MOP013, pp. 508-511.
- [7] A. Burov and V. Lebedev, “Coupling and its effect on beam dynamics”, in *Proc. 42nd ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams (HB’08)*, Nashville, TN, USA, Aug. 2008, paper WGA14, pp. 85-88.