LCLS-1 CAVITY BPM ALGORITHM FOR UNLOCKED DIGITIZER CLOCK

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Abstract

Cavity BPMs commonly use the fundamental TM010 mode (excited either in the x/y cavity itself or in a separate "reference" cavity) which is insensitive to beam position as a reference signal, not only for amplitude normalization but also as a phase/time reference to facilitate synchronous detection of the signal derived from the position-sensitive TM110 mode. When taking these signals into the digital domain the reference and position signals need to be acquired by a synchronous clock. However, unless this clock is also locked to the accelerating RF absolute, timing information is lost which affects the relative phase between reference and position signals (assuming they are not carefully tuned to the same frequency).

This contribution presents a method for estimating the necessary time of arrival information based on the sampled reference signal which is used to make the signal detection insensitive to the phase of the digitizer clock. Running an unlocked digitizer clock allows for considerable simplification of infrastructure (cabling, PLLs) and thus decreases cost and eases maintenance.

INTRODUCTION

Cavity Beam-Position Monitors (BPMs) inherently offer a very high resolution [1, 2]. A beam of charged particles passes a cylindrical cavity and excites the electro-magnetical eigenmodes of the device. The coupling of the beam to some of these modes, in particular the "dipole-mode" TM110 is very sensitive to the transversal beam position. The structures to extract the signals from the cavity are carefully designed to be sensitive to TM110 only and reject other modes [3].

TM110 is also excited by a centered but oblique beam trajectory and "slanted" bunches [1] but the resulting signal is in phase-quadrature to the position signal.

The fundamental TM010 mode of a second, "reference", cavity which is largely insensitive to the beam-position is also measured so that the position-sensitive signal can be normalized to the beam charge and phase.

Figure 1 shows the typical hardware employed to acquire the cavity-BPM signals. Three RF signals (e.g, X-band), originating at the X- and Y-ports of the main cavity as well as the output of the reference cavity are fed into a analogue receiver and subsequently digitized. The receiver uses multiple mixing stages and/or an image-rejecting configuration.

In order to maintain the highest possible resolution of the system and to reject (or, depending on the application: detect) the effects of an oblique trajectory (or bunch) a phase-synchronous detection algorithm is commonly used [1, 2, 4] with the reference cavity establishing the necessary time or phase reference.

Obviously, the three channels must use a common LO as well as a common ADC clock in order to maintain phase-synchronicity among the channels.

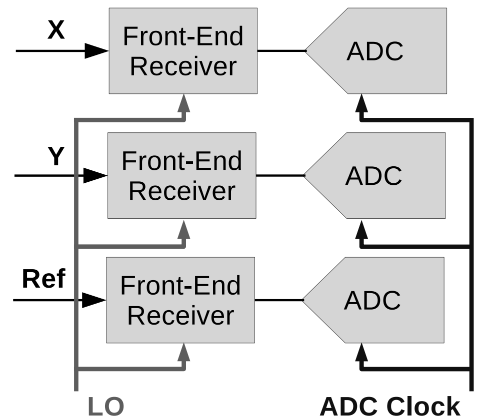

Figure 1: Cavity BPM receiver hardware block diagram.

A synchronous detection algorithm amounts to the estimation of the amplitude of a "known" signal (shape) in the presence of noise [5]. A generic, linear and time-discrete detector for a signal $s(t)$ which is assumed to be time- and band-limited (i.e., it can reasonably approximated by a suitable periodic continuation) can be described by Eq. (1):

$$\hat{A} = \frac{1}{N} \Sigma_{n=0}^{N-1} s(nT_s - t_o) f(nT_s) \qquad (1)$$

i.e., the signal is *correlated* with a (normalized[1]) "test"-function $f(t)$.

In addition to the test-function we also must know or estimate the "starting" time t_o of the signal which is also called "Time of Arrival" or TOA.

The subject of this paper is a method for TOA estimation which is suitable for cavity BPMs with a free-running ADC clock. The ADC is usually triggered by the timing system but the TOA depends on the phase of the ADC clock and is unknown to at least $\pm\frac{1}{2}$ sampling period.

REVIEW OF COMMON METHODS FOR TOA ESTIMATION

In the context of cavity BPMs it is important to consider that one of the advantages of a synchonous detector is its superior SNR when the signal amplitude is small, which is the case when the beam passes close to the electrical center of the main cavity.

[1] so that $\frac{1}{N} \Sigma_{n=0}^{N-1} f^2(nT_s) = 1$.

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This observation suggests that the signal of the *reference* cavity which is always available with a good SNR is the preferable target for TOA estimation – provided that it can be assumed that the relative timing between reference and positional signal remains constant *all the way* down to the ADCs.

Several methods have and can be used:

- Tune the cavities to the same frequency (e.g. [4], [6], [7]). By using a complex exponential for the test function in Eq. (1), t_o will cancel out.
- Phase-lock the ADC clock to the beam and thus maintain a fixed timing (probably – the publications are not always very explicit – used by several authors; e.g., [8]).
- Observe an auxiliary signal (e.g., crystal-detector, [1]) and use for TOA estimation.
- Use an algorithm to estimate TOA from the reference signal.

The last approach has the advantage that neither careful tuning nor an expensive clock distribution infrastructure is required. Since it can use the phase information contained in the reference signal we can also expect this method to yield a better estimate than a crystal-detector.

ALGORITHM FOR TOA ESTIMATION

We shall now proceed to present the algorithm used at LCLS. In order to simplify the mathematical notation we will use a continuous-time representation but assume that all signals are time- and band-limited (see above) so that carrying the results into the discrete-time domain is straightforward.

TOA Definition

Consider a causal signal $s(t)$ and a "suitable", normalized test-function $f(t)$ so that the correlation integral:

$$R(\hat{\tau}) = \int_0^{\infty} s(t+\hat{\tau}) f(t)\, \mathrm{d}t \tag{2}$$

exists. We then define the *time of arrival*, τ_A, *with respect to the test-function* $f(t)$, as the value of $\hat{\tau}$ which maximises the above correlation, Eq. (2).

If $f(t)$ is chosen to be proportional to the signal shape then $R(\hat{\tau})$ is proportional to the signal autocorrelation and has a single maximum at $\hat{\tau} = 0$ (because the signal is assumed to be time-limited it cannot be periodic). In some respects (e.g., maximization of SNR in white noise) such a choice is indeed optimal [5] but any $f(t)$ for which Eq. (2) has a maximum is suitable.

Spectral Representation

The Fourier-transform of Eq. (2) is

$$R(\hat{\tau}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(\omega) F^*(\omega) \mathrm{e}^{-j\omega\hat{\tau}}\, \mathrm{d}\omega \tag{3}$$

where the asterisk denotes the complex conjugate of $F(\omega)$, the Fourier-transform of $f(t)$.

At this point we must pay some attention to the fact that the reference-signal of a cavity BPM does not only have an unknown TOA with respect to the digitizer clock – any (unlocked) LO(s) in the system also introduce an unknown *phase shift*. Such a phase shift can be described in the frequency-domain

$$\phi(\omega) = +j\mathrm{sign}(\omega)\phi$$

Adding an estimated phase shift $\hat{\phi}$ to Eq. (3) and making use of the hermititian symmetry yields

$$R(\hat{\tau}, \hat{\phi}) = \frac{1}{\pi} \int_0^{+\infty} |S(\omega)F(\omega)| \cos(\Psi(\omega) - \underline{\hat{\phi}} - \omega\hat{\tau})\, \mathrm{d}\omega \tag{4}$$

where Ψ is the phase between S and F^*. The estimated phase $\hat{\phi}$ must be zero for $\omega = 0$ and is constant elsewhere (due to the sign function vanishing at $\omega = 0$). We use the symbol $\underline{\hat{\phi}}$ to emphasize the special behavior of $\hat{\phi}(0)$. This is important to remember when moving to a discrete-time representation.

TOA and Phase Estimation

We can now proceed to search for the time $\hat{\tau}$ and phase $\hat{\phi}$ which maximise Eq. (4). We omit the explicit dependence of S, F and Ψ on ω, the integration boundaries, introduce $A = |SF|$, take the partial derivatives to $\hat{\phi}$ and $\hat{\tau}$, respectively, and set equal to zero:

$$\frac{\partial R(\hat{\tau}, \hat{\phi})}{\partial \hat{\phi}} = 0 = -\frac{1}{\pi} \int A \sin(\Psi - \underline{\hat{\phi}} - \omega\hat{\tau})\, \mathrm{d}\omega \tag{5}$$

$$\frac{\partial R(\hat{\tau}, \hat{\phi})}{\partial \hat{\tau}} = 0 = -\frac{1}{\pi} \int A \sin(\Psi - \underline{\hat{\phi}} - \omega\hat{\tau})\, \omega\, \mathrm{d}\omega \tag{6}$$

Our next assumption is that close to the optimum, the phase error

$$\Delta(\omega) = \Psi(\omega) - \underline{\hat{\phi}} - \omega\hat{\tau} \tag{7}$$

is very small so that we can approximate $\sin(x) \approx x$. It is very important to note that we do *not have to assume that the estimated phase nor the estimated TOA be small!* We just assume that the linear phase $\hat{\phi} - \omega\hat{\tau}$ "tracks" the phase Ψ of the correlation well.

If, e.g., we use the signal shape itself for the test function $f(t)$ then Ψ becomes a linear function of the LO phase and the unknown TOA, and Eq. (7) simplifies to

$$\Delta(\omega) = \Psi(\omega) - \underline{\hat{\phi}} - \omega\hat{\tau} = \underline{\phi}^{LO} + \omega\,\tau_A - \underline{\hat{\phi}} - \omega\hat{\tau}$$

i.e., when the estimated $\hat{\phi}$ and $\hat{\tau}$ match the 'true' values then Δ vanishes everywhere.

The linearized system of equations can be stated

$$\begin{array}{ccccccc} \hat{\phi} & \int A\, \mathrm{d}\omega & + & \hat{\tau} & \int A\, \omega\, \mathrm{d}\omega & = & \int A\, \Psi\, \mathrm{d}\omega \\ \hat{\phi} & \int A\omega\, \mathrm{d}\omega & + & \hat{\tau} & \int A\, \omega^2\, \mathrm{d}\omega & = & \int A\, \Psi\omega\, \mathrm{d}\omega \end{array} \tag{8}$$

and trivially solved for $\hat{\phi}$ and $\hat{\tau}$. Again: when moving into the discrete-time domain then we must be careful with the DC term and remember that $\mathrm{sign}(\omega) = 0$.

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However, the estimator is even useful if the phase Ψ of the correlation SF^* deviates from a linear phase. Because phase- and time-shift are linear operations and the estimator is also linear this means that a *differential* estimation between two "shots" of a signal with each one having a random phase- and time-delay is still possible (see Appendix).

RESULTS

The proposed algorithm has been implemented in the BPM processing software at SLAC. An AltiVec™ [9] co-processor is employed to FFT the raw signals and to compute the various sums in Eq. (8). The phase and TOA of the positional signals are corrected in the frequency-domain and then correlated with a filter response (still in frequency domain). This operation is equivalent with a digital downconversion as it is used by several authors [1], [4] and yields the desired complex amplitude from which position and trajectory angle can be extracted. The details are, however, beyond the scope of this paper.

Figure 2 shows the digitized signal of a LCLS reference cavity for two beam pulses. The effect of the unlocked ADC can easily be seen.

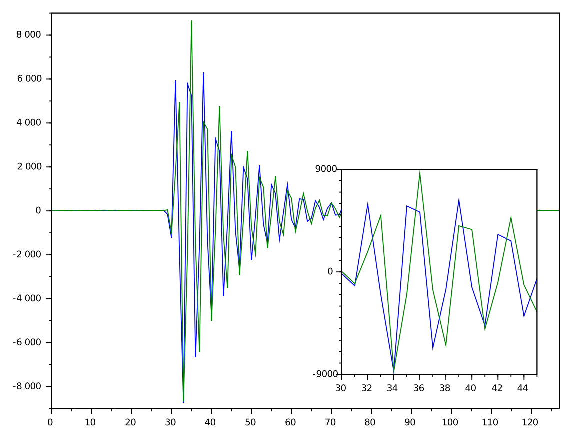

Figure 2: Reference signal of two beam pulses with unlocked ADC and LO. The sub-plot zooms to samples #30 – 45.

As an example, we used a test function $f(t)$ which has the frequency-response of an ideal band-pass and covers a bandwidth of 20 frequency bins (see Fig. 3). The time-domain response is depicted in Fig. 4.

The result of applying the phase- and TOA estimation to the original waveforms is presented in Fig. 5. In addition, the waveforms were normalized to the beam-charge – something that would not be done in case of a position calculation but was performed here to show how the two waveforms match up after correcting for TOA.

A time-shift by a (not necessarily integer-) multiple of the sampling interval can trivially be performed in the frequency-domain.

CONCLUSION

A method for TOA estimation has been presented which can be used by the signal-processing of a cavity BPM system in order to correct for the effect of an unlocked digitizer clock even if the reference- and position cavities are not tuned to exactly the same frequency.

Relaxing the tuning requirements and obsoleting the need for a clock- distribution infrastructure results in considerable savings.

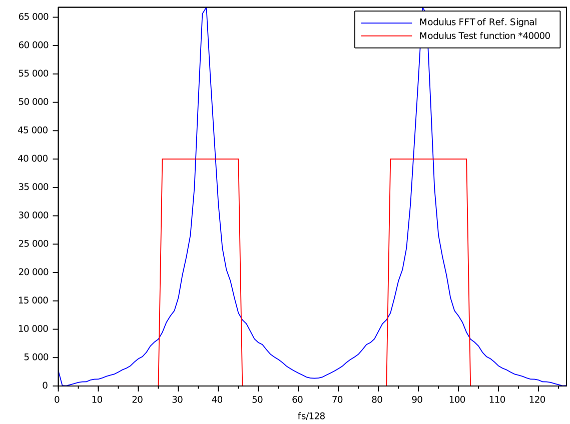

Figure 3: FFT (modulus) of the beam pulses and response of the test function (multiplied by 40000 to make it visible).

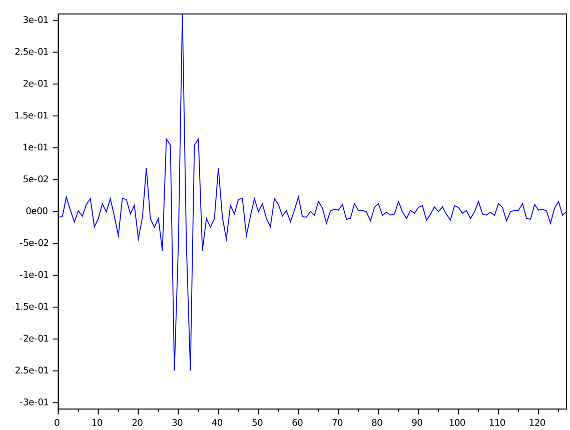

Figure 4: Time-domain response of the example test function.

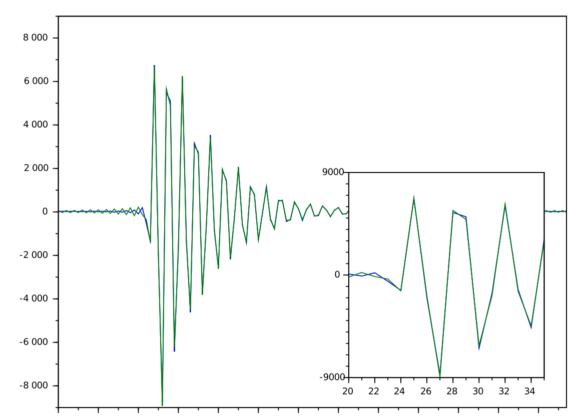

Figure 5: Reference signal of two beam pulses after applying the correction for $\hat{\tau}$ and $\hat{\phi}$.

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APPENDIX

We can abbreviate Eq. (8) using matrix notation

$$M\ \hat{x} = y$$

with $\hat{x}$ a column-vector containing the estimated values, M the matrix which depends only on the amplitude function $A(\omega)$ and y which involves integrals of the phase but is linear in the phase

$$y(\Psi + \gamma) = y(\Psi) + y(\gamma)$$

In particular, computing $y(\phi)$ with ϕ = const (arbitrary phase shift) and $y(\omega\tau)$ (arbitrary time shift) we note that the coefficients of y become identical with the coefficients of M and thus

$$M\ \hat{x} = y(\Psi) + M\ x$$

with the vector x containing ϕ, τ. Therefore, a non-vanishing $y(\Psi)$ merely introduces a phase and time-offset and the difference in estimation between two "shots" with unknown x_1 and x_2 reduces to

$$\begin{aligned} M\ \hat{x}_2 - M\ \hat{x}_1 &= y_2 - y_1 = y(\Psi) - y(\Psi) + M\ x_2 - M\ x_1 \\ M\ (\hat{x}_2 - \hat{x}_1) &= M\ (x_2 - x_1) \end{aligned}$$

which shows that the estimator yields the "exact" result (in absence of measurement errors, noise etc.) for phase- and time-*differences*.

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