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MICROBUNCHED ELECTRON COOLING (MBEC) FOR FUTURE ELECTRON-ION COLLIDERS*

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Abstract

The Microbunched Electron Cooling (MBEC) is a promising cooling technique that can find applications in future hadron and electron-ion colliders. In this paper we give a qualitative derivation of the cooling rate for MBEC and estimate the cooling time for the eRHIC electron-ion collider. We then argue that MBEC with two plasma amplification stages should be sufficient to overcome the emittance growth due to the intra-beam scattering in eRHIC.

INTRODUCTION

The idea of coherent electron cooling has been originally proposed by Ya. Derbenev [1] as a way to achieve cooling rates higher than those provided by the traditional electron cooling technique [2, 3]. The mechanism of the coherent cooling can be understood in a simple setup shown in Fig. 1. An electron beam with the same relativistic γ-factor as the hadron beam co-propagates with the hadrons in a section of length L_m called the "modulator". In this section, the hadrons imprint microscopic energy perturbations onto the electrons via the Coulomb force. After the modulation, the electron beam passes through a dispersive chicane section, $R_{56}^{(e)}$, where the energy modulation of the electrons is transformed into a density fluctuation referred to as "microbunching"[1]. Meanwhile, the hadron beam passes through its dispersive section, $R_{56}^{(h)}$, in which more energetic particles move in the forward direction with respect to their original positions in the beam, while the less energetic particles trail behind. When the beams are combined again in a section of length L_k called the "kicker", the electric field of the induced density fluctuations in the electron beam acts back on the hadrons. With a proper choice of the chicane strengths, the energy change of the hadrons in the kicker leads, over many passages through the cooling section, to a gradual decrease of the energy spread of the hadron beam.

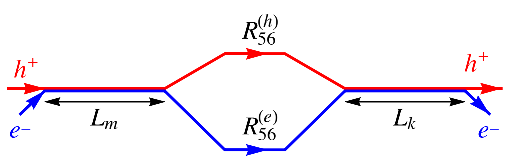

Figure 1: Schematic of the microbunched electron cooling system. Blue lines show the path of the electron beam, and the red lines indicate the trajectory of the hadron beam.

The transverse cooling is achieved in the same scheme by introducing dispersion in the kicker for the hadron beam.

In most cases, the cooling rate in the simple setup shown in Fig. 1 is not fast enough for practical applications. It can be considerably increased if the fluctuations in the electron beam are amplified on the way from the modulator to the kicker. Litvinenko and Derbenev proposed to use for this purpose the gain mechanism of the free electron laser (FEL) [5]. While this may be sufficient for some applications, one of the drawbacks of this approach is a narrow-band nature of the FEL amplifier that may not provide enough gain before the amplified signal saturates [6]. Following an earlier study by Schneidmiller and Yurkov [7] of microbunching dynamics for generation of coherent radiation, Ratner proposed a broadband amplification mechanism [8] in which the amplification is achieved through a sequence of drifts and chicanes such that the density perturbations in the drifts execute a quarter-wavelength plasma oscillation. In a recent paper [9], Litvinenko and co-authors put forward an idea to use a parametric instability in the electron beam causes by a periodic variation of the transverse size of the beam when it propagates through the cooling system.

In this paper, using order of magnitude estimates, we first derive a formula for the cooling rate in the system shown in Fig. 1. We then estimate the cooling rate for the parameters of eRHIC and show that the simple setup of Fig. 1 does not provide a sufficient cooling rate for the electron-ion collider without amplification in the electron channel. Finally, we estimate the amplification through a quarter-period plasma oscillation and argue that two plasma amplification stages should be enough to make the cooling time in eRHIC below one hour.

We use the Gaussian system of units throughout this paper.

QUALITATIVE DERIVATION OF MBEC COOLING RATE

For the hadron-electron interaction we adopt a model in which the interaction is treated as if a hadron were a disk of charge Ze with an axisymmetric Gaussian radial distribution of the rms transverse size Σ. The electron is also modeled by a Gaussian disk of charge $-e$ with the same transverse profile. A similar Gaussian-to-Gaussian interaction model was used in 1D simulations of a longitudinal space charge amplifier in Ref. [10].

The interaction between two charged slices of transverse size $\sim \Sigma$ is efficient only if they are close to each other. If the distance between them is smaller than $\Delta z \lesssim \Sigma/\gamma$, where γ is the Lorentz factor, the electric field of a hadron of charge Ze can be estimated as Ze/Σ^2, and the interaction force between an electron and a hadron is $\sim Ze^2/\Sigma^2$. For

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[1] In a long modulator section the microbunching can be generated directly in the modulator when the energy modulation is converted into a density fluctuation through plasma oscillations [4].

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$\Delta z \gtrsim \Sigma/\gamma$, the interaction force decays as $\sim Ze^2/\Delta z^2\gamma^2$. So for estimates we will assume that, for a given hadron, the dominant contribution to the cooling comes from electrons located within the distance

$$\Delta z \sim \frac{\Sigma}{\gamma}. \qquad (1)$$

We use the notation η for the relative energy deviation $\Delta E/E_0$ where E_0 is the nominal energy of the beam. Using the interaction force Ze^2/Σ^2, a relative energy modulation η_e induced by a hadron in the modulator of length L_m can be estimated as

$$\eta_e \sim \frac{Ze^2}{\Sigma^2} L_m \frac{1}{\gamma m_e c^2} \sim \frac{cZeL_m}{\gamma\Sigma^2 I_A}, \qquad (2)$$

where $I_A = m_e c^3/e = 17$ kA is the Alfvén current. Here we assume that a hadron and an electron do not shift longitudinally during the interaction on the length L_m.

The energy perturbation (2) is converted into a density perturbation when the electron beam passes through the chicane $R_{56}^{(e)}$. The optimal value of $R_{56}^{(e)}$ is found from the requirement that the electrons are longitudinally shifted in the chicane by the interaction distance $\sim \Delta z$:

$$R_{56}^{(e)} \sim \frac{\Delta z}{\sigma_\eta^{(e)}} \sim \frac{\Sigma}{\sigma_\eta^{(e)}\gamma}, \qquad (3)$$

where $\sigma_\eta^{(e)}$ is the rms relative energy spread in the electron beam. Electrons whose energy is perturbed by η_e due to the interaction with a hadron will have an additional shift $\delta z \sim R_{56}^{(e)}\eta_e$, and this will cause a density perturbation of the order of

$$\delta n_e \sim \frac{\delta z}{\Delta z} n_e \sim \frac{\eta_e}{\sigma_\eta^{(e)}} n_e. \qquad (4)$$

Here δn_e and n_e refer to the number of electrons in the beam per unit length. This density perturbation creates an electric field in the kicker,

$$E \sim e\delta n_e \Delta z \sim e\frac{\eta_e}{\sigma_\eta^{(e)}}\frac{\Sigma}{\gamma} n_0. \qquad (5)$$

With the optimal choice of the value of the hadron chicane, $R_{56}^{(h)} \sim \Delta z/\sigma_\eta^{(h)} \sim \Sigma/\sigma_\eta^{(h)}\gamma$, where $\sigma_\eta^{(h)}$ is the rms relative energy spread in the hadron beam, the hadron energy change in the kicker, $\sim ZeEL_k$, works against the hadron beam energy spread. This gives the following estimate for the inverse cooling time expressed in the revolution periods,

$$N_{\rm cool}^{-1} \sim \frac{ZeEL_k}{\gamma m_h c^2 \sigma_\eta^{(h)}} \sim \frac{ZeL_k}{\gamma m_h c^2 \sigma_\eta^{(h)}} e \frac{\eta_e}{\sigma_\eta^{(e)}}\frac{\Sigma}{\gamma} n_e \sim \frac{(Ze)^2 L_k L_m}{\gamma^3 m_h c^2 \Sigma^3 I_A \sigma_\eta^{(e)}\sigma_\eta^{(h)}} ecn_e\Sigma^2. \qquad (6)$$

Replacing $ecn_e\Sigma^2$ in this formula by the electron beam current I_e and using the notation $r_h = (Ze)^2/m_h c^2$ for the classical hadron radius, we arrive at the following result:

$$N_{\rm cool}^{-1} \sim \frac{L_k L_m I_e r_h}{\gamma^3\Sigma^3 I_A \sigma_\eta^{(e)}\sigma_\eta^{(h)}}. \qquad (7)$$

As we will see below, this estimate, within a numerical factor, agrees with the result of an accurate theoretical analysis.

RESULTS OF RIGOROUS THEORETICAL ANALYSIS OF THE PROBLEM

A rigorous theoretical analysis of the cooling rate in the model of MBEC outlined in the Introduction was carried out in Ref. [11]. Here we present the main results of that analysis.

As was already mentioned, for the Coulomb interaction of beam particles we used a model of Gaussian slices assuming the transverse charge distribution with the rms size Σ. The longitudinal Coulomb force between two such slices located at distance z is given by the following formula:

$$F_z(z) = -\frac{Ze^2}{\Sigma^2}\Phi\left(\frac{z\gamma}{\Sigma}\right). \qquad (8)$$

The plot of function Φ is shown in Fig. 2. This function

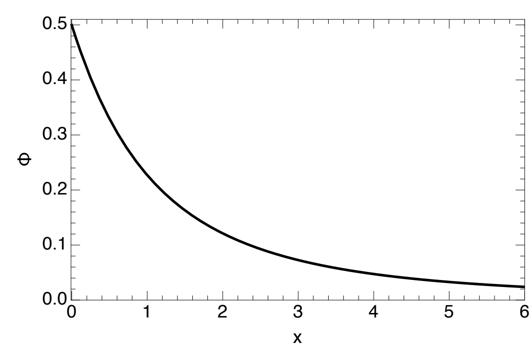

Figure 2: Function $\Phi(x)$ for positive values of the argument.

is odd, and for a negative argument $x < 0$ it is defined by $\Phi(-x) = -\Phi(x)$. The value of this function at the origin is $\Phi(0^+) = \frac{1}{2}$; for $x \gg 1$, we have $\Phi(x) \approx 1/x^2$. For a quick numerical evaluation of this function we found the following interpolation formula,

$$\Phi(x) \approx b\frac{e^{-ax}+1}{2+dx+bx^2}, \qquad (9)$$

with $a = 1.60081$, $b = 0.499606$ and $d = 0.14579$.

Using this model for the particle interaction, in Ref. [11], we derived the following expression for cooling time $N_{\rm cool}$ evaluated in terms of the revolution periods:

$$N_{\rm cool}^{-1} = \frac{4}{\pi} F \frac{I_e r_h L_m L_k}{\Sigma^3\gamma^3 I_A \sigma_\eta^{(h)}\sigma_\eta^{(e)}}, \qquad (10)$$

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where the form-factor F depends on the strength of the chicanes $R_{56}^{(h)}$ and $R_{56}^{(e)}$. Analysis shows that for the optimal cooling the ratios $q_e = R_{56}^{(e)}\sigma_\eta^{(e)}\gamma/\Sigma$ and $q_h = R_{56}^{(h)}\sigma_\eta^{(h)}\gamma/\Sigma$ should be made equal, $q_e = q_h = q$; in this case the form-factor F depends only on the parameter q. The plot of the function $F(q)$ is shown in Fig. 3.

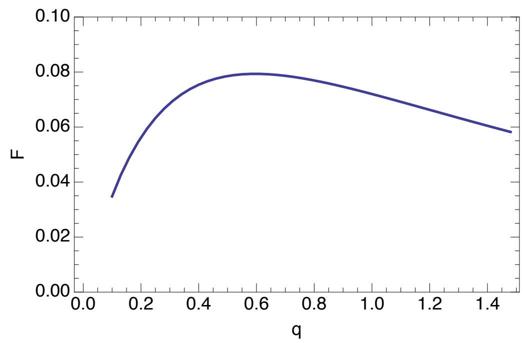

Figure 3: Plot of function $F(q)$ versus q.

We see that the maximum value of function F is reached at $q = 0.6$ and is equal to 0.079. Substituting this value into Eq. (10) we find for the optimized cooling rate:

$$N_{\text{cool}}^{-1} = \frac{0.10}{\sigma_\eta^{(h)}\sigma_\eta^{(e)}}\frac{1}{\gamma^3}\frac{I_e}{I_A}\frac{r_h L_m L_k}{\Sigma^3}. \qquad (11)$$

This formula has the same structure as Eq. (7), with an additional numerical factor 0.1. At the optimal cooling rate the chicane strengths are

$$R_{56}^{(e)} = 0.6\frac{\Sigma}{\sigma_\eta^{(e)}\gamma}, \qquad R_{56}^{(h)} = 0.6\frac{\Sigma}{\sigma_\eta^{(h)}\gamma}. \qquad (12)$$

Because the cooling rate (11) depends on the local electron beam current I_e that varies within the electron bunch, one has to average Eq. (11) taking into account the finite electron bunch length which we denote by $\sigma_z^{(e)}$. Assuming a Gaussian current distribution in the electron beam, $I_e = [Q_e c/\sqrt{2\pi}\sigma_z^{(e)}]\exp[-z^2/2(\sigma_z^{(e)})^2]$, where Q_e is the electron beam charge, it is straightforward to calculate that the average electron current a hadron feels over many passages through the electron beam is equal to

$$\bar{I}_e = \frac{Q_e c}{\sqrt{2\pi}[(\sigma_z^{(e)})^2 + (\sigma_z^{(h)})^2]^{1/2}}. \qquad (13)$$

For an electron beam several times shorter than the hadron one, we can neglect $\sigma_z^{(e)}$ in this formula in comparison with $\sigma_z^{(h)}$. In this limit, replacing I_e in Eq. (11) by $\bar{I}_e$, we obtain for the cooling rate

$$N_{cool}^{-1} = 0.10\frac{1}{\gamma^3\sigma_\eta^{(h)}\sigma_\eta^{(e)}}\frac{Q_e c}{\sqrt{2\pi}\sigma_z^{(h)} I_A}\frac{r_h L_m L_k}{\Sigma^3}. \qquad (14)$$

ESTIMATES FOR ERHIC COLLIDER

As a numerical illustration of the general theory presented in the previous sections we will estimate the optimized cooling rate for the nominal parameters of the electron-hadron collider eRHIC [12]. The parameters of the proton beam in eRHIC and of the electron beam in a possible MBEC cooling system are given in Table 1. Substituting these parameters into Eq. (14) gives for the cooling time

$$N_c = 1.5 \times 10^{10}, \qquad (15)$$

which, with the revolution period in the RHIC ring of 13 μs, corresponds to 51.5 hours. The optimal parameters of the electron and proton chicanes are $R_{56}^{(h)} = 0.41$ cm and $R_{56}^{(e)} = 2.4$ cm. Of course, such a long cooling time is not sufficient for the eRHIC collider, where the intra-beam scattering (IBS) time scale for the emittance doubling is estimated in the range of 2 hours. We conclude that a simple setup shown in Fig. 1 needs to be augmented by some kind of amplification in the electron channel, as mentioned in the Introduction.

Table 1: Parameters of the eRHIC Collider with a Hypothetical MBEC Cooling Section

Parameter	Value
Proton beam energy	275 GeV
RMS length of the proton beam, $\sigma_z^{(h)}$	5 cm
RMS relative energy spread of the proton beam, $\sigma_\eta^{(h)}$	6×10^{-4}
Peak proton beam current, I_h	23 A
RMS transverse size of the beam in the cooling section, Σ	0.7 mm
Electron beam charge, Q_e	1 nC
RMS relative energy spread of the electron beam, $\sigma_\eta^{(e)}$	1×10^{-4}
Modulator and kicker length, L_m and L_k	40 m

Our assumption that the hadron-electron interaction results only in the energy perturbation of electrons in the modulator, and not their density, is justified if plasma oscillations in the electron beam can be ignored. Plasma oscillations convert energy perturbations in the beam into density modulations and vice versa in a quarter of the plasma wavelength λ_p, so these effects can be ignored if $\frac{1}{4}\lambda_p$ is much larger than the modulator and kicker lengths. To estimate $\frac{1}{4}\lambda_p$ in the electron beam we can use the following formula, (see, e.g., Ref. [7]),

$$\frac{1}{4}\lambda_{\text{pl}} \sim \gamma^{3/2}\Sigma\sqrt{\frac{I_A}{I_e}}. \qquad (16)$$

Substituting parameters from Table 1 in this formula, we find that $\frac{1}{4}\lambda_p \lesssim L_m, L_k$ if the electron beam current is limited by $I_e \lesssim 30$ A, which for the given electron beam charge of 1 nC imposes a constrain on the electron bunch length, $\sigma_z^{(e)} \gtrsim 4$ mm.

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AMPLIFICATION BY PLASMA OSCILLATIONS IN THE BEAM

To increase the cooling rate, one can add amplification stages in the electron channel [8] as shown in Fig. 4. One

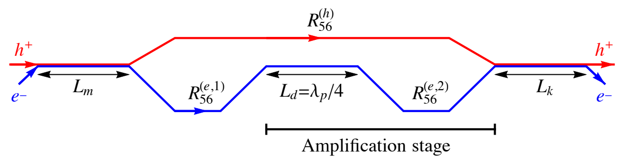

Figure 4: MBEC cooler with one plasma amplification stage.

stage consists of a drift of $\frac{1}{4}$ of the period of the plasma oscillations in the beam followed by another chicane with the dispersion strength $R_{56}^{(e,2)}$.

The mechanism of the plasma amplification can be understood as follows. A perturbation of the electron beam density generated after the chicane $R_{56}^{(e,1)}$ (see Fig. 4), through the Fourier transformation, can be decomposed into sinusoidal density perturbations in the electron beam. Each such perturbation propagating through the drift executes a quarter-wavelength plasma oscillation, which converts the initial density perturbation into a sinusoidal energy modulation. When the beam passes through the chicane $R_{56}^{(e,2)}$ the energy modulation is converted back into a density perturbation with an amplitude that can be larger than the initial one [7, 10]. We can easily estimate the gain factor G of one plasma cascade.

In a cold beam with a sinusoidal plasma oscillation with the wavenumber k the energy perturbation η is related to the density perturbation δn_e by the following relation,

$$\eta \sim \frac{k_b\sqrt{\gamma_e}}{k}\frac{\delta n_e}{n_e}, \tag{17}$$

where $k_b = \sqrt{4\pi r_e n_e/\Sigma^2}$. An initial density perturbation δn_e after a quarter of plasma period is converted to an energy perturbation given by this equation. The chicane $R_{56}^{(e,2)}$ at the end of the amplification stage then converts this energy modulation into a density one with the amplitude

$$\delta n_e' \sim \frac{\eta}{\sigma_\eta^{(e)}} n_0 \sim \frac{1}{\sigma_\eta^{(e)}}\frac{k_b\sqrt{\gamma_e}}{k}\delta n_e. \tag{18}$$

We will use for the wavenumber k the characteristic value $1/\Delta z$ of the interaction distance, $k \sim 1/\Delta z \sim \gamma/\Sigma$. We then find for the amplification factor $G = \delta n_e'/\delta n_e$,

$$G \sim \frac{1}{\sigma_\eta^{(e)}}\frac{k_b\sqrt{\gamma_e}}{k} \sim \frac{1}{\sigma_\eta^{(e)}}\sqrt{\frac{I_e}{\gamma I_A}}. \tag{19}$$

The last equation agrees with the result of Ref. [7]. For the parameters from Table 1 and $I_e = 10$ A we find the amplification factor $G \approx 14$ and hence two amplification stages (that is a chicane-drift-chicane-drift-chicane configuration in the electron channel) should be enough to lower the cooling time below 1 hour limit.

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