# BEAM SHAPE RECONSTRUCTION USING SYNCHROTRON RADIATION INTERFEROMETRY 

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## Abstract

Synchrotron Radiation Interferometry (SRI) through a double-aperture system is a well known technique to measure the transverse beam size using visible light. In many machines the beam is tilted in the transverse plane, but the SRI technique only allows to directly measure the size of the projection of the beam shape along the axis connecting the two apertures. A method to fully reconstruct the beam in the transverse plane using SRI has been developed and successfully tested at the ALBA synchrotron light source. This report shows the full beam reconstruction technique and presents the results at different couplings. We also discuss how this technique could improve the measurement of very small beam sizes, improving the resolution of standard SRI.

## INTRODUCTION

Transverse beam size measurements are used to monitor the beam quality in accelerators. In synchrotron light sources, a direct image of the beam transverse plane can be provided by x-ray pinholes [1], while direct imaging using the visible part of the synchrotron radiation cannot be performed due to diffraction limitations.

Another widely used method to measure the transverse beam size is the Synchrotron Radiation Interferometry (SRI), which is based on the analysis of the spatial coherence of the synchrotron light and has been used in accelerators since the late 90s [2].

As opposed to the imaging techniques like the x-ray pinhole camera, the standard SRI technique using a doubleaperture system only provides the projected beam size (in the aperture axis direction), and therefore information about possible beam tilt is lost.

A method to reconstruct the full transverse beam profile using a rotating double-aperture system, which allows to properly measure the beam size and relative beam tilt angle has been developed. The technique can also be used to perform ultra-low vertical beam size measurements, crucial for the newest machines.

## SYNCHROTRON RADIATION INTERFEROMETRY AT ALBA

The SRI setup is located in the ALBA diagnostic beamline Xanadu [3]. The radiation is produced by a bending magnet, the visible part is extracted and imaged by a Younglike interferometry.

The interferometry system is composed by two pinholes, a lens to focus the interference fringes, a telescopic ocular to magnify the interferogram, a narrow band color filter and a polarizer to select the wavelength and polarization of the
light. The final results are captured by a CCD. A sketch of the measurement setup is presented in Fig. 1.


Figure 1: Sketch of the SRI experimental setup: The synchrotron radiation (SR) produced by the beam passes through the double-pinhole system and is imaged through a lens and an objective ocular to the CCD. The radiation polarization and the wavelength are selected through a polarizer (Pol.) and a color filter (Col. Filt.).

The formula describing the interferogram intensity along the direction parallel to the axis passing through the pinholes $(x)$ is:

$$
\begin{equation*}
I(x)=I_{0}\left\{\frac{\mathrm{~J}_{1}\left(\frac{2 \pi a x}{\lambda f}\right)}{\left(\frac{2 \pi a x}{\lambda f}\right)}\right\}^{2} \times\left\{1+V \cos \left(\frac{2 \pi D x}{\lambda f}\right)\right\} \tag{1}
\end{equation*}
$$

where $I_{0}$ is the light intensity, $a$ is the pinholes radius, $\lambda$ is the radiation wavelength, $f$ is the focal length of the imaging system, $D$ is the distance between the pinholes and $V$ is the visibility. The visibility is the contrast of the interferogram fringes: $V=\frac{I_{M a x}-I_{\text {Min }}}{I_{M a x}+I_{M i n}}$, where $I_{M a x}$ and $I_{\text {Min }}$ are respectively the maximum and the minimum of the interferogram fringe at the center.

Equation 1 is used to fit a slice of the measured interferogram, letting the visibility as free parameter. Assuming that the beam has a Gaussian distribution along the direction of the pinholes axis, the beam size is obtained as:

$$
\begin{equation*}
\sigma=\frac{\lambda L}{\pi D} \sqrt{\frac{1}{2} \log \left(\frac{1}{V}\right)} \tag{2}
\end{equation*}
$$

where $L$ is the distance between the source point and the pinholes. The larger is the visibility the smaller is the beam size. Examples of interferograms used to measure the beam size projections are presented in Fig. 2.

## FULL BEAM RECONSTRUCTION

Since the SRI only measures the projection of the beam on the double aperture axis, we proceed to measure the projection along different axis by rotating the pinholes system. Figure 2 presents interferograms obtained for pinholes rotated at $0^{\circ}, 45^{\circ}, 90^{\circ}$ and $135^{\circ}$, and the corresponding fit.


Figure 2: Interferograms and fits for the pinholes axis rotated of $0^{\circ}, 45^{\circ}, 90^{\circ}$ and $135^{\circ}$.

Each measurement provides a pair of parallel lines tangent to the beam transverse section in two points. Merging the pairs of lines the full beam shape is reconstructed. The precision of the reconstruction is limited by the number of projection measured.

A full beam reconstruction was performed taking data every $5^{\circ}$ in the range $\left[0^{\circ}, 200^{\circ}\right.$ ], the result is presented in Fig. 3. The reconstruction clearly shows a tilted elliptic beam.


Figure 3: Beam reconstruction calculating the tangent lines from the projections.

## Elliptic Beam

Assuming an elliptic beam, an analytical expression can be inferred to express the projected beam size $\sigma_{p}$ with respect to the double-pinhole rotation angle $\theta$. A detailed explanation is presented in [4], next we show some guide-lines about how to obtain the $\sigma_{p}(\theta)$ expression.

The equation of an ellipse is written as a function of the polar angle $\theta$ as:

$$
\begin{array}{r}
x(\theta)=\sigma_{u} \cos (\theta) \\
y(\theta)=\sigma_{v} \sin (\theta) \tag{3}
\end{array}
$$

where $\sigma_{u}$ and $\sigma_{v}$ are the ellipse's horizontal and vertical semi-axis and correspond to the effective beam sizes or the ellipse eigenvalues.

The measured projection at each angle $\sigma_{p}$ is given by [4]:

$$
\begin{equation*}
\sigma_{p}(\theta)=\sqrt{x^{2}(\theta)+y^{2}(\theta)} \tag{4}
\end{equation*}
$$

If the ellipse is tilted the maximum and the minimum projections will not be on axis ( $\theta=0$ and $\theta=\frac{\pi}{2}$ ), and a phase $\Phi$ must be added:

$$
\begin{equation*}
\theta \rightarrow \theta+\Phi \tag{5}
\end{equation*}
$$

The phase $\Phi$ provides the tilt of the ellipse.
The final equation describing the projection as a function of $\theta$ is finally given by substituting Eq. 3 and 5 in Eq. 4:

$$
\begin{equation*}
\sigma_{p}(\theta)=\sqrt{\sigma_{u}^{2} \cos ^{2}(\theta+\Phi)+\sigma_{v}^{2} \sin ^{2}(\theta+\Phi)} \tag{6}
\end{equation*}
$$

We can use Eq. 6 to fit the measurements shown in Fig. 3, with $\sigma_{u}, \sigma_{v}$ and $\Phi$ as free parameters. The result is shown in the top plot of Fig. 4. Having three parameters and using a Least Squares fit method, the minimum number of measurements needed to reconstruct the ellipse is four.

From the same set of data, the four projections for $\theta$ equal to $0^{\circ}, 45^{\circ}, 90^{\circ}$ and $135^{\circ}$ were extracted. Data and the fit using Eq. 6 are presented in the middle plot of Fig. 4. In both cases the results for the beam sizes and the tilt are:

$$
\begin{equation*}
\sigma_{u}=54 \mu \mathrm{~m}, \quad \sigma_{v}=29 \mu \mathrm{~m}, \quad \Phi=11^{\circ} \tag{7}
\end{equation*}
$$

which are compatible with the values expected for this parameters at the Xanadu source point.


Figure 4: Beam projection as a function of the pinhole rotation angle $\theta$. In the first plot all the 40 measurements, while in the second the projections at $0^{\circ}, 45^{\circ}, 90^{\circ}$ and $135^{\circ}$ are presented. The third plot (black line) is the difference between the fits obtained with 40 and 4 points as a percentage of the 40 points curve.

Finally, in the bottom plot of Fig. 4 the difference between the two fit curves as a percentage of the first is presented. The maximum deviation is less than $1 \mu \mathrm{~m}$. This value is smaller with respect to the statistical error associated with standard SRI beam size measurements $(\simeq 1 \mu \mathrm{~m})$, thus the beam shape reconstruction using only 4 point is reliable.

## Coupling Scan

In order to proof the reconstruction technique consistency, measurements were performed at five different values of the machine coupling. In this way the tilt, the horizontal, and the vertical beam sizes changes. A LOCO [5] was launched at each coupling in order to compare the results obtained by the measurements with the theoretical values.

LOCO provides the beam ellipse eigenvalues and the tilt at the source point. The same values were measured using the 4 points SRI reconstruction. The reconstructed ellipses are presented in Fig. 5. Figure 6 presents the comparison between the SRI reconstruction and LOCO. Results for the beam sizes are consistent at least for the first four values of the coupling. The measured value of tilt has some discrepancy always lower than $5^{\circ}$ but still within the error bar. Considering that the pinholes were tilted without any precision check on the starting alignment with respect to the rotation stage, the result is considered consistent.


Figure 5: Reconstruction of the beam at different couplings. Black lines are calculated from real data while the red ellipse is drown from the result of the fit.

## ULTRA-SMALL BEAM SIZE MEASUREMENT

In future light sources the vertical beam size will be so small that could not be measurable even with SRI technique A possibility to obtain the smallest vertical beam size is to use Eq. 6 using SRI reconstruction, and infer the beam ellipse eigenvalues. Simulations have been performed using SRW [6] to study the feasibility of the method. An elliptic beam with zero tilt ( $\Phi=0^{\circ}, \sigma_{u}=\sigma_{x}$, and $\sigma_{v}=\sigma_{y}$ ) and an horizontal beam size of $55 \mu \mathrm{~m}$ has been generated. The vertical beam size has been changed from 2 to $10 \mu \mathrm{~m}$. To perform the beam reconstruction, projections measurements have been simulated sampling the rotation angle every $15^{\circ}$. The vertical projection (corresponding to $90^{\circ}$ ) has not been considered.

The projections as function of the angles have been finally fitted using Eq. 6 to obtain the beam ellipse parame-


Figure 6: Comparison between the SRI reconstruction and the results from LOCO for the horizontal and vertical beam size and the beam tilt at the Xanadu location.
ters. As an example, Figure 7 presents the results obtained by simulated a non-tilted beam with horizontal and vertical beam sizes respectively of $55 \mu \mathrm{~m}$ and $5 \mu \mathrm{~m}$. Results are compatible with the expected values.


Figure 7: SRW simulated $5 \mu \mathrm{~m}$ vertical beam size reconstruction using the rotated SRI technique. Black dots are data, the error bar is fixed to $1 \mu \mathrm{~m}$, black dashed line is the theoretical curve expected, red line is the result of the fit using Eq. 6 .
Results for the different vertical beam sizes, at a fixed horizontal beam size of $55 \mu \mathrm{~m}$ and at $0^{\circ}$ tilt angle, are listed in Table 1.

Table 1: Results of simulations of small vertical size reconstruction.

| $\sigma_{y}$ Theo. $(\mu \mathrm{m})$ | $\sigma_{y}$ Rec. $(\mu \mathrm{m})$ |
| :---: | :---: |
| 2 | 2.2 |
| 5 | 5.4 |
| 7 | 6.9 |
| 10 | 9.7 |

## CONCLUSION

In this report a method to perform a full reconstruction of the transverse beam shape using the SRI technique has been proposed.
The ALBA beam shape has been reconstructed measuring its projection along several axis by rotating the double pinhole system.
It has been proved that a simplification of the technique can be applied for elliptic beam. In this case, only four projections are needed to reconstruct the full beam transverse shape. To verify the results, measurements have been performed at different couplings.

Finally a new method to indirectly measure ultra-small beam sizes has been proposed based on the same technique. SRW simulations has been performed in order to confirm the validity of the method.

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