LONGITUDINAL DIAGNOSTICS METHODS AND LIMITS FOR HADRON LINACS

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Abstract

A summary of the longitudinal diagnostics for linacs is presented based on the Spallation Neutron Source (SNS) linac example. It includes acceptance phase scans, Bunch Shape Monitors (BSM), and a method based on the analysis of the stripline Beam Position Monitors' (BPM) signals. The last method can deliver the longitudinal Twiss parameters of the beam. The accuracy, applicability, and limitations of this method are presented and discussed.

INTRODUCTION

The SNS linac accelerates H^- ions up to 1 GeV. It has two parts: a normal temperature linac and a SCL which is the world's first of the kind high power hadron superconducting linac. The SCL accelerates negative hydrogen ions from 186 MeV to 1 GeV with 81 six-cell niobium elliptical superconducting RF cavities [1]. The SNS power ramp up started in 2006, and in 2009 SNS reached 1 MW level. During this time, an unexpected beam loss in an SCL was encountered. This beam loss was reduced to the acceptable level by empirically lowering the field gradients of the SCL quadrupoles without understanding the loss mechanism. That led to efforts by the accelerator physics group to understand and to control the beam sizes in all three dimensions in the SNS superconducting linac. Later the mechanism of the unexpected beam loss was identified as the Intra Beam Stripping (IBSt) process [2,3]. This explained our success in the loss reduction, but future improvements depend on our ability to measure and control the linac bunch sizes along the SCL including the bunch length. To measure this parameter the new method of non-invasive longitudinal diagnostics was developed [4].

In the present paper we are going to describe the new approach, its accuracy, conditions of applicability in hadron linacs, and its limitations. In the beginning, we will give the overview of traditional longitudinal diagnostics in the SNS linac. Then we will discuss the possibility and conditions of using the Beam Position Monitor's (BPMs') signal for the bunch length measurements. After that, we will describe the scheme of the new method where we combine a short RF cavity, a drift space, and the BPM to measure the longitudinal Twiss. We are going to present formulas for estimating parameters of the cavity and a drift length necessary for successful application of the method. At the end we will discuss the results of application of this method to the SNS linac.

SNS BUNCH LENGTH DIAGNOSTICS

At SNS the direct measurements of the bunch length in the linac and transport lines are performed by the Bunch Shape Monitors (BSM) [5]. The SNS linac has 4 BSMs in the warm section right before the SCL part. These BSMs were used to check the bunch shape at the entrance of SCL. The measurements showed that we have a longitudinally unmatched beam in this section, and the longitudinal emittance at the SCL entrance is substantially higher than the design value. The last BSM was also used to benchmark a new BPM-based method [4]. Unfortunately, the BSMs as beam intercepting devices are not used in the superconducting linac because of a possibility of a cavities' surface contamination.

Another method for the bunch length measurement is the widely used acceptance phase scan. This method was used at SNS for the Drift Tube Linac cavities in the warm linac and for SCL [6]. The classical variant of the acceptance scan uses a Faraday cup with energy degrader to measure the transmission of the beam through a long accelerator cavity as a function of the cavity phase. In the case of the SNS superconducting linac the combination of the beam current monitors and beam loss monitors was used [6]. For the SCL this method is very time consuming, creates a lot of beam loss in the SCL during the scan, and it has errors that cannot be evaluated.

The new suggested method uses the BPMs to measure the longitudinal bunch length. The next section will discuss the conditions for reliable measurements of this parameter.

BUNCH LENGTH AND BPM SIGNALS

The analysis of the spectral density of the sum signal of all four BPM's quadrants was performed a long time ago [7]. The Fourier amplitude of the surface charge density on a beam pipe is defined by geometry, relativistic parameters of the beam, and a Fourier amplitude of the longitudinal density of the bunch [7]

$$u_\omega \propto \frac{A_\omega}{I_0\left(\dfrac{R\cdot\omega}{\gamma\cdot\beta\cdot c}\right)} \qquad (1)$$

where ω is BPM's frequency, R is the beam pipe radius, c is the velocity of light, A_ω is the Fourier amplitude of the longitudinal density of the bunch, I_0 is the modified Bessel function, and γ, β are relativistic parameters.

The modified Bessel function in the formula (1) describes the attenuation of the signal for higher frequencies because of the pure geometry. We can get the detailed longitudinal shape of the bunch only in the ultra-relativistic case, when $\gamma \to \infty$, and the useful frequencies will be limited only by the external circuit. In the

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realistic cases of hadron accelerators ($\beta < 1$) our frequency response will be limited. Let's estimate the region of the bunch length for which the BPM's signal with a particular frequency can be used for measurements.

Let's assume that the bunch has a Gaussian longitudinal charge density distribution

$$\lambda(z) = q \cdot N \cdot \frac{1}{\sqrt{2\pi\sigma_z^2}} \cdot \exp\left(-\frac{z^2}{2 \cdot \sigma_z^2}\right) \quad (2)$$

where $\lambda(z)$ is the longitudinal charge density along the longitudinal coordinate z, q and N are the charge and the number of particles in the bunch, and σ_z is the RMS length of the bunch in meters. The amplitude of the Fourier harmonic of (2) for a particular frequency ω is

$$A_\omega(\sigma_\varphi) = A_{\max} \cdot \exp\left(-\frac{\sigma_\varphi^2}{2}\right) \quad (3)$$

where $A_{\max}$ is maximal value of the amplitude, and σ_φ is the RMS length of the bunch in radians.

After inversion, formula (3) will give us the bunch length as function of the BPM's amplitude signal if we assume the constant relativistic parameters in the equation (1)

$$\sigma_\varphi = \sqrt{2 \cdot \ln\left(\frac{A_{\max}}{A_\omega}\right)} \quad (4)$$

To find the acceptable range of BPM amplitudes for the bunch length measurements we have to estimate the relative error of σ_φ from the formula (4)

$$\frac{\delta\sigma_\varphi}{\sigma_\varphi} = \frac{1}{2 \cdot x \cdot \ln(x)} \cdot \frac{\delta A_\varphi}{A_{\max}} \; ; \; x = \frac{A_\varphi}{A_{\max}} \quad (5)$$

The relative error of the bunch length measurements as a function of BPM amplitude is shown on Fig. 1 for the case when $\delta A_\omega / A_{\max} = 1\%$.

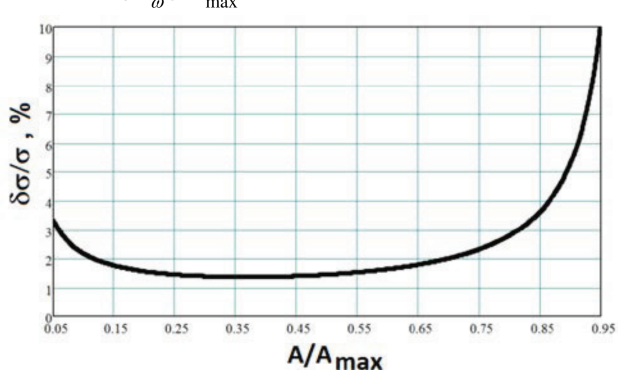

Figure 1: The relative error of bunch length measurements by using the formula (4). The BPM amplitude error is assumed 1%.

The Fig 1 shows that acceptable range of the BPM's amplitudes is between 10 and 90 % of the maximal value of the amplitude. If the BPM's amplitude is too big or too small we cannot extract the bunch length with a good accuracy. The Fig. 2 shows a corresponding range of the RMS bunch lengths by using equation (3). We can see that the measurable RMS bunch lengths are between 30 and 120 degrees.

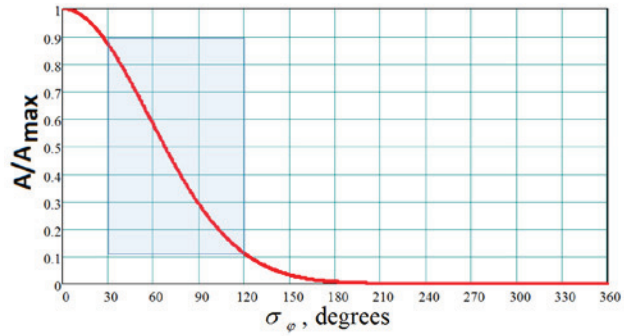

Figure 2: The range of measurable bunch lengths for the BPM's amplitude in 10%-90% range of the maximal value.

Typical bunches in the linac are much shorter (only several degrees) because of the necessity to provide effective acceleration and to avoid the big nonlinearities and beam loss. It means that BPM cannot be used as a device for the RMS bunch length measurements in linacs under normal circumstances. Nevertheless, if we allow beam debunching when the RMS bunch length will reach the region shown in Fig. 2, we can use BPMs to measure this length. In [4] it was suggested to use a combination of three elements (a short RF cavity, a drift space and a BPM) to measure not only the RMS bunch length, but also the longitudinal Twiss at the entrance of the RF cavity. In the following sections we will discuss the necessary parameters of this system.

PHYSICAL VARIABLES DESCRIPTION

In our analysis we will describe the bunch as an ensemble of particles with particular phases φ and energy deviations dE relative to the synchronous particle. The bunch as a whole is characterized by the second order correlations over the ensemble of particles

$$\langle \varphi^2 \rangle = \sigma_\varphi^2 \; ; \; \langle dE^2 \rangle = \Delta E^2 \quad (6)$$

$$\langle \varphi \cdot dE \rangle = K_{corr} \cdot \sigma_\varphi \cdot \Delta E \quad (7)$$

The first two (in formula (6)) are squares of the RMS longitudinal size and the energy spread respectively, and the last one (in formula (7)) is a phase-energy correlation. The different types of correlations are shown in Fig. 3.

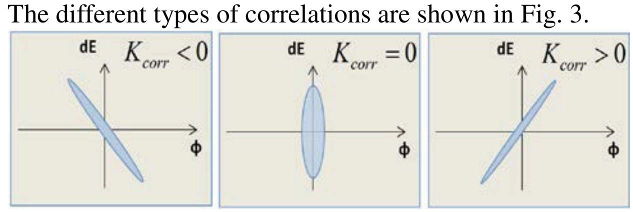

Figure 3: The longitudinal phase space for different types of phase-energy correlations.

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The second order statistical correlations are related to the more recognizable Twiss parameters (emittance, alpha, and beta)

$$\varepsilon_{rms} = \sqrt{\langle \varphi^2 \rangle \cdot \langle dE^2 \rangle - \langle \varphi \cdot dE \rangle^2} \qquad (8)$$

$$\alpha_{Twiss} = -\frac{\langle \varphi \cdot dE \rangle}{\varepsilon_{rms}} \quad ; \quad \beta_{Twiss} = \frac{\langle \varphi^2 \rangle}{\varepsilon_{rms}} \qquad (9)$$

These parameters are often used as input of beam envelope tracking accelerator models. The correct model and the initial Twiss parameters allow predicting the RMS sizes for the whole linac and performing the beam matching procedure to reduce halo growth and beam loss.

RF CAVITY+DRIFT+BPM LATTICE

Our goal is to find the initial longitudinal Twiss parameters by using a system that consists of an RF cavity, a drift space which is long enough to allow for the necessary beam debunching, and the BPM for the bunch length measurements. In the beginning, we will perform our analysis for the case with negligible space charge effects. The final (at the BPM's position) and initial (at the RF cavity's entrance) longitudinal coordinates of the particles are related through the transport matrix for the RF cavity and the drift

$$\begin{pmatrix} \varphi_{BPM} \\ dE_{BPM} \end{pmatrix} = M_{drift} \cdot M_{RF} \cdot \begin{pmatrix} \varphi_0 \\ dE_0 \end{pmatrix} \qquad (10)$$

where the drift and the short (few accelerating gaps) RF cavity transport matrices are

$$M_{Drift} = \begin{pmatrix} 1 & \frac{L}{\bar{\lambda}_{RF}} \cdot \frac{1}{m\gamma^3\beta^3} \\ 0 & 1 \end{pmatrix}; \quad \bar{\lambda}_{RF} = \frac{\beta \cdot c}{\omega_{RF}} \qquad (11)$$

$$M_{RF} = \begin{pmatrix} 1 & 0 \\ -qV_0 \cdot \sin(\phi_{RF}) & 1 \end{pmatrix} \qquad (12)$$

where L is the length of the drift, m is the mass of the particle, and $\omega_{RF}, qV_0, \phi_{RF}$ are the RF frequency, the maximal energy gain provided by the RF cavity, and the synchronous particle phase.

After substituting (11, 12) into (10) we get the expression for final phase of the arbitrary particle in the bunch

$$\varphi_{BPM} = m_{1,1} \cdot \varphi_0 + m_{1,2} \cdot dE_0 \qquad (13)$$

where the transport matrix components are

$$m_{1,1} = 1 - \frac{L}{\bar{\lambda}_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3} \qquad (14)$$

$$m_{1,2} = \frac{L}{\bar{\lambda}_{RF}} \frac{1}{m\gamma^3\beta^3} \qquad (15)$$

After we calculate the square of both sides of the equation (13) and average over the ensemble, we get the final RMS longitudinal size at the BPM position

$$\sigma^2_{BPM}(\phi_{RF}) = \left(1 - \frac{L}{\bar{\lambda}_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3}\right)^2 \sigma_0^2$$
$$+ 2\left(1 - \frac{L}{\bar{\lambda}_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3}\right)\left(\frac{L}{\bar{\lambda}_{RF}} \frac{K_{corr}\sigma_0 \Delta E}{m\gamma^3\beta^3}\right) \qquad (16)$$
$$+ \left(\frac{L}{\bar{\lambda}_{RF}} \frac{\Delta E}{m\gamma^3\beta^3}\right)^2$$

The final RMS bunch length according to the formula (16) will have three components. The first and last ones are defined by the initial size and the energy spread correspondingly, and the middle component is the phase-energy correlation contribution. Let's consider these components one by one.

DRIFT LENGTH

In the beginning, we assume zero energy spread in the bunch. In this case the final longitudinal size will be defined by the RF cavity parameters and the initial size. To get the substantial size growth $\sigma_{BPM} >> \sigma_0$ we have to provide the length of the drift no less than

$$L \geq \bar{\lambda}_{RF} \cdot \frac{m\gamma^3\beta^3}{qV_0} \cdot \frac{\sigma_{BPM}}{\sigma_0} \qquad (17)$$

In this formula we assume the maximal defocusing effect from the cavity when ϕ_{RF} = -90^0 for the particles with positive charge. Let's estimate the drift length for beginning of the SNS superconducting linac. The SCL RF cavities have the following parameters $\bar{\lambda}_{RF}$ = 0.06 m, qV_0 = 10 MeV, β = 0.55. The typical initial bunch length is about 3^0, and at the BPM position we want to have at least 60^0. For these parameters the minimal drift length is 32 meters. We can easily create the drift like this by switching off all downstream RF cavities.

Formula (17) shows the fast growth of the necessary drift length with the energy of the beam. For the SNS case, L should be more than 200 meters when the beam reaches energy 400 MeV at the end of the medium beta part of the superconducting linac. This dependency is one of the serious limitations of the method. Some of the possible ways to mitigate this condition will be simultaneous scans of several cavities.

ENERGY SPREAD

If we assume that there is no phase-energy correlation in formula (16), there will be only two contributions: one from the initial bunch length and another from the energy spread. Comparing them we can have two extreme cases

$$qV_0 \cdot \sigma_0 << \Delta E \text{ or } qV_0 \cdot \sigma_0 >> \Delta E \qquad (18)$$

where the bunch length σ_0 is in radians.

In the first case of equation (18) we will be able to extract from the BPM's data only the bunch length, and for

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the second case it will be only the energy spread. Both these cases are unfortunate for us, because our goal is to get the full set of the longitudinal Twiss parameters (8, 9). It will be possible if our values approximately satisfy the following condition

$$qV_0 \cdot \sigma_0 \approx \Delta E \tag{19}$$

In this case we will get from our data both the initial bunch length and the initial energy spread. In Fig. 4 it is shown the dependency of the bunch length at two positions of the BPMs for the same parameters as for formula (17) in the previous section and the initial RMS energy spread 235 keV. We can clearly see the base value defined by the energy spread and a variable part approximately proportional to the $\sin^2$ function for both distances. To plot these graphs we used formula (16). The shift of 90^0 was chosen to emphasize a resemblance of the simulation to the real data which will be shown later. The bunch length in Fig. 4 is calculated for BPM's frequency 402.5 MHz which is a half of the RF cavity frequency that was used in the estimation after equation (17).

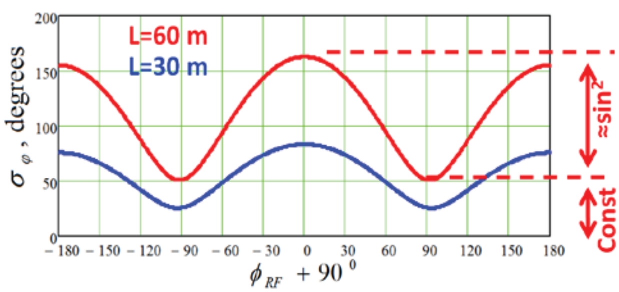

Figure 4: RMS longitudinal sizes of the bunch at two distances as function of the synchronous phase of the first SCL cavity.

In Fig. 5 it is shown the BPMs' amplitudes for the bunch lengths from Fig. 4. In Fig. 5 we can see that the distance 30 meters as a BPM position can be used for bunch length measurements only for particular cavity phases, because the BPM amplitude signal should be in the 10% - 90% range of the maximal amplitude. The distance 60 meters is good for all phases. The doubling of the necessary drift length relative to the estimation in the previous section is explained by the reduced BPM's frequency used at the SNS SCL. This selection of the BPM's frequency was necessary to reduce the noise level in the BPM electronics from cavities' RF.

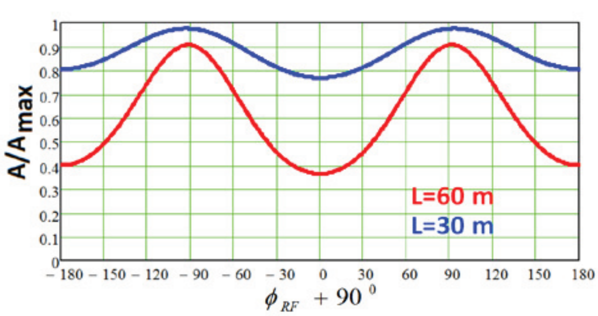

Figure 5: Amplitudes of BPMs at two distances as function of the synchronous phase of the first SCL cavity.

PHASE-ENERGY CORRELATION

In the presence of a phase-energy correlation in the ensemble of particles in the bunch the curves shown in Fig. 5 will be deformed. To plot these curves we have to use the formula (16) with a non-zero correlation coefficient K_{corr}. Fig. 6 shows the deformation of the curves for positive and negative correlation coefficients relative to the case of no-correlations shown in Fig. 5. It also means that we can extract the information about the correlation from measurements to complete the longitudinal Twiss parameters that we want to find.

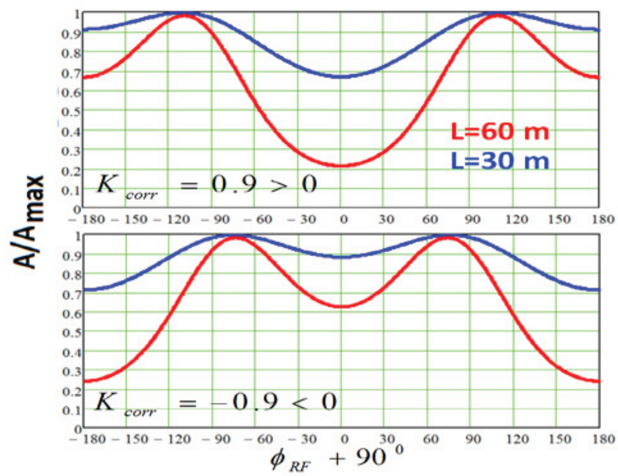

Figure 6: Amplitudes of BPMs at two distances for two cases of the phase-energy correlation

TWISS PARAMETERS CALCULATION

Formula (16) could be rewritten in a new form

$$\sigma_{BPM}^2(\phi_{RF}) = F(\phi_{RF})\sigma_0^2 + B(\phi_{RF})\langle \varphi dE \rangle + C\Delta E^2$$

On the left side of this equation we have the value that we will measure with BPMs, and on the right side we have three unknown values: the squared RMS bunch length, the correlation, and the squared energy spread. It means that we need measurements for at least three cavity phases. In this case we will have a linear system of three equations and three unknowns. In reality we can have more equations and represent them in the following form

$$\begin{bmatrix} \sigma_{BPM}^2(\phi_{RF}^{(1)}) \\ \ldots \\ \sigma_{BPM}^2(\phi_{RF}^{(N)}) \end{bmatrix} = \begin{bmatrix} F(\phi_{RF}^{(1)}) & B(\phi_{RF}^{(1)}) & C \\ \ldots & \ldots & \ldots \\ F(\phi_{RF}^{(N)}) & B(\phi_{RF}^{(N)}) & C \end{bmatrix} \begin{bmatrix} \sigma_0^2 \\ \langle \varphi dE \rangle \\ \Delta E^2 \end{bmatrix}$$

When the number of measurement points N is more than 3, we can solve this system by using the Least Square Method. The algorithm including the parameters error estimation for our particular task is described in [4]. After we extract the unknown second order correlations, the longitudinal Twiss parameters can be calculated from formulas (8, 9).

SPACE-CHARGE EFFECTS

All formulas in the previous sections can be used directly if we do not have substantial space charge effects.

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If these effects are present we can use these formulas to estimate the parameters of the system. The final analysis should be done by using the transport matrices (11, 12) generated by the one of the envelope computer simulation codes like Trace3D. In this case the solution process will include application iteration or general fitting algorithms.

APPLICATION FOR SNS SCL

The suggested method was applied for the SNS superconducting linac. We performed the phase scan of the first cavity in the SCL with all other cavities switched off. The signals from all downstream BPMs were recorded and analysed by using the transport matrix generated by a code similar to Trace3D. The total number of BPMs was 14. With the phase scan from -180^0 to +180^0 and the phase step 5^0 we had 1008 equations in the system from the previous section of this paper. The accuracy of the longitudinal Twiss parameters was 1-3%. A picture of the BPM amplitudes during the phase scan of the first SCL cavity is shown in Fig. 7. We can see a clear resemblance to the picture in Fig. 6. An important point in these measurements and analysis was the calibration of the BPMs' amplitudes. We performed this calibration by using the production setup for the SCL linac, so we can be sure that we have a very short bunch in the whole SCL. The observed BPMs' amplitudes at the production were considered A_{max} values. We also took into account the Bessel function with the correct energies along SCL when we translated these amplitudes from production energies to the phase scan conditions.

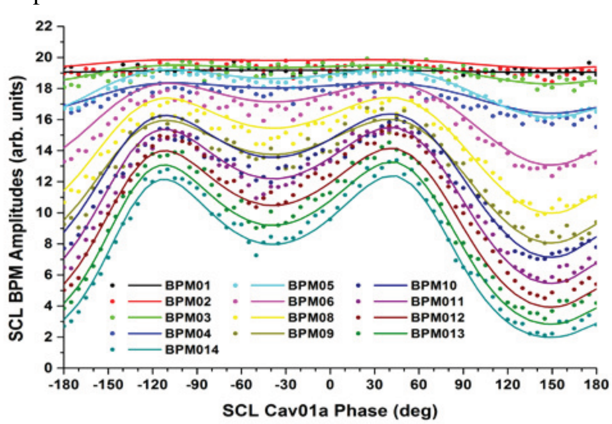

Figure 7: The BPM amplitudes during the first SCL cavity phase scan. Points are measurements; lines are from the envelope model.

As we mentioned above, we can use every cavity in the SCL as a measuring device if we have enough drift space before the BPM for beam to debunch. Fig. 8 shows the bunch length along the first half of SCL. The line on Fig. 8 is a result of simulation with initial Twiss from the analysis of the first cavity data. The Fig. 8 shows a good agreement between measurements and the model.

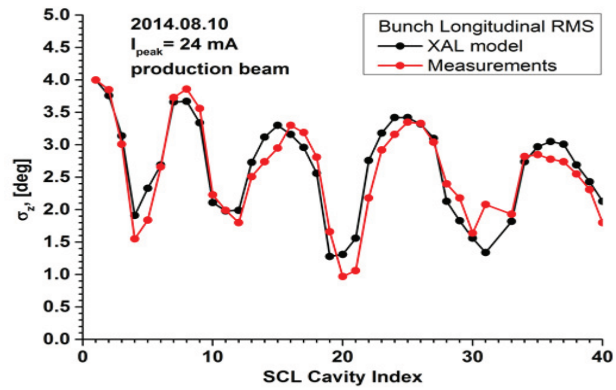

Figure 8: The RMS bunch length along the SCL. Measurements were performed by using the method described here.

CONCLUSION

The accuracy, applicability, and limitations of the original method (RF+drift+BPM) of the longitudinal Twiss parameters measurement was analysed. The successful application of this method to the SNS superconducting linac has been demonstrated.

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