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LONGITUDINAL DYNAMICS OF LOW ENERGY SUPERCONDUCTING LINAC*

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Abstract

The superconducting linac is composed of short independent cavities, and the cavity occupies only a small portion (1/4 to 1/6) of the machine compared with the normal conducting one. When phase advance per period is greater than 60 degrees, the smooth approximation is no longer valid and the longitudinal motion has to be described by time dependent system. With the help of Poincare map, the single particle nonlinear time dependent longitudinal motion is investigated. The study shows that when phase advance per period is less than 60 degrees, the system can be well described by smooth approximation, that means there is a clear boundary (separatrix) between stable and unstable area; when phase advance is greater than 60 degrees, the system shows a quite different dynamic structures and the phase acceptance is decreased significantly compared with the smooth approximation theory predicated, especially when phase advance per period is greater than 90 degrees. The results show that even for low current machine, the zero current phase advance should be kept less than 90 degrees to make sure there is no particle loss because of the shrink of the longitudinal acceptance.

INTRODUCTION

Keeping the zero current phase advance per period less than 90 degrees to avoid the envelope instability driven by space charge force has been widely accepted as one of the fundamental design principles of the high current linear accelerators [1], but for low current machine, should we still keep the zero current phase advance per period less than 90 degree? As the advance of the superconducting technology, more and more long pulse or continues wave ion accelerators adopt the superconducting acceleration structures just behind the RFQ because of their excellent properties, such as low AC power consumption, large beam tubes, great potential in terms of reliability and flexibility thanks to its independently-powered structures. The superconducting cavity can provide much higher acceleration field compared with the normal conducting one and can get higher acceleration efficiency, but at the same time the beam also suffers much stronger transverse defocusing from the higher electromagnetic field in the superconducting cavities, so there must be enough transverse focusing elements to confine the beam within the aperture, especially at low energy part, where it usually needs one focusing elements per cavity. However, the existence of the static magnetic field will increase the surface resistance of the superconducting cavity and may cause it to quench, so the cavity needs to be well screened from any static magnetic field, which makes it impossible to integrate the transverse focusing lens with the cavity just as the normal conducting Alvarez DTL cavity does. As a consequence, the focusing period length will be much larger than the normal conducting one. The long period length, high acceleration gradient to fully utilize the potential of the superconducting cavities and large synchronous phase for large acceptance, all these makes the zero current phase advance per period greater than 90 degrees. In this paper, we proposed a model that can describe the longitudinal motion of low energy superconducting linac properly, and the longitudinal motion of low energy superconducting linac is explored.

MODEL DESCRIPTION

The longitudinal motion in linac is usually described by the following equations [2],

$$w' = \frac{dw}{ds} = B(cos\phi - cos\phi_s) \tag{1}$$

$$\phi' = \frac{d\phi}{ds} = -Aw \tag{2}$$

where $w \equiv \delta\gamma = \frac{W-W_s}{mc^2}, A \equiv \frac{2\pi}{\beta_s^3\gamma_s^3\lambda}, B \equiv \frac{qE_0T}{mc^2}$.

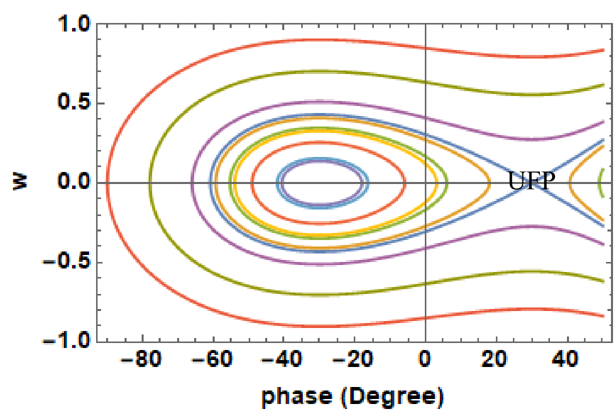

Figure 1: Phase portrait of smooth approximation longitudinal motion, the blue line is the separatrix.

The longitudinal motion equations are derived based on thin gap approximation and average in one period, they can also be directly derived from traveling wave approximation. When acceleration rate is small and the parameters A and B can be looked as constant, then the dynamics system described by equations (1) and (2) is time independent and integrable. The first motion constant is the energy or Hamiltonian of the system

$$\frac{A}{2}w^2 + B(sin\phi - \phi cos\phi_s) = H_\phi, \tag{3}$$

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and they are a good approximation of the longitudinal motion of the normal conducting linac, such as DTL structures. The most significant characteristic property of the system is that the trajectory of the particle in phase space is confined on the curve of (3), and the stable area is defined by the curve which pass the unstable fixed point, the boundary of the stable and unstable area is called separatrix. The stable and unstable area has a clear boundary and the phase portrait is shown in Fig. 1.

For the low superconducting accelerators, the lattice is shown in Fig. 2, the cavity filling factor which is defined as

$$\eta = L_c/L, \tag{4}$$

where L_c is the effective length of the cavity and L is the period length. Because of the existence of the long drifts between cavities, the cavity filling factor for low energy superconducting linac is very small, usually between 0.2 to 0.25 and the validation of the smooth approximation is questionable. We propose that the longitudinal motion in low energy superconducting linac can still be described by equation (1) and (2), while the parameter B is time dependent defined as,

$$\mathrm{B} = \begin{cases} \frac{qE_0T}{mc^2}, & 0 < s < \eta L \\ 0, & \eta L < s < L \end{cases}, \tag{5}$$

Then the system is time dependent nonlinear system.

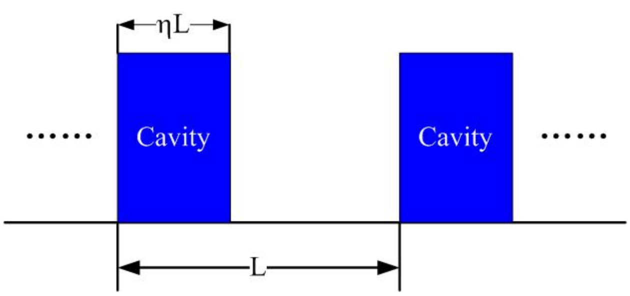

Figure 2: Longitudinal lattice structure.

LINEAR DYNAMICS

In order to investigate the time dependent nonlinear system, we first linearize the equation (2) at the stable fixed point $(\phi_s, 0)$ and the linear motion equation is

$$\mathrm{x}'' + k^2 x = 0, \tag{6-1}$$

and

$$k^2 = \begin{cases} -AB\sin\phi_s, & 0 < s < \eta L \\ 0, & \eta L < s < L \end{cases}. \tag{6-2}$$

where $x = \phi - \phi_s$. The lattice is equivalent to a periodic focusing channel composed by two elements, a solenoid and a drift space. By transform matrix, we can get the linear property of the system. The period transform matrix of the lattice is,

$$T = T_c T_d$$

where Tc and Td is the transform matrix of cavity and drift space, respectively and they are

$$T_d = \begin{pmatrix} 1 & (1-\eta)L \\ 0 & 1 \end{pmatrix}, \tag{7}$$

$$T_c = \begin{pmatrix} \cos\theta & \frac{L_c}{\theta}\sin\theta \\ -\frac{\theta}{L_c}\sin\theta & \cos\theta \end{pmatrix}. \tag{8}$$

and

$$\theta = \sqrt{k} L_c \tag{9}$$

is the focusing angle of the cavity. From the transform matrix of the system we can deduce the relation between phase advance per period σ and the main parameters of the focusing lattice,

$$\cos\sigma = \cos\theta - \frac{1}{2}\frac{1-\eta}{\eta}\theta\sin\theta \tag{10}$$

we see if the cavity filling factor is 1, that means the linac is composed by cavities just like DTL structure, then $\sigma = \theta$ and is obviously true. If filling factor is less than 1, then $\sigma > \theta$.

When $\sigma \ll 1$ and $\theta \ll 1$, we can get

$$\sigma = \theta/\sqrt{\eta} \tag{11}$$

which is equivalent to the lattice that is composed by cavity with filling factor 1 and the acceleration gradient ηE_0, that is the smooth approximation. In conclusion, when smooth approximation is valid, the phase advance per period is proportional to focusing angle of the cavity, and the proportional parameters is $1/\sqrt{\eta}$. The phase advance per period as function of cavity focusing angle is shown in Fig.3, where the dotted line is the relation of equation (11), i.e. the smooth approximation results and the solid line is the relation of equation (10). We can see when phase advance is greater than 60 degrees, the relation (10) and (11) shows obvious difference in case of filling factor is less 1. From the discussion above, we can conclude that the smooth approximation is only valid when phase advance is less than 60 degrees.

When acceleration gradient and length of the cavity is fixed, as the filling factor is decreased, the phase advance is increased quickly and which will impose an important limitation of the applicable cavity voltage for high current machine, where the phase advance should be less than 90 degrees. And with the filling factor decrease, the focusing strength is decreasing with sqrt of filling factor:

$$k_l = \frac{\sigma}{L} \approx \frac{\theta}{L_c}\sqrt{\eta}.$$

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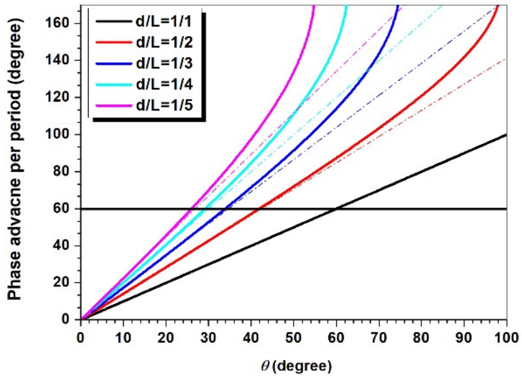

Figure 3: Phase advance per period as function of filling factor and focusing angle of cavity.

NONLINEAR DYNAMICS

If we define a vector as

$$\vec{x} = (\phi, w),$$

then equation (1) can be write as a vector differential equation

$$\frac{d\vec{x}}{ds} = f(\vec{x}, s). \tag{12}$$

The function f is periodic function of s with period of L. introducing the new variable t=s/L, system (12) transforms into the autonomous system

$$\begin{cases} \frac{d\vec{x}}{ds} = f(\vec{x}, tL) \\ \frac{dt}{ds} = 1/L \end{cases} \tag{13}$$

in dimensional 3. The flow in phase space intersect with the plane

$$t = n, n \in Integer,$$

and we project the intersection point onto the plane t=0, the trajectory of the intersect point will reveal the dynamics structure of the system, just as Fig. 4 shows.

By applying the method mentioned above, we can get some information of the dynamics structures of the system (13). The results are summarized as following:

1) The dynamics structure is directly depended on the phase advance per period. When phase advance is less than 60 degrees, the dynamics structures is identical with that of smooth approximation time independent one, i.e., there is a clear boundary between stable and unstable area, and the size of the stable area is exactly same as the time independent system, just as Fig. 5 shows. It also proves that smooth approximation is valid when phase advance is less than 60 degrees;

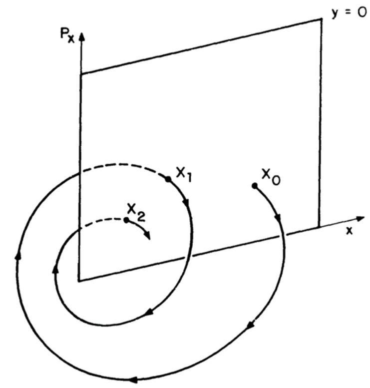

Figure 4: Poincare map in extended phase space.

2) As phase advance per period increasing, the stable area is shrinking, especially when $\frac{\sigma}{2\pi} = q/p$, where p and q are integers, the islands around the stable area appear and the strongly reducing the limit of stability around the origin. This can be explained as the existing of the high order fixed points [3]. The phase portrait with phase advance of 90 degrees, 110 degrees and 120 degrees are shown in Figs. 6-9;

3) When phase advance is 120 degrees, there are six third order fixed points, 3 of them are centre type, and 3 of them are saddle type. When phase advance is 120 degrees, the saddle type fixed points collides with the origin and the stable area shrinks to zero, i.e., there is no stable area in phase space.

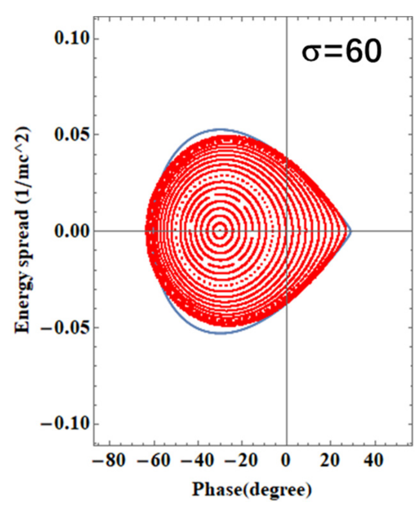

Figure 5: Phase portrait with phase advance 60 degrees.

PHASE ACCEPTANCE

We have calculated numerically the limit of stability of the time dependent system of longitudinal motion along phase axis by checking that the orbit remain bounded after 1000 iterates. The results are depicted in Fig. 9. We can the sudden shrink of the stable area at the rational tune. The phase acceptance remained constant for phase advance less than 60 degrees, then at around 70 degrees, the stable phase

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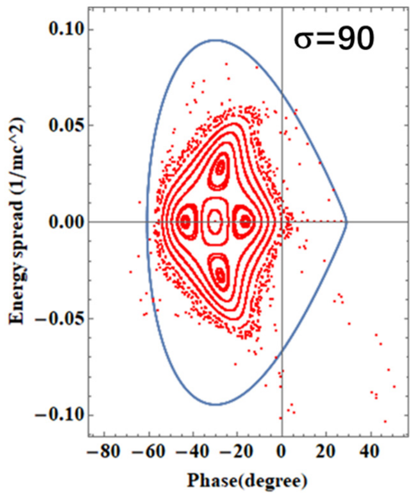

Figure 6: Phase portrait with phase advance 90 degrees.

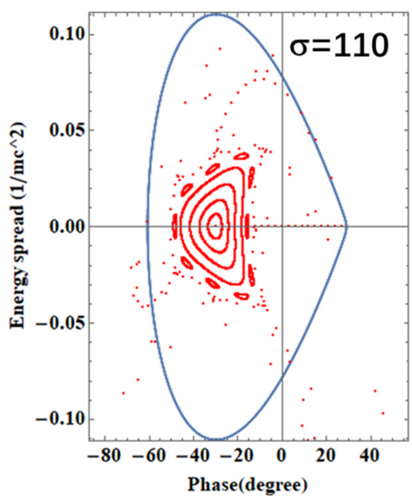

Figure 7: Phase portrait with phase advance 110 degrees.

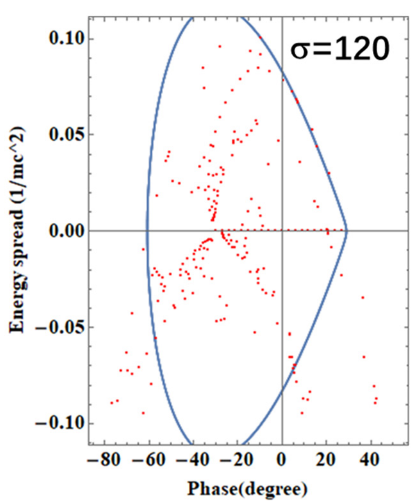

Figure 8: Phase portrait with phase advance 120 degrees.

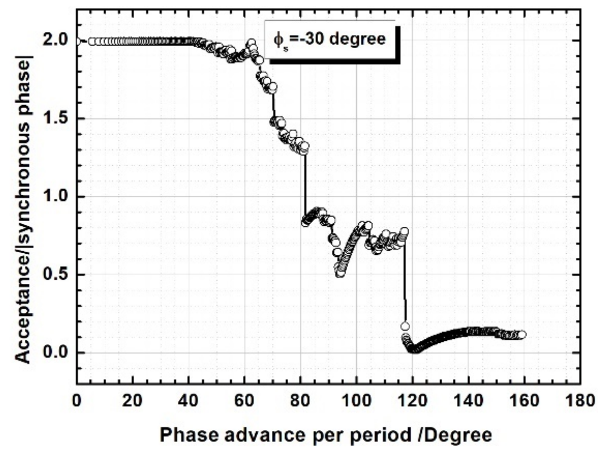

Figure 9: Phase acceptance as function of phase advance per period.

boundary decreases to about 1.5 times the synchronous phase, and at about 80 degrees, the boundary is decreased to about 0.8 times the synchronous phase, and when phase advance is greater than 120 degrees, the stable area is almost zero, and the motion becomes unstable universally. Further study shows, the aperture is only the function of phase advance, the trend is almost same for different synchronous phase

CONCLUSION

The longitudinal motion of linac composed with short independent cavities separated by long drifts should be described by the time dependent motion equation of (5), the smooth approximation is only valid when phase advance is less than 60 degrees. As phase advance increase, the stable area is shrinking and when phase advance is greater than 120 degrees, there is almost no stable area and the motion becomes unstable. This is very important in linac design especially at low energy and phase jump point, where phase acceptance is very critical and special attention should be payed to avoid the particle loss because of the small phase acceptance at these points.

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