MEASUREMENT OF COUPLING IMPEDANCES USING A GOUBAU LINE

F. Stulle*, J. Bergoz, Bergoz Instrumentation, Saint-Genis-Pouilly, France
H. W. Glock, Helmholtz-Zentrum für Materialien und Energie GmbH, Berlin, Germany

Abstract

Longitudinal coupling impedances can be deduced from S-Parameter measurements performed on a Goubau Line. The Goubau Line, also known as single wire line, is a variant of the coaxial wire method. Both setups use a wire for mimicking the particle beam. Coaxial tapers at the wire ends adapt the wave impedance to the 50 Ω impedance of coaxial cables, sources and receivers. But for guiding the electromagnetic wave, the Goubau Line relies on the realistic boundary conditions imposed by an insulated wire instead of using a coaxial shield. Equations for the deduction of longitudinal coupling impedances are reviewed and applied to Goubau Line measurements. Goubau Line measurements and CST Studio simulations are compared, showing good agreement.

INTRODUCTION

The coaxial wire method is a well-established technique for the deduction of coupling impedances [1-3]. The wire is mimicking the particle beam. A coaxial shield is used to guide the electromagnetic fields. Coaxial tapers on both ends adapt the wave impedance to the 50 Ω impedance of coaxial cables, sources and receivers. Using a vector network analyzer, S-parameters of the setup with and without device under test (DUT) are obtained. These measurements are sufficient to mathematically deduce coupling impedances of the DUT with high accuracy.

The Goubau Line [4] is a variant of the coaxial wire method. The important difference is that for guiding the electromagnetic wave it relies on the realistic boundary conditions of an insulated wire, instead of using a coaxial shield. That a single wire in open space can act as a waveguide had already been shown in the early days of electrodynamics [5-7].

Such a setup allows for more flexibility because it does not need to be adapted to the DUT geometry. Additionally, the DUT can be easily placed off-axis. On the other hand, it cannot be considered lossless, which is a usual assumption when analyzing S-parameters obtained by the coaxial wire method. Consequently, the standard equations, which are used to analyze data taken with the coaxial wire method, should be carefully examined before applying them to Goubau Line measurements.

We review the calculation of the longitudinal coupling impedance from S-parameter measurements. Assumptions and simplifications are discussed in view of their applicability to Goubau Line measurements. Afterwards, longitudinal coupling impedances are compared which were obtained from Goubau Line measurements and by CST Studio wakefield simulations [8].

*stulle@bergoz.com

The DUT was a pair of vacuum flanges intended to house a current transformer (Fig. 1). However, no current transformer was installed inside.

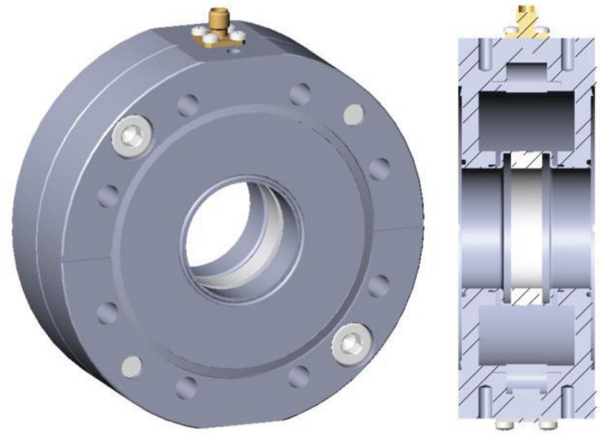

Figure 1: Drawing of the tested vacuum flanges.

Of interest is the coupling impedance induced by the ceramic gap and the surrounding cavity, which can be considered a single lumped impedance. For the particle beam, the metal parts left and right of the gap are like pieces of the vacuum chamber.

Note that such a DUT was chosen to demonstrate Goubau Line measurements and to facilitate simulations. The results are not representative for the real beam instrumentation installed in an accelerator.

CALCULATION OF LONGITUDINAL COUPLING IMPEDANCES

Coupling impedances are the frequency-domain equivalent to the time-domain wake potentials. One can be obtained from the other by (inverse) Fourier transform.

The impact of these coupling impedances on a highly relativistic particle beam is like that of a complex impedance on a current flowing in an electronic circuit. Consequently, they can be analyzed by using scattering parameters (S-parameters) (Fig. 2).

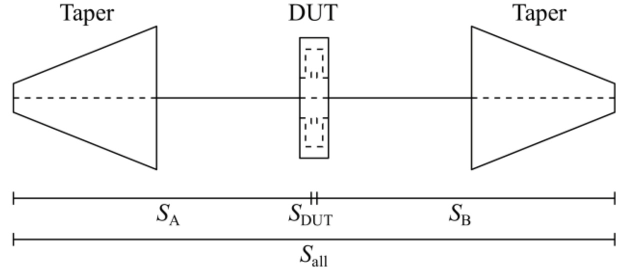

Figure 2: Definition of scattering matrices. The setup is in general asymmetric, i.e. S_A and S_B differ.

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A single, lumped impedance Z_{DUT} leads to a reflection coefficient

$$S_{11,\mathrm{DUT}} = \frac{Z_{\mathrm{DUT}}}{Z_{\mathrm{DUT}} + 2\,Z_0}.$$

Z_0 is the wave impedance of the measurement setup at the position of the DUT. The DUT can be considered symmetric:

$$S_{22,\mathrm{DUT}} = S_{11,\mathrm{DUT}}$$
$$S_{12,\mathrm{DUT}} = S_{21,\mathrm{DUT}}.$$

And the relation $S_{21,\mathrm{DUT}} = 1 - S_{11,\mathrm{DUT}}$ applies, which is valid for any two-port network consisting of an arbitrary series impedance.

Consequently, the scattering matrix of the DUT can be written as:

$$S_{\mathrm{DUT}} = \begin{pmatrix} S_{11,\mathrm{DUT}} & 1 - S_{11,\mathrm{DUT}} \\ 1 - S_{11,\mathrm{DUT}} & S_{11,\mathrm{DUT}} \end{pmatrix}.$$

The part of the measurement setup left of the DUT is described by the scattering matrix

$$S_{\mathrm{A}} = \begin{pmatrix} S_{11,\mathrm{A}} & S_{12,\mathrm{A}} \\ S_{21,\mathrm{A}} & S_{22,\mathrm{A}} \end{pmatrix}.$$

The part of the measurement setup right of the DUT is described by the scattering matrix

$$S_{\mathrm{B}} = \begin{pmatrix} S_{11,\mathrm{B}} & S_{12,\mathrm{B}} \\ S_{21,\mathrm{B}} & S_{22,\mathrm{B}} \end{pmatrix}.$$

In general, S_{A} and S_{B} will differ. Both will be reciprocal, i.e. $S_{12,\mathrm{A}} = S_{21,\mathrm{A}}$ and $S_{12,\mathrm{B}} = S_{21,\mathrm{B}}$.

By transforming the scattering matrices (S-parameters) to scattering transfer matrices (T-parameters) and matrix multiplication, the scattering transfer matrix of the overall setup can be calculated:

$$T_{\mathrm{all}} = T_{\mathrm{A}}\,T_{\mathrm{DUT}}\,T_{\mathrm{B}}.$$

S_{all} is then obtained from T_{all} by back-transformation.

To deduce Z_{DUT} two sets of measurements need to be performed. One reference, S_{ref}, without the DUT. And another, S_{all}, with the DUT. Either the measured transmission coefficients S_{21} or the reflection coefficients S_{11} can be used:

$$Z_{\mathrm{DUT,T}} = 2\,Z_0 \frac{\left(S_{21,\mathrm{all}} - S_{21,\mathrm{ref}}\right)\left(S_{22,\mathrm{A}}\,S_{11,\mathrm{B}} - 1\right)}{S_{21,\mathrm{all}}\left(1 - S_{22,\mathrm{A}}\right)\left(1 - S_{11,\mathrm{B}}\right)} \quad (1)$$

and

$$Z_{\mathrm{DUT,R}} = 2\,Z_0 \frac{S_{22,\mathrm{A}}\,S_{11,\mathrm{B}} - 1}{1 - S_{11,\mathrm{B}}}$$
$$\times \left(1 - S_{22,\mathrm{A}} + \frac{S_{21,\mathrm{A}}^2\left(1 - S_{11,\mathrm{B}}\right)}{\left(S_{11,\mathrm{all}} - S_{11,\mathrm{ref}}\right)\left(S_{22,\mathrm{A}}\,S_{11,\mathrm{B}} - 1\right)}\right)^{-1} \quad (2)$$

Eqns. 1 and 2 share the dependence on the reflection coefficients $S_{22,\mathrm{A}}$ and $S_{11,\mathrm{B}}$. But eqn. 2 additionally depends on the transmission coefficient $S_{21,\mathrm{A}}$; meaning that more information is required to obtain correct results.

In general, $S_{22,\mathrm{A}}$, $S_{11,\mathrm{B}}$ and $S_{21,\mathrm{A}}$ need to be estimated from system knowledge. Overall accuracy of Z_{DUT} will depend on the quality of this estimation.

For well-matched coaxial wire setups, $S_{22,\mathrm{A}}$ and $S_{11,\mathrm{B}}$ can be neglected, $S_{21,\mathrm{A}}$ can be considered a lossless delay and the measured value of $S_{11,\mathrm{ref}}$ will result zero. In such a case, eqns. 1 and 2 simplify to the well-known equations by Hahn and Pedersen [3].

A first-order Taylor expansion of eqns. 1 and 2 yields for small $S_{22,\mathrm{A}}$ and $S_{11,\mathrm{B}}$:

$$Z_{\mathrm{DUT,T}} \approx 2\,Z_0 \frac{\left(S_{21,\mathrm{ref}} - S_{21,\mathrm{all}}\right)\left(S_{22,\mathrm{A}} + S_{11,\mathrm{B}} + 1\right)}{S_{21,\mathrm{all}}}$$

and

$$Z_{\mathrm{DUT,R}} \approx 2\,Z_0 \frac{S_{22,\mathrm{A}} + S_{11,\mathrm{B}} + 1}{\frac{S_{21,\mathrm{A}}^2}{S_{11,\mathrm{all}} - S_{11,\mathrm{ref}}}\left(S_{22,\mathrm{A}} - S_{11,\mathrm{B}} + 1\right) - 1}.$$

GOUBAU LINE MEASUREMENTS

Since a Goubau Line has open boundary conditions, some signal power is always lost due to radiation. Additionally, reflections occur at the transitions from the cones to the wire and at the transitions to the DUT. Hence, establishing exact knowledge of $S_{22,\mathrm{A}}$, $S_{11,\mathrm{B}}$ and $S_{21,\mathrm{A}}$ is difficult, hindering an accurate determination of Z_{DUT}.

These S-parameters can be approximated when exploiting system knowledge. The Goubau Line cones follow a known shape. The corresponding impedance profile can be calculated. The characteristic wave impedance along the wire can be obtained from analytical equations given in [4]. The position of the DUT can be measured and its aperture diameter is in any case a known value.

By combining all values, the wave impedance at any point along the Goubau Line can be approximated. Thus S-parameters can be estimated allowing the application of eqns. 1 and 2 without further simplifications. Nevertheless, it should be noted that different approximations are possible leading to similar but not the same results.

Such an approach is not only applicable for Goubau Line measurements, but also for measurements performed on similar setups, including the coaxial wire setup using an outer shield. To limit the influence of errors on the S-parameter approximation, it will always be beneficial if reflection coefficients remain small and transmission parameters remain large. Both is fulfilled for the Goubau Line set up at Bergoz Instrumentation over a wide range of frequencies; even if the DUT is placed off-axis.

The Goubau Line used for the measurements consisted of a single cone and a 0.5 mm diameter enamel coated copper wire. Behind the DUT, the RF wave was absorbed by RF absorbers (Fig. 3). Since such a setup does not allow transmission measurements, all Goubau Line results are derived from reflection measurements.

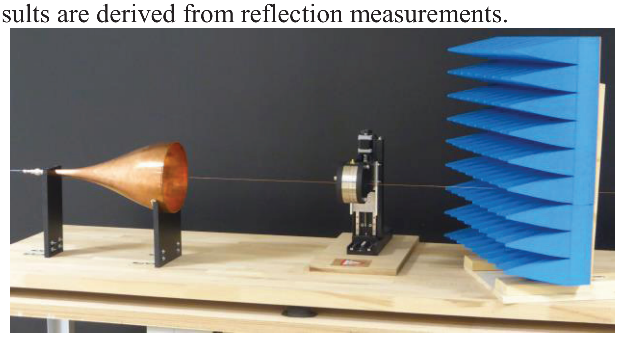

Figure 3: Goubau Line used for reflection measurements. The DUT is supported by micro-movers, allowing accurate positioning.

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WAKEFIELD SIMULATIONS

Wakefield and impedance simulations were performed using CST Studio [8]. Applying the CST ParticleStudio wakefield solver these simulations resemble the passage of a charged particle bunch in the time domain using an explicit algorithm. This is done by discretizing the empty space, the dielectric components and the metallic boundaries on a hexagonal grid, where the integral representations of Maxwell's equations are transferred to a discrete matrix-vector equation.

The fields are excited by imposing a charge and current distribution equivalent to that of a Gaussian-shaped bunch traversing the structure on the beam axis. The wake potential is registered and by Fourier transformation converted into an impedance spectrum. The temporal evolution is done in small time steps, for every time step the field values and their time derivatives are updated.

Since this numerical model does not contain any kind of a central wire it is best suited for comparison to Goubau Line measurements.

The actual simulations used a hexagonal grid of 42.53 million mesh points. The length of the Gaussian excitation was 5 mm (1σ). The wake was computed for a length of 20 m.

SIMULATION AND MEASUREMENT RESULTS

The DUT was a pair of empty current transformer flanges with a total length of 40 mm and 34.9 mm aperture diameter (Fig. 1). They included a ceramic gap made of Alumina. For the Goubau Line measurements, the DUT was placed 1 cm off-axis with respect to the wire. For the wakefield simulations, the excitation was placed 1 cm off-axis with respect to the DUT axis.

The wakefield simulations were performed using three different values for the relative permittivity ε_r of the Alumina gap; $\varepsilon_r = 9.9$, 10.7 and 12.0. This parameter has some influence on the frequency of the resonances. A lower ε_r results higher resonance frequencies, a higher ε_r results lower resonance frequencies (Fig. 4).

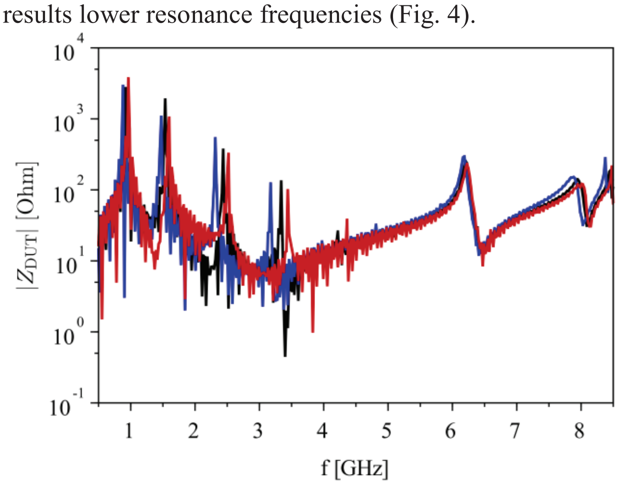

Figure 4: Simulated Z_{DUT} using $\varepsilon_r = 9.9$ (red), $\varepsilon_r = 10.7$ (black) and $\varepsilon_r = 12.0$ (blue).

For comparison to the Goubau Line measurements $\varepsilon_r = 10.7$ was chosen since this value matched measured resonance frequencies very well. It lies within the range of $\varepsilon_r \approx 9 - 12$ found in literature. The real ε_r of the Alumina gap is not known.

The reference measurement $S_{11,ref}$ was done with the ceramic gap shorted by copper tape. $S_{11,all}$ was measured after removing this tape. Care was taken that the DUT position remained unchanged. Differences in position would add a strong background to the calculated coupling impedances, deforming resonances or even hiding weaker ones. Figure 5 shows the measured magnitudes of $S_{11,ref}$ and $S_{11,all}$.

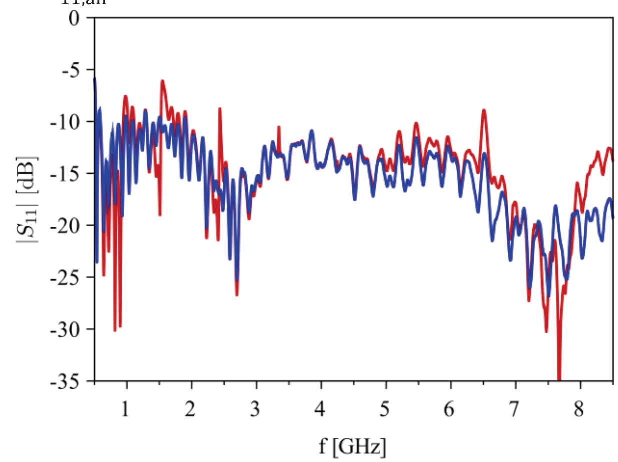

Figure 5: Measured reflection coefficients $S_{11,ref}$ (blue) and $S_{11,all}$ (red).

While some differences are visible, which hint to an influence of the DUT, these are not striking. Only after analysis using eqn. 2 their real importance becomes visible. Figure 6 compares Z_{DUT} obtained from above measurements to CST Studio simulations.

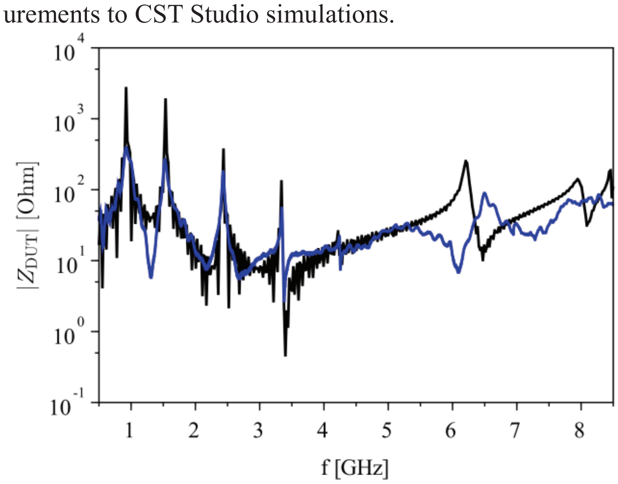

Figure 6: Z_{DUT} obtained by measurement (blue) and by simulation for $\varepsilon_r = 10.7$ (black).

Up to about 5 GHz, simulated and measured Z_{DUT} agree very well. Both contain the same resonances, which are all located at the same frequencies and generally have similar amplitudes. Though the measured resonances are usually weaker.

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One resonance around 6.3 GHz shows up in the simulations and measurement with inverted amplitude, which is not understood.

Above 7.5 GHz the simulations contain resonances which in the measurements lead to a smoothly increased Z_{DUT} only.

Despite the observed differences and the fact that in the simulations ε_{r} had been tuned to match measured resonance frequencies, the similarities are significant. This emphasizes that simulation and measurement results are valid. Both allow not only to identify resonance frequencies, but also to examine resonance strength.

On-Axis DUT

A measurement has also been performed with a centered DUT (Fig. 7). Comparing it to the previous off-axis results (Fig. 6) reveals the importance of the DUT position. Consequently, this comparison reveals the importance of performing measurements or simulations at different DUT positions.

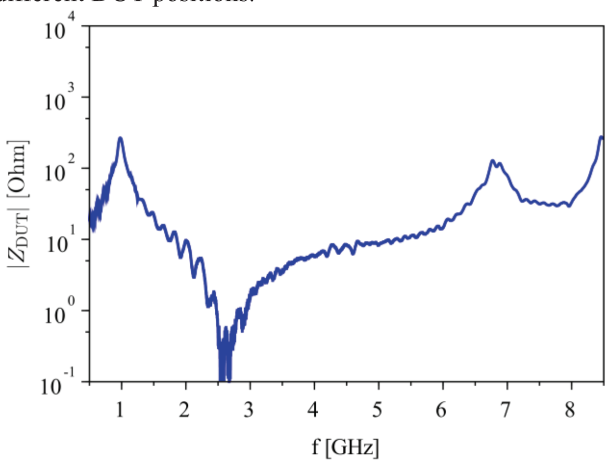

Figure 7: Measured coupling impedance for a centered DUT.

Only the lowest frequency resonance close to 1 GHz remains at similar strength. Up to 6.5 GHz the spectrum does not show any sign of other resonances.

CONCLUSION

The longitudinal coupling impedance of a pair of current transformer flanges has been deduced from Goubau Line measurements and CST Studio simulations. Up to about 5 GHz both approaches resulted the same resonance frequencies and similar resonance amplitudes. At higher frequencies differences could be identified which remain to be understood.

Since the relative permittivity of the Alumina gap was not known, this value had to be scanned in the CST Studio simulations. A value of $\varepsilon_{\mathrm{r}} = 10.7$ resulted in best agreement to the measurements.

The results show that Goubau Line measurements can be used to identify resonances in vacuum components. Despite the fact that knowledge about Goubau Line characteristics remains limited, accuracy is sufficient to distinguish different resonance strengths and to resolve even weak resonances. Furthermore, the Goubau Line working principle, namely its open boundaries, allows for easy off-axis measurements.

Flexibility regarding the DUT geometry and position is the major advantage of a Goubau Line over conventional coaxial wire setups.

Further studies are planned to understand the observed discrepancies. Transmission measurements will be performed using a second cone instead of the RF absorbers. Other devices will be tested.

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