THEORY OF X-RAY TRANSITION RADIATION FROM GRAPHENE FOR TRANSITION RADIATION DETECTORS*

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Abstract

We present the theory of transition radiation for monolayers in X-ray domain from the first principles and consider the pros and cons of using graphene-monolayer in transition radiation detectors.

INTRODUCTION

Transition Radiation Detectors (TRD) are used in many modern experiments like ATLAS and ALICE in CERN, FENIX in BNL, AMS-02 experiment in space, etc. Usually they are used to separate particles with gamma factor less than 500 and more than 1500. However, to increase separation threshold TRD technique requires significant improvement.

Cut-off frequency of the transition radiation (TR) spectrum is proportional to the Lorenz-factor γ, which extends the spectrum to the X-ray domain, see Fig. 1. The cut-off frequency is defined as:

$$\omega_c = \gamma\omega_p. \tag{1}$$

This frequency is proportional to the plasma frequency ω_p of the material which depends on the electron density and the effective mass of electron m_e :

$$\omega_p^2 = 4\pi NZe^2/m_e, \tag{2}$$

with e being the electron charge, NZ being the number of electrons in the unit volume of the target.

In paper [1] it was suggested to use TR radiators based on graphene-monolayer. The idea is based on the fact that some fraction of conductivity electrons in graphene has zero (or very little) mass and, hence, as plasma frequency is inversely proportional to the mass of electron it could shift the TR spectrum to the region of higher frequencies with respect to the ordinary materials. Indeed, it is seen from Eq. (2) that if the effective electron mass goes to zero, then both the plasma frequency and the cut-off frequency from Eq. (1) go to infinity:

$$m_e \to 0, \quad \Rightarrow \quad \gamma\omega_p \to \infty. \tag{3}$$

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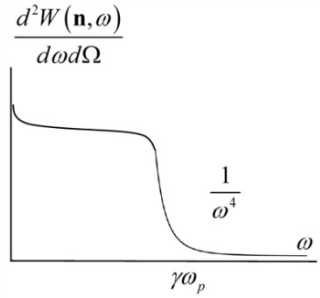

Figure 1: Cut-off of the transition radiation spectrum at high frequencies.

Calculation of the characteristics of the radiation from any monolayer can not be a limiting case of zero-thickness film from the theory for the film of arbitrary thikness, because the latter uses normally boundary conditions, which is not the case for monolayer.

Below we present the theory of transition radiation for monolayers in X-ray domain from the first principles [2, 3] and consider the pros and cons of using graphene-monolayer in TRDs.

MICROSCOPIC THEORY OF RADIATION FROM A MONOLAYER

Let us consider the generation of TR by a single charged particle, for example, by an electron. The target is a monolayer consisting of N atoms or molecules or any small particles with the size less than the wavelength.

Microscopic Maxwell's equations for such a system have the form:

$$\begin{cases} rot\mathbf{H}^{mic}(\mathbf{r},\omega) = \dfrac{4\pi}{c}\mathbf{j}'(\mathbf{r},\omega) - \dfrac{i\omega}{c}\mathbf{E}^{mic}(\mathbf{r},\omega), \\ rot\mathbf{E}^{mic}(\mathbf{r},\omega) = \dfrac{i\omega}{c}\mathbf{H}^{mic}(\mathbf{r},\omega), \\ div\mathbf{H}^{mic}(\mathbf{r},\omega) = 0, \\ div\mathbf{E}^{mic}(\mathbf{r},\omega) = 4\pi\left(\rho^0(\mathbf{r},\omega) + \rho^{mic}(\mathbf{r},\omega)\right), \end{cases} \tag{4}$$

where

$$\mathbf{j}'(\mathbf{r},\omega) = \mathbf{j}^0(\mathbf{r},\omega) + \mathbf{j}^{mic}(\mathbf{r},\omega). \tag{5}$$

The exact solution of the system Eq. (4) is :

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$$\mathbf{E}^{mic}(\mathbf{r},\omega)=\mathbf{E}^{0}(\mathbf{r},\omega)+$$
$$+\frac{4\pi i}{\omega}\int d^3q e^{i\mathbf{qr}}\frac{k^2\mathbf{j}^{mic}(\mathbf{q},\omega)-\mathbf{q}\left(\mathbf{q},\mathbf{j}^{mic}(\mathbf{q},\omega)\right)}{q^2-k^2}, \quad (6)$$

which can be represented as:

$$E_i^{mic}(\mathbf{r},\omega)=E_i^{0}(\mathbf{r},\omega)+\hat{A}_{ij}E_j^{mic}(\mathbf{r},\omega), \quad (7)$$

where $\hat{A}_{ij}$ is the operator. As the first approximation the field acting upon the particles at the points $\mathbf{r}=\mathbf{R}_a$ can be exchanged by some average field that we call the local field, i.e.

$$E_j^{mic}(\mathbf{r},\omega)\big|_{\mathbf{r}=\mathbf{R}_a}\approx E_j^{loc}(\mathbf{r},\omega). \quad (8)$$

Therefore, from Eq. (7) it follows that

$$E_i^{mic}(\mathbf{r},\omega)=E_i^{0}(\mathbf{r},\omega)+\hat{A}_{ij}E_j^{loc}(\mathbf{r},\omega). \quad (9)$$

The main problem is to express the $E_j^{loc}(\mathbf{q},\omega)$ through the field of the external sources, i.e. the field of moving electrons $\mathbf{E}^{0}(\mathbf{r},\omega)$.

To do this it is necessary to average the field in two different ways.

First, let us average it over the positions of all N particles the monolayer:

$$E_i(\mathbf{R}_a,\omega)=\left\langle E_i^{mic}(\mathbf{R}_a,\omega)\right\rangle=$$
$$=\int d^3R_b\,g(\mathbf{R}_b)E_i^{mic}(\mathbf{R}_a-\mathbf{R}_b,\omega), \quad (10)$$

where $\langle ...\rangle$ stands for the averaging. The result can be written in the general form:

$$E_i=E_i^{0}+\hat{B}_{ij}E_j^{loc}. \quad (11)$$

Second, let us average it over the positions of all the particles relatively the given one:

$$E_i^{loc}(\mathbf{R}_a,\omega)=\int\prod_{s=1}^{N-1}d^3R_{an_s}w\left(\mathbf{R}_{an_s}\right)E_i^{mic}(\mathbf{R}_a;\mathbf{R}_n,\omega), \quad (12)$$

where $\mathbf{R}_n=\left(\mathbf{R}_{n_1},...,\mathbf{R}_{n_{N-1}}\right)$. The result of the averaging can be written as:

$$E_i^{loc}=E_i^{0}+\hat{C}_{ij}E_j^{loc}. \quad (13)$$

The macroscopic field in any point can be obtained from Eq. (11) as

$$E_i(\mathbf{r},\omega)=E_i^{0}(\mathbf{r},\omega)+\frac{\alpha(\omega)}{2\pi^2}\times$$
$$\times\left\langle\sum_b\int d^3p\frac{p^2u_{is}(\mathbf{p})}{p^2-k^2}E_s^{loc}(\mathbf{R}_b,\omega)\exp\left[i\mathbf{p}(\mathbf{r}-\mathbf{R}_b)\right]\right\rangle, \quad (14)$$

where

$$u_{is}(\mathbf{p})=\delta_{is}-p_ip_s/p^2,\quad k=\omega/c. \quad (15)$$

To find the field of radiation in the wave zone, we use the well-known asymptotic formula:

$$\int d^3p\frac{f(\mathbf{p})}{p^2-k^2-i0}\exp[i\mathbf{pr}]=2\pi^2\frac{\exp[ikr]}{r}f\left(k\frac{\mathbf{r}}{r}\right), \quad (16)$$

which is correct at $kr\gg1$. As the result, the radiation field can be obtained in form:

$$E_i^{r}(\mathbf{r},\omega)=\alpha(\omega)k^2\times$$
$$\times\left\langle\sum_b\frac{\exp\left[ik|\mathbf{r}-\mathbf{R}_b|\right]}{|\mathbf{r}-\mathbf{R}_b|}u_{is}\left(\frac{\mathbf{r}-\mathbf{R}_b}{|\mathbf{r}-\mathbf{R}_b|}\right)E_s^{loc}(\mathbf{R}_b,\omega)\right\rangle. \quad (17)$$

For $r\gg R_b$ Eq. (17) takes the form:

$$E_i^{r}(\mathbf{r},\omega)=\frac{\exp[ikr]}{r}\alpha(\omega)k^2u_{is}(\mathbf{n})\times$$
$$\times\left\langle\sum_b E_s^{loc}(\mathbf{R}_b,\omega)\exp[-i\mathbf{kR}_b]\right\rangle, \quad (18)$$

where $\mathbf{k}=\mathbf{n}\omega/c$, $\mathbf{n}=\mathbf{r}/r$.

The distribution of the radiation energy per unit solid angle and per unit frequency, emitted by all particles in the layer, can be written as

$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega d\omega}=cr^2\left|\mathbf{E}(\mathbf{r},\omega)\right|^2. \quad (19)$$

Let us average Eq. (18) over all positions of the particles in the layer $z=0$ and supposing that the particles of the monolayer are distributed uniformly in that plane :

$$\left\langle\sum_b\mathbf{n}\times\mathbf{E}^{loc}(\mathbf{R}_b,\omega)\exp\{-i\mathbf{kR}_b\}\right\rangle=$$
$$=n_1\iint dX_bdY_b\mathbf{n}\times\mathbf{E}^{loc}(\mathbf{R}_b,\omega)\exp\{-i\mathbf{kR}_b\}=$$
$$=n_1(2\pi)^2\mathbf{n}\times\mathbf{E}^{loc}(k_x,k_y,Z_b=0,\omega). \quad (20)$$

Substituting Eq. (20) into Eq. (19) we can obtain:

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$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega d\omega} = (2\pi)^4 \frac{\omega^4}{c^3} n_1^2 |\alpha(\omega)|^2 \times \times \left|\mathbf{n}\times\mathbf{E}^{loc}(\mathbf{K}, Z_b = 0, \omega)\right|^2, \quad (21)$$

where $\mathbf{K} = (k_x, k_y)$. The relation between the local field and the Coulomb field of the moving electrons for the case of monolayer was obtained in [2, 3]:

$$E_i^{loc}(\mathbf{q}, z, \omega) = = \left\{ \frac{\delta_{ij} - e_i e_j}{1 - \pi n_1 a \alpha(\omega)} + \frac{e_i e_j}{1 + 2\pi n_1 a \alpha(\omega)} \right\} E_j^0(\mathbf{q}, z, \omega). \quad (22)$$

As the result we obtain the spactral-angular distribution of radation from a monolayer in form

$$\frac{d^2W_m(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{4e^2\omega^4 n_1^2}{c^3 v_z^2 \left[K^2 - \frac{\omega^2}{c^2} + \frac{(\omega - \mathbf{Kv})^2}{v_z^2}\right]^2} \times \times \left| \frac{\mathbf{n}\times\left((\mathbf{v} - v_z\mathbf{e})\frac{\omega}{c^2} - \mathbf{K}\right)}{\alpha^{-1}(\omega) - \pi a n_1} + \frac{\mathbf{n}\times\mathbf{e}\left(\frac{\omega v_z}{c^2} - \frac{\omega - \mathbf{Kv}}{v_z}\right)}{\alpha^{-1}(\omega) + 2\pi a n_1} \right|^2. \quad (23)$$

Here a is the parameter characterizing the structur of the monolayer [3].

In case $\theta \ll 1$ for the normal incidence neglecting corrections of the order of γ^{-2} we get more simple expression:

$$\frac{d^2W_m(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{4e^2 n_1^2 (\omega^2/c^3)}{\left|\alpha^{-1}(\omega) - \pi a n_1\right|^2} \frac{\theta^2}{\left(\theta^2 + \gamma^{-2} + \frac{\omega_{p\,eff}^2}{\omega^2}\right)^2}. \quad (24)$$

Here the effective frequency

$$\omega_{p\,eff} = \begin{cases} 0, & vacuum \\ \omega_p, & matter \end{cases} \quad (25)$$

describes the case when the radiation from a monolayer is formed inside the matter being characterized by some ω_p.

As we work at rather high frequencies, the polarization of the particles the monolayer consists of takes the form

$$\alpha(\omega) = Z\frac{e^2}{m\omega^2}, \quad (26)$$

where Z is the number of electrons in the particle.

Therefore, for X-ray TR from a separate monolayer covering the substrate we obtain from Eq. (24) the final expression:

$$\frac{d^2W_m(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{4e^2 n_1^2 (\omega^2/c^3)}{\left|\frac{m\omega^2}{Ze^2} - \pi a n_1\right|^2} \frac{\theta^2}{\left(\theta^2 + \gamma^{-2} + \frac{\omega_{p\,eff}^2}{\omega^2}\right)^2}. \quad (27)$$

To figure out what the result looks like, let us compare Eq. (27) with the standard one for X-ray TR

$$\frac{d^2W_{film}(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{e^2}{c}\frac{2}{\pi^2}\left(\frac{\omega_{p\,eff}}{\omega}\right)^4 \times \times \frac{\theta^2 \sin^2\left[\frac{l\omega}{4c}\left(\theta^2 + \gamma^{-2} + \frac{\omega_{p\,eff}^2}{\omega^2}\right)\right]}{\left(\theta^2 + \gamma^{-2} + \frac{\omega_{p\,eff}^2}{\omega^2}\right)^2 \left(\theta^2 + \gamma^{-2}\right)^2}. \quad (28)$$

Indeed, the value

$$\left(\frac{l\omega}{4c}\right)^2 \left(\theta^2 + \gamma^{-2} + \frac{\omega_{p\,eff}^2}{\omega^2}\right)^2 = \left(\frac{l}{L_f}\right)^2, \quad (29)$$

where

$$L_f = \frac{4c}{\omega}\frac{1}{\theta^2 + \gamma^{-2} + \omega_{p\,eff}^2/\omega^2} \quad (30)$$

is the formation length for radiation in the substance. In the simplest case $\theta = 0$, $\omega = \gamma\omega_p$ one gets $L_f = \gamma^2\lambda/\pi$, which for thin films used in TRD usually exceeds considerably the film thickness l.

The ratio is then can be written as

$$\frac{d^2W_{film}(\mathbf{n},\omega)}{d^2W_m(\mathbf{n},\omega)} = \frac{(\omega_{p\,eff}/\omega)^4 \left|\frac{m\omega^2}{Z\pi e^2 n_1} - a\right|^2}{2(\omega^2/c^2)(\theta^2 + \gamma^{-2})^2} \left(\frac{l}{L_f}\right)^2. \quad (31)$$

Here we consider two cases. The first one is

$$\frac{m\omega^2}{\pi e^2 n_1} < a. \quad (32)$$

This implies rather low frequencies, which is not the case for TR detectors. The second case is

$$\frac{m\omega^2}{\pi e^2 n_1} > a, \quad (33)$$

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for which we have

$$\frac{d^2W_{film}(\mathbf{n},\omega)}{d^2W_m(\mathbf{n},\omega)} = \frac{c^2}{\omega^2}\frac{2n^2}{Z^2n_1^2\gamma^{-4}}\left(\frac{l}{L_f}\right)^2 = \frac{1}{2}\left(\frac{ln}{Zn_1}\right)^2. \quad (34)$$

Taking $n/n_1 = 10^3\,\mu m^{-1}$, $l = 0{,}6\mu m$, $Z = 6$ we obtain

$$d^2W_m(\mathbf{n},\omega) = \left(2\times10^{-4}\right)d^2W_{film}(\mathbf{n},\omega). \quad (35)$$

which is very small, but in principle can be measured.

CONCLUSION

Thus, we show that at high frequencies the monolayer radiate less intensively, than a thin film. Numerical estimations show, however, that in principle the contribution of monolayers could be measured comparing with that for macroscopically thick film.

The second interesting thing is that the conditions of formation of the radiation field are different for the cases when the graphene monolayer is towards the film or downwards, and this should lead to the asymmetry in case of the stacks of films relatively the direction of the flight of the electrons.

Along with that there is no any extension of the spectrum of X-ray TR, as it can be expected from general consideration, just because the condition of zero-mass for the conductivity electrons in graphene does not work at high frequencies. For all these reasons we can say, that graphene, instead of its interesting physical properties, is not of practical interest for transition radiation based detectors.

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