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IBS NEAR TRANSITION CROSSING IN NICA COLLIDER

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Abstract

Intrabeam scattering (IBS) of charged particles in a particle beam results in an exchange of energy between different degrees of freedom. That results in an increase of average energy of particles in the beam frame and an increase of the 3D-emittance. The paper considers calculations of beam emittance growth rates for different options of NICA collider and IBS effects in close vicinity of the transition.

INTRODUCTION

Intrabeam scattering (IBS) is a Coulomb scattering of charged particles in a beam. It causes an exchange of energy between various degrees of freedom resulting in an increase of average energy of particles in the beam frame and an increase of the total beam emittance in the 6D phase space.

Anton Piwinski was first who derived equations describing IBS [1]. These equations neglect derivatives of the beta-functions and therefore, strictly speaking, are accurate for rings with "smooth focusing" only. They also represent a good approximation for weak focusing rings. Later, Bjorken and Mtingwa derived equations applicable to the general case [2] where the motion in the transverse planes is still considered being uncoupled. These equations were rederived in Ref. [3] which derivation is based on the Landau kinetic equation [4] and the IBS rates were expressed through symmetric elliptic integrals [5]. This work also showed how the equations can be extended to the case of motion coupled in all three degrees of freedom [5]. These results were used in the calculations of the IBS growth rates for different lattice options of the NICA collider [6] during work on its conceptual design [7]. The maximum energy of the stored ions in the collider is close to the transition energy ($\gamma \approx 5.8$, $\gamma_{tr} \approx 7.1$). That required a detailed study of the IBS phenomena near transition. The obtained results had essential influence on the final lattice design and major parameters of the collider.

IBS GROWTH RATES

We introduce two coordinate systems. The first one is the standard local coordinate frame (LF) for a ring (x, θ_x, y, θ_y, s θ_s,); and the second one is the beam frame (BF) moving with the beam where additionally the axes are rotated to coincide with the axes of 6D beam ellipsoid in the 6D phase space (x, v_x, y, v_y, z, v_z). For both systems we assume that the center of the system coincides with the beam center of "gravity".

Sequence of major steps used for an estimation of the IBS heating rates is shortly summarized as following:

- using known Twiss parameters and the emittances find 6D particle ellipsoid in the BF and its rotation relative to the LF;
- calculate along the ring the growth rates for rms velocities in the BF;
- convert the velocity growth rates to the emittance growth rates in the LF;
- average the obtained data over entire machine circumference to obtain the overall IBS rates.

IBS FOR SMOOTH FOCUSING BELOW AND ABOVE TRANSITION

To understand the how the IBS works let us consider first the IBS in vicinity of the transition energy in the smooth optics for unbunched beam. In this approximation we assume that:

- Twiss parameters are constant along the ring
- vertical dispersion D_y=0
- L is the ring circumference, which in the case of bunched beam is related with the rms bunch length by following equation: $\sigma_z \to L/\left(2\sqrt{\pi}\right)$.

Using formalism of Ref. [3] we obtain the matrix of velocity second moments' in the BF:

$$\Sigma_v = \gamma \cdot \beta \cdot c \cdot \begin{pmatrix} \theta_x^2 & 0 & 0 \\ 0 & \theta_y^2 & 0 \\ 0 & 0 & \theta_p^2 \end{pmatrix},$$

where

$$\sigma_x = \sqrt{\varepsilon_x \beta_x + \sigma_p^2 D^2} \quad \sigma_y = \sqrt{\varepsilon_y \beta_y} \quad \theta_{x,y}^2 = \frac{\varepsilon_{x,y}}{\beta_{x,y}} \quad \theta_p^2 = \sigma_p^2 \frac{\varepsilon_x \beta_x}{\gamma \sigma_x^2}$$

Accounting that there is no rotation of the BF relative to the LF and performing transition from the BF to the LF we obtain the emittance growth rates:

$$\frac{d}{dt}\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \sigma_p^2 \end{pmatrix} = \frac{\sqrt{\pi}}{2\sqrt{2}} \cdot \frac{e^4 N L_c}{M^2 c^3 \sigma_x \sigma_y L \beta^3 \gamma^5 \sqrt{\theta_x^2 + \theta_y^2 + \theta_p^2}} \begin{pmatrix} \beta_x \Psi_{IBS}(\theta_x, \theta_y, \theta_p) + \gamma^2 \frac{D^2}{\beta_x} \Psi_{IBS}(\theta_p, \theta_x, \theta_y) \\ \beta_y \Psi_{IBS}(\theta_y, \theta_p, \theta_x) \\ 2\gamma^2 \Psi_{IBS}(\theta_p, \theta_x, \theta_y) \end{pmatrix}$$

where e is the ion electrical charge, N is the number of ions in the beam, L_c is the Coulomb logarithm, M is the ion mass, c is the speed of light, β and γ are the relativistic factors, D is the horizontal dispersion, and the functions $\Psi_{IBS}(\ldots)$ are expressed through the symmetric elliptic integral of the second kind [5].

In the equilibrium all "local temperatures" are equal (Θ_x=Θ_y=Θ_p) and do not change with time. That yields:

$$\Psi_{IBS}(\theta_x, \theta_y, \theta_p) = \Psi_{IBS}(\theta_y, \theta_p, \theta_x) = \Psi_{IBS}(\theta_p, \theta_x, \theta_y) = 0 .$$

Consequently, this requires:

$$\frac{\varepsilon_x}{\beta_x} = \frac{\varepsilon_y}{\beta_y} = \frac{\sigma_p^2}{\gamma^2} \cdot \frac{\varepsilon_x \beta_x}{\varepsilon_x \beta_x + \sigma_p^2 D^2} .$$

That yields:

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$$\frac{\sigma_p^2}{\gamma^2} = \frac{\varepsilon_x}{\beta_x} \frac{1}{1 - \frac{\gamma^2}{\gamma_{tr}^2}}$$

where we took into account that for smooth optics $D=R/Q_x^2$, $\beta=R/Q_x$, and R is the ring radius. The last equation can be fulfilled only below the transition energy when

$$1 - \frac{\gamma^2}{\gamma_{tr}^2} > 0 \ .$$

This means that above the transition the beam cannot be at the thermal-equilibrium and its 6D-emittance will grow for any combination of beam emittances. However below the transition there is a state where a thermal equilibrium exists and the beam emittances are conserved.

In a real machine optics, where the Twiss parameters change along the ring and the equilibrium does not exist, the beam still may be in a quasi-equilibrium below transition where a temperature exchange between degrees of freedom is absent and the overall emittance growth is suppressed: closer the beam optics is to the smooth optics smaller the emittance growth rates are.

IBS STUDY FOR NICA COLLIDER

The Nuclotron-based Ion Collider fAcility (NICA) [6] is under construction at the JINR. It is aimed to the collider experiments with ions and protons and has to provide the ion-ion (Au^{+79}) collisions in the energy range of 1÷4.5 GeV/nucl as well as the proton-ion and polarized proton-proton and collisions. Two collider rings of 503 m circumference are designed to achieve the required luminosities at two interaction points. Each ring has a racetrack shape with two bending arcs and two long straight sections.

A collider lattice development overcame several iterations which took into account an interdependency of major machine parameters. The IBS is one of the major phenomena limiting the luminosity lifetime. It determines the requirements to the beam cooling systems, and thus has a direct influence on the machine operation scenario. An IBS study was performed [7] in the course of optimization of the NICA collider lattice structure.

In transition from the smooth to strong focusing optics one has to average the local IBS rates over the ring:

$$Rate = \sum_i \frac{rate_i \, ds}{C_{ring}}$$

where $rate_i$ is the local rate of the emittance (ε_x, ε_y or σ_p^2) growth in a sufficiently short length element i where the Twiss parameters may be considered constant. The quasi-equilibrium point for an optics structure is defined by the equality of these averaged IBS rates $\tau_{IBSx} = \tau_{IBSy} = \tau_{IBSp}$.

Initially, the IBS rates were calculated for pure FODO- and ODFDO- (triplet) lattices for the beam energy of 4.5 GeV/nucl. The following conditions were assumed:

- a storage ring consists of from FODO- or ODFDO-cells only (see Figure 1)
- the ring circumference is fixed
- the bunch length and the number of the particles in the beam is fixed
- for a fixed horizontal emittance ε_y and σ_p are adjusted to obtain equal growth rates in all degrees of freedom.

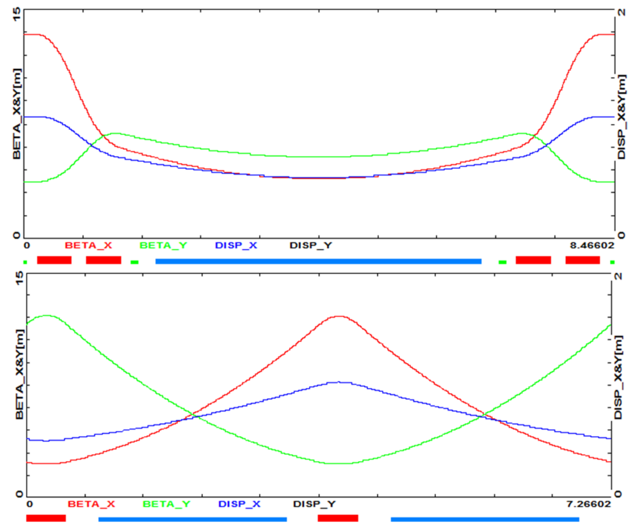

Figure 1: Structures of regular ODFDO- (top) and FODO- (bottom) cells used in the IBS rates calculations.

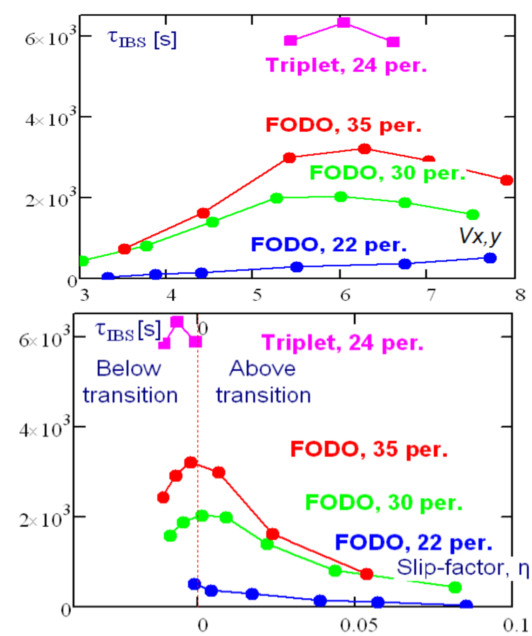

Figure 2: The beam size growth times due to IBS versus ring betatron tune (top), the same data presented as a function of the slip-factor landscape (bottom). The transition (η=0) is marked by vertical dashed line, where η- is the ring slip-factor.

One can see in Figure 2 that:

- the triplet optics yields about 2 times larger IBS-time than the best of considered FODO optics;
- the minimal beam heating is achieved near transition.

Thus, for “ideal” storage ring without interaction points (IPs) the triplet optics looks preferable from the IBS point of view.

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Then, the entire ring optics including IPs (Figure 3) was investigated.

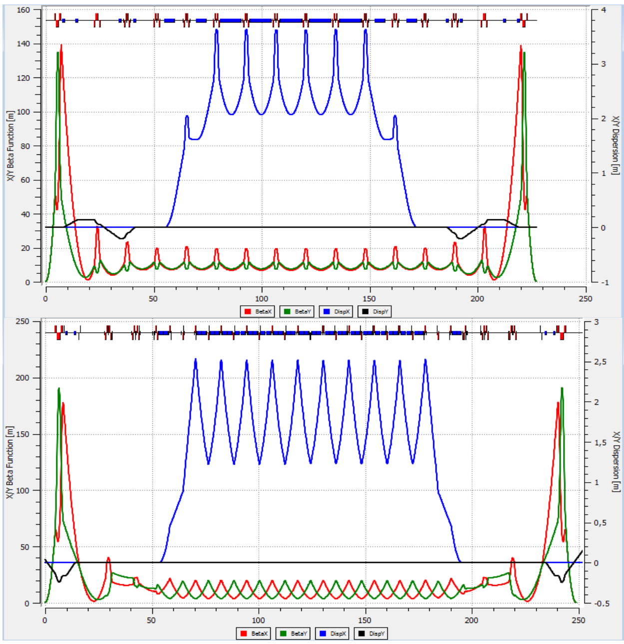

Figure: 3 Collider ring optics (with IPs): ODFDO- (above) and FODO- (below) used for IBS calculations.

A transition from the ideal ring to the collider optics with low-β straight sections increases β-functions in the IP vicinity. That yields an increase of IBS rates.

Finally, the collider ring lattice based on FODO-cells has only ~1.5 times larger rates: the growth time of ~890 s versus ~1350 s for the luminosity of $6{\cdot}10^{27}$ cm^{-2}s^{-1}.

The contribution to 6D IBS heating for ODFDO structure (Figure 4) is dominated by straight sections of the ring. The local contribution has the maximum value in the collision point.

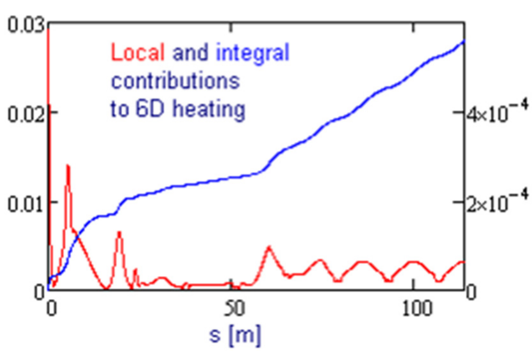

Figure 4: Local (red) and integral (blue) contributions to 6D IBS heating for a quarter of ODFDO structure.

Change of β^* leads to an additional shift out from the quasi-equilibrium point. The IBS times dependence on β-function value at the IPs for triplet and FODO optics are presented in Figure 5.

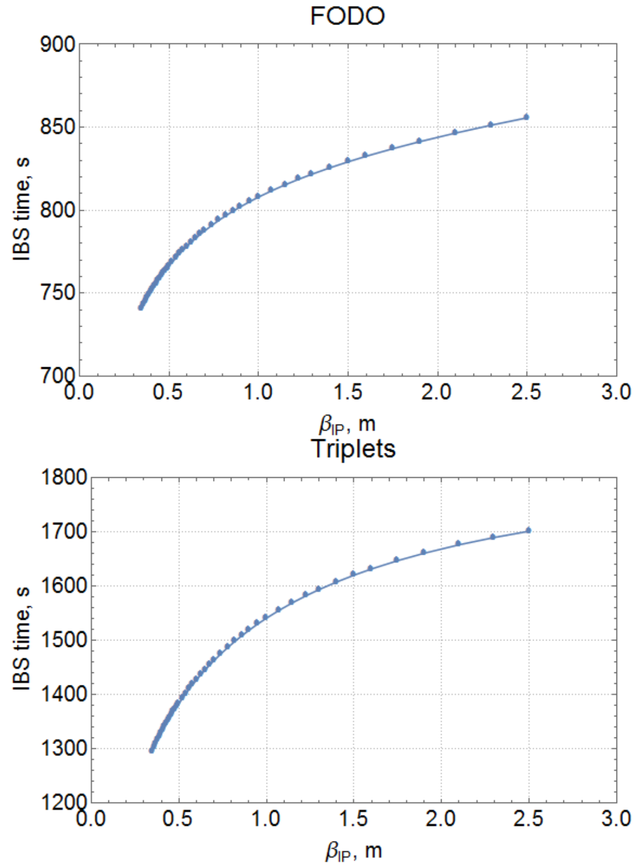

Figure 5: Dependence of IBS times versus β^* for FODO- (upper) and ODFDO- (lower) ring lattice options.

Basing on the results of the IBS studies the FODO structure with 24 cells was chosen as the basis for NICA collider optics. The vertical emittance and the longitudinal momentum spread for $^{197}Au^{79+}$ beam at quasi-equilibrium are shown in Figure 6 for different energies. The horizontal emittance was chosen to be 1.1mm·mrad. The requirements to get at least 6σ aperture determines the ring geometrical acceptance of 40mm·mrad.

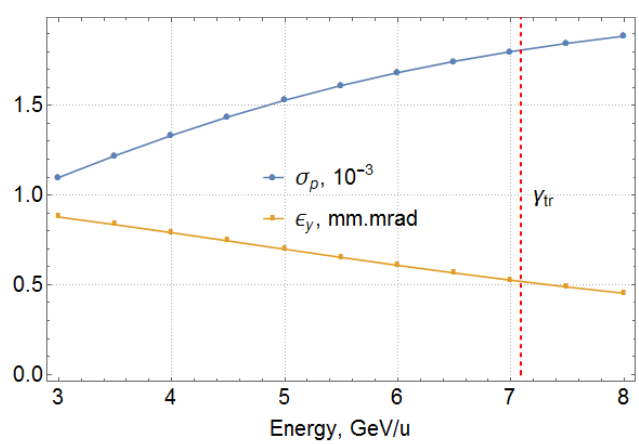

Figure 6: The energy dependence of emittances at equal IBS rates: blue dots for momentum spread, yellow – for vertical emittance.

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The IBS growth rates for the beam intensity of $N = 2{\cdot}10^9$ particles per bunch are shown in Figure 7.

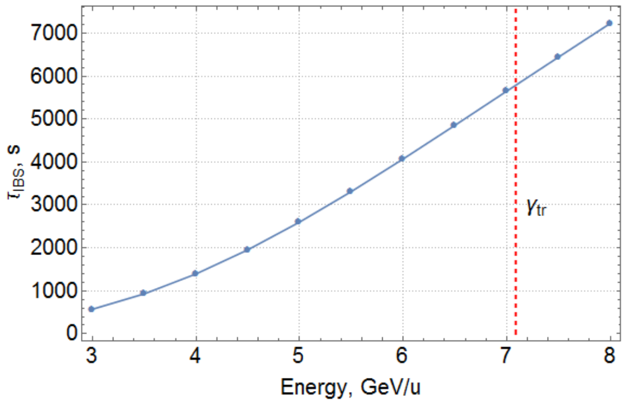

Figure 7: IBS time versus energy for NICA collider lattice at quasi-equilibrium.

CONCLUSIONS

An IBS numerical study was carried out at the development of the NICA collider conceptual proposal. The following conclusions were drawn out:

1. Beam thermal equilibrium for the smooth focusing structures exists only below transition energy.
2. For strong focusing storage rings with regular lattices (no IP insertions) the ODFDO-option gives twice larger IBS times than the FODO- one.
3. IPs' insertions into the lattice structure moves the beam far from the quasi-equilibrium and decreases growth times. That reduces the difference between the triplet and FODO focusing resulting in the difference in the growth rates of only 30%.
4. The FODO-structure was chosen as the base for the NICA collider optics.
5. In the case of real optics the IBS times have no irregular behavior in vicinity of transition energy.

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