EXPERIENCE AND PERSPECTIVE OF FFAG ACCELERATOR*

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Abstract

A number of Fixed Field Gradient(FFAG) Accelerator has been developed and built after the world first proton FFAG was developed at KEK in 1999. In this paper, the experiences of the operational FFAG accelerators mostly constructed in Japan and also, the perspective for high intensity beam with a novel scheme are described.

INTRODUCTION

An idea of fixed field alternating gradient (FFAG) accelerator was proposed by Ohkawa in 1953, After this, several electron models were developed at MURA in 19060's. In 1999, the world first proton FFAG model(pop FFAG) with rf acceleration shown in Fig. 1 was developed at KEK [1,2]. Since then, various types of FFAG accelerators have been developed and constructed.

The FFAG accelerators, which are fully operational at the moment, are mostly scaling type. The scaling type of FFAG accelerator has an unique feature where the beam focusing is zero-chromatic. This defeats the problems caused by the betatron resonances in the beam motions during acceleration, which could lead fast acceleration or even cw beam acceleration. The first proton model (pop FFAG) almost satisfied the zero chromatic constraint. However, the real machines, sometimes, this situation could not be perfectly satisfied because of the field defects and errors.

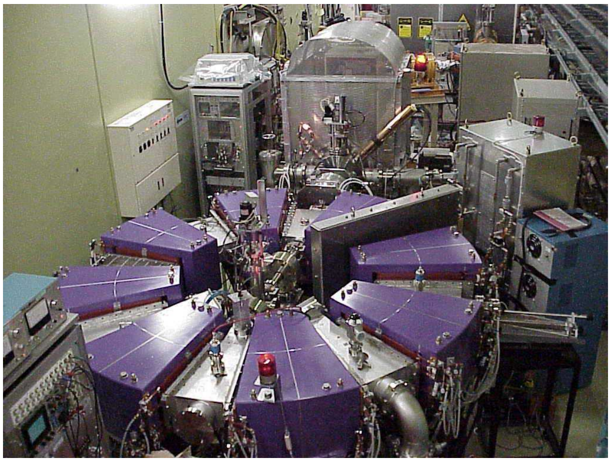

Figure 1: The first proton scaling FFAG model (pop FFAG) developed at KEK.

To overcome these problems practically, techniques of the betatron tune control and/or the fast resonance crossing should be needed.

Recently, we have proposed a new type of strong focusing ring accelerator, named "Harmotron"(Harmonictron) for high intensity beam acceleration [3].

The requirements in beam optics and behaviors of realizing a 100 MW class of beam power in medium energy hadron(proton/deuteron) accelerators should be as follows; (1) continuous wave (cw) beam acceleration, (2) strong beam focusing in 3D space, and (3) ease of beam extraction. There is no such circular hadron accelerator exists to satisfy these requirements so far . Only a linear accelerator can do.

On the other hand, "Hamotron" could satisfy all of these requirements. The Harmotron is based on a vertical scaling FFAG and, for beam acceleration, harmonic jump acceleration (HNJ) [4,5] is applied with constant rf frequency acceleration. The HNJ acceleration in vertical scaling FFAG, allows a strong phase focusing without having a transition energy because the momentum compaction is zero in the vertical scaling FFAG, and brings also a large turn separation at the highest energy to make beam extraction easier.

This paper presents the issues experienced in the operational FFAG accelerators and also gives the perspective of future high intensity FFAG, "Harmotron".

OPERATION OF SCALING FFAG

In the scaling FFAG accelerator, there are two types:one is a horizontal type and the other a vertical one. Each type has a different shape of the magnetic field configuration to satisfy the zero chromaticity. In the horizontal scaling FFAG,

$$B_y = B_0(R/R_0)^k, \qquad (1)$$

where k is a geometrical field index. Most of the present operational scaling FFAG accelerators are horizontal type.

The first proton model (pop FFAG), which is also a horizontal type, almost satisfied the zero chromatic constraint and the variation of betatron tunes during beam acceleration either for horizontal or vertical direction were less than 0.05 as shown in Fig. 2.

Since the field shapes in the real machine should not be perfect because the unexpected construction mistakes and errors happen, the betatron tunes are not always constant during beam acceleration.

In the 150MeV proton FFAG accelerators built at KEK and Kyushu Univ., the betatron tunes vary during beam acceleration as shown in Fig. 3. As can be seen from this figure, the betatron tunes cross two resonance lines of the normal third integer resonances: $3Q_x = 11$ and $Q_x - 2Q_y = 1$. On the other hand, the scaling FFAG with FDF lattice has a good tunability to control the betatron tunes. The vertical tune, in particular, can move largely by changing the magnetic field strength of F and D magnets (F/D ratio) .

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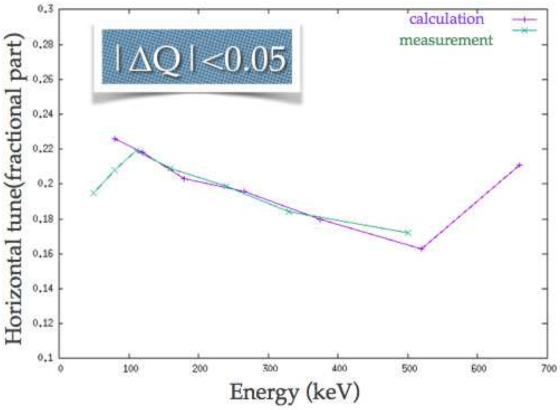

Figure 2: Measured horizontal tune variation of proton scaling FFAG model (pop FFAG) developed at KEK.

The vertical tune depends approximately on the F/D ratio (flutter) as follows.

$$Q_y = (-k^2 + F^2)^{1/2} : F^2 = \frac{1+\alpha^2}{1-\alpha^2}. \tag{2}$$

Here, α shows a F/D ratio and F is a flutter.

This scheme was used to avoid the resonance line of $Q_x - 2Q_y = 1$. As shown in Fig. 4, the vertical tune was slightly (+0.02) pushed up by decreasing the F/D ratio, so that the beam loss caused by crossing the resonance could be cured. Although the tune control with this technique is useful, there is a side effect. The closed orbits are changed when the F/D ratio(flutter) varies and displacements of the injection and extraction orbits occur. It requires sometimes harmful works to minimize the injection and extraction optics errors.

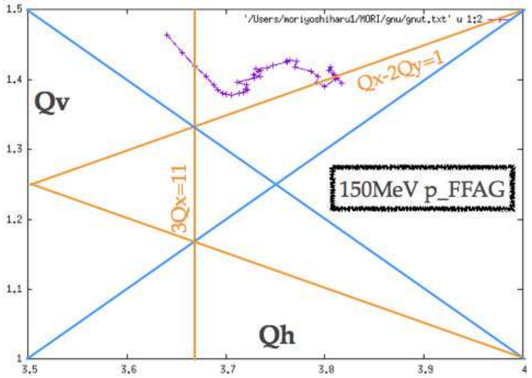

Figure 3: Tune variation of 150MeV proton scaling FFAG developed at KEK.

For the resonance of $3Q_x = 11$, it was difficult to avoid it with controlling the tunes, except increasing the injection energy. Thus, fast resonance crossing was only choice to overcome it. In crossing of the non-linear resonance, the emittance growth during crossing is determined by the crossing speed and the effects from the driving term of the non-linear forces and non-linear detuning, which presents an adiabatic parameter as shown below.

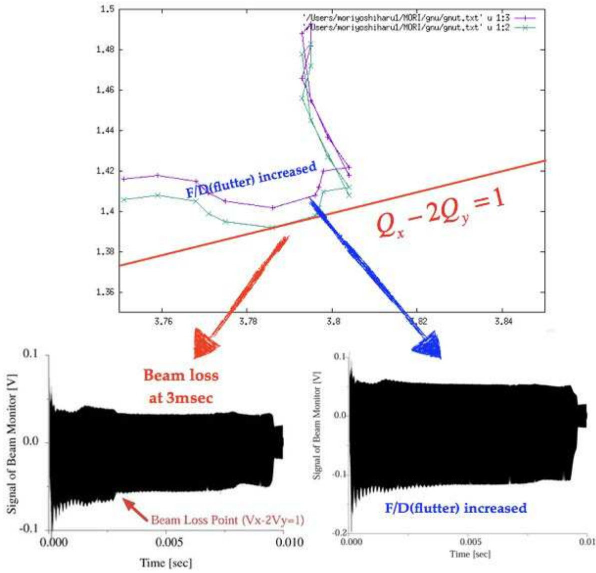

Figure 4: Tune control with F/D ratio(flutter) manipulation and curing the beam loss.

$$\alpha_a = \left(\frac{\epsilon}{4\pi\Delta_N\Delta_e}\right)^{\frac{3}{2}}. \tag{3}$$

Here, Δ_N is a non-linear tune shift(detuning) and Δ_e is an excitation width (driving term), ϵ is a change of tune per revolution (resonance crossing speed). If the parameter, α_a, is greater than 7 or so, the emittance growth caused by the resonance crossing could be eliminated [6] . In case of 150MeV proton FFAG shown in Fig.3, α_a»10, thus, there was no significant effect in crossing the resonance line of $3Q_x = 11$.

PERSPECTIVE FOR HIGH INTENSITY

There are some restricted requirements for beam behaviors and technical issues in realizing a future high intensity medium energy hadron accelerators. The requirements are,

(1) Continuous wave (cw) beam acceleration,
(2) Strong beam focusing in 3D space, and
(3) Ease of beam extraction.

In order to realize the cw beam acceleration, the guiding(focusing) magnetic field must be static and the rf frequency of rf acceleration should be constant. As for the beam focusing, alternating gradient(AG) focusing in transverse direction and phase focusing (synchrotron oscillation) in longitudinal direction allow the strong focusing in 3D space. In circular accelerator realizing the cw beam operation, a large turn separation is essential for making the beam extraction easier.

There is no circular accelerator to satisfy all these requirements so far. Cyclotron cannot satisfy the requirement of (2) because of no phase focusing in isochronous acceleration. Synchrotron is a pulse operated accelerator, which is impossible to accelerate the beam continuously. The fixed field alternating gradient (FFAG) accelerator cannot satisfy all these requirements, either. Directing the strong focusing in 3D space, constant rf frequency acceleration becomes impossible and a pulsed operation is inevitable like synchrotron. If the cw operation is aimed, then, the strong focusing in longitudinal direction must be given up just like cyclotron.

In order to overcome these difficulties, a new scheme of accelerator, named "Harmotron (Harmonicttron)", has been proopsed. The details are shown in our recent paper [3].

The Harmotron consists of a vertical scaling FFAG accelerator with harmonic number jump (HNJ) acceleration. The idea of the vertical scaling FFAG accelerator was originally proposed by Ohkawa [7] in 1955 and analyzed in detail by Brooks recently [8]. The feature that orbit radius is always constant means the zero-momentum compaction and no transition energy exists in the vertical scaling FFAG.

In the relativistic energy range, where particle velocity almost equals light velocity, a light mass particle such as the electron can be accelerated with constant frequency rf field in the vertical scaling FFAG accelerator. Thus, Ohkawa named it "electron cyclotron". Even non-relativistic particle such as proton can be accelerated by the rf field which synchronizes a time revolution elapsing around the ring for each turn as in the ordinary proton synchrotron. In the vertical scaling FFAG accelerator, the momentum compaction becomes always zero as mentioned above because the orbit radius is constant during acceleration (Fig. 5). Thus, the transition energy is infinite, in other words, no transition energy exists, and the beam is accelerated always below transition in the vertical scaling FFAG so that many problems caused by the transition energy can be avoided.

The idea of HNJ acceleration has been proposed byRuggiero in 2006 and developed recently [4, 5]. The scheme of HNJ acceleration, however, has some difficulties to accelerate heavy particles such as protons or deuterons for a wide range of medium (non-relativistic) energy because the transition energy exists where the slippage factor becomes zero. In order to eliminate the transition energy inherently, momentum compaction in beam dynamics must be zero like linear accelerator. A vertical scaling FFAG accelerator makes the momentum compaction zero because of a constant orbit radius whatever the beam energy.

The magnetic field strength changes exponentially in the vertical direction to keep a zero chromatic beam optics with constant orbit radius in the vertical scaling FFAG shown as,

$$B_y = B_0 exp(my). \tag{4}$$

Here, a characteristic number m is expressed with a field index, n, as,

$$m = n\rho. \tag{5}$$

The linearized particle motion in the transverse direction which is subject to a skew quadrupole magnetic field can be expressed by the betatron equations in skew coordinates under the approximation of no orbit curvature effect ($\rho \rightarrow$ large).

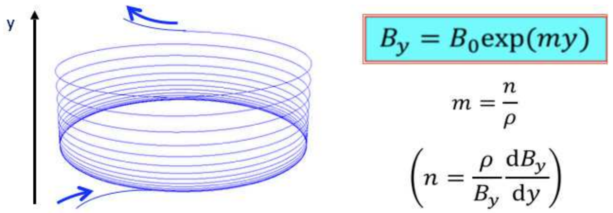

Figure 5: Schematic drawing of vertical FFAG and the magnetic field configuration for vertical direction.

The characteristic number of m specifies the orbit displacement, y_d, between initial momentum(p_i) and final beam momentum (p_e) as $m = (1/y_d)ln(p_e/p_i)$. If p_e/p_i equals 3 and y_d is less than 1 m, then, m should be more than 1.1 m^{-1}. The typical machine parameters of a vertical scaling FFAG accelerator which accelerates proton from 50 MeV to 500 MeV has been examined and the details are shown in the paper [3].

Applying HNJ acceleration to the vertical scaling FFAG, heavy particles such as the proton can be accelerated in wide range of the non-relativistic energies with a fixed frequency rf acceleration which allows a cw operation. We name this new type of accelerator based on vertical scaling FFAG with HNJ acceleration as "Harmotron".

From the synchronization condition of HNJ acceleration, the required energy gain to jump an integer harmonic number $\Delta_i h$ of harmonics between the turns i and i+1 can be expressed with [5] ,

$$E_{i+1} - E_i = \frac{\Delta_i h}{f_{rf}(\frac{dT}{dE})_{E=E_i}}. \tag{6}$$

Here, T is a revolution time of piecewise linearized around the particle energy.

The term dT/dE of required energy gain in Eq. (6) can be expressed in the following equation with a slippage factor.

$$\frac{dT}{dE} = \frac{\eta\gamma^2 C}{M_0 c(\gamma^2 - 1)^{3/2}}, \tag{7}$$

where C is the circumference of the ring, c is light velocity and M_0 is rest mass energy. Since the momentum compaction is zero in Harmotron (vertical scaling FFAG), Eq. (6) can be expressed as,

$$E_{i+1} - E_i = -\frac{\Delta_i h M_0 c(\gamma^2 - 1)^{3/2}}{f_{rf}}\frac{c}{C}. \tag{8}$$

As can be seen from this equation, the required energy gain per turn is a function of γ_i since f_{rf} is constant and $\Delta_i h$ should be a negative value for acceleration.

The rf voltage and/or phase have to be changed to satisfy the energy gain per turn shown in Eq. (8) in HNJ acceleration. A couple of schemes have been proposed to change the rf voltage or phase during acceleration by Ruggiero in his original paper [4], however, practical difficulties arise for realizing them.

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In HNJ acceleration of medium energy heavy particle, the energy change per each turn is so large that adiabatic condition in longitudinal focusing (synchrotron oscillation) may not be satisfied enough to keep within the large longitudinal beam acceptance. Thus, preserving the adiabatic condition of synchrotron oscillation during acceleration is important to keep a large phase space acceptance.

The criterion of adiabaticity for rf acceleration can be expressed as [9],

$$\Omega_s \gg \frac{1}{\Omega_s}\frac{d\Omega_s}{dt}, \tag{9}$$

where $\Omega_s/2\pi$ is a synchrotron frequency. When this condition is satisfied, the particles are well trapped by a rf bucket and accelerated. The above condition can be evaluated with the adiabatic parameter which is given by the following equation when the rf phase is constant π [10, 11].

$$n_{ad} = \frac{\Omega_s T_r}{1 - [V_i/(V_i + \Delta V)]^{1/2}}. \tag{10}$$

Here, V_i is the total rf voltage of i-th turn and ΔV is the increment of rf voltage derived by the rf cavity after i-th turn, T_r is a transit time of the rf cavity gap. The parameter, n_{ad}, counts the adiabaticity of the system showing how slow is the change of the bucket height with respect to the synchrotron frequency. When $n_{ad} \gg 1$, the system can be adiabatic.

The adiabatic condition in HNJ acceleration could be satisfied by distributing the multi rf cavities in the ring (see Fig. 6) and tuning the frequency of each rf cavity [5, 12]. If the rf system consists of N rf cavities, the adiabatic parameter shown in Eq. (10) becomes approximately $N/2$ times bigger than that for a single rf cavity.

The rf frequency of each rf cavity distributed homogeneously around the ring can be obtained with the following equation [5].

$$f_{i,j} = f_{ref}\left[1 + \frac{2j + N + 1}{2N}\frac{\Delta_i h}{h_i}\right]^{-1}, \tag{11}$$

where i is the turn number, j is the cavity number, h_i is a harmonic number and f_{ref} is a reference rf frequency.

As long as h_i is larger than its variation $\Delta_i h$, the frequency of each cavity is independent of the turn number and is approximately given as,

$$f_i \simeq f_{ref}[1 - (\frac{2j+1}{2N} + \frac{1}{2})\frac{\Delta_i h}{h_i}], \tag{12}$$

Thus, the rf frequency of each cavity is independent of the turn number and increases monotonically when $\Delta_i h$ is negative as a function of the cavity number. Moreover, if $h_i >> \Delta_i h$, then, f_i becomes f_{ref}.

When the rf voltage is constant, the rf phase in HNJ acceleration can be varied during beam acceleration. If the longitudinal adiabatic condition is satisfied during beam acceleration, the particles could be well captured by the rf bucket and accelerated around the stable phase.

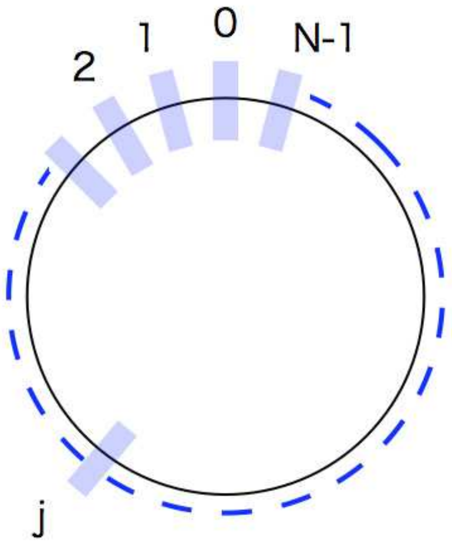

Figure 6: Adiabatic condition in rf acceleration can be introduced with multi rf cavities distributed around the ring, which allows to satisfy the adiabatic condition.

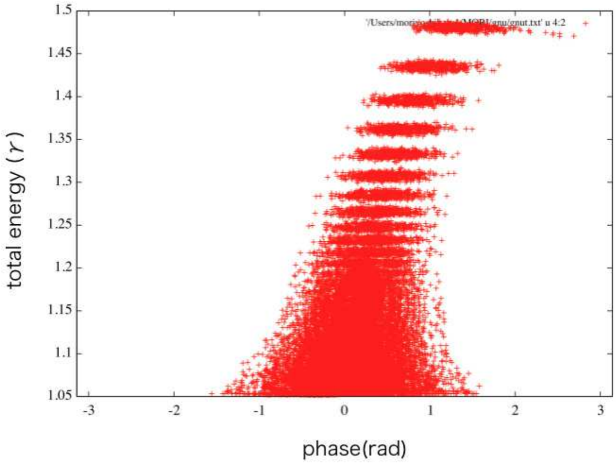

Figure 7: The beam tracking simulation results for the longitudinal beam motions for the phase spread of σ=1 rad at initial beams.

The beam tracking simulation results for the longitudinal beam motions for different phase spread initial beams are presented in Fig. 7. In this case, the number of rf cavities is 32 which are homogeneously distributed around the ring, and the rf voltage of 1.41 MV is constant during the beam acceleration. As can be seen from this figure, the particles are well captured and accelerated up to the maximum energy following the rf stable phase, and the phase acceptance at the beam injection is quite large, which is more than 70% of 2π. This means that an adiabatic beam capture process is fulfilled in the HNJ acceleration using many rf cavities with a small rf voltage which are distributed around the ring. The particles are captured adiabatically and well accelerated in a bucket with harmonic number jump.

SUMMARY

Since the world first proof of principle proton FFAG model was developed at KEK, various types of FFAG accelerators have been built. The operational FFAG accelerators are mostly scaling type and, in the real machines, practical techniques of manipulating the betatron tunes and/or the fast resonance crossing are needed because sometimes large magnetic field errors exist.

For future high intensity medium-energy hadron accelerator, a new scheme consisting of a vertical FFAG and harmonic number jump(HNJ) acceleration, named "Harmotron", has been proposed.

The Harmotron has a couple of unique features. Since no transition energy exists in Harmotron, a wide range of beam energy becomes possible with a monotonic change of harmonic number in HNJ acceleration. By keeping enough adiabaticity in longitudinal motions to capture and accelerate the particles efficiently by distributing many rf cavities around the ring, HNJ acceleration with a constant rf voltage becomes possible, so that the cw operation with large longitudinal acceptance can be realized.

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