INCOHERENT AND COHERENT POLARIZATION RADIATION AS INSTRUMENT OF THE TRANSVERSAL BEAM SIZE DIAGNOSTICS*

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Abstract

Polarization radiation, which includes diffraction radiation (DR), transition radiation (TR), Smith-Purcell radiation, and others, can be a good instrument for beam diagnostics. All information about the beam size is contained in the so-called form-factor of the beam. The form-factor represents the sum of two parts corresponding to the coherent and incoherent radiation. Contrary to the general opinion the incoherent part does not always equal unity. In this report we give theoretical description of the incoherent and coherent parts of the form-factor both for Gaussian and uniform distribution of the ultrarelativistic particles in the bunch. The theory constructed describes also the case of beam skimming the target, which leads to mixing of DR and TR. We show that the incoherent part depends on the transversal size of the beam, and dependence differs for different distributions. The role of the incoherent part of the form-factor of the bunch for different parameters is discussed.

INTRODUCTION

Diffraction radiation (DR), Smith-Purcell radiation (SPR), transition radiation (TR) have the similar nature: they arise due to the dynamic polarization of the target material by the Coulomb field of a charged particle. So, they can be called polarization radiation. The theory of polarization radiation from a single particle are well developed, except for X-ray polarization radiation.[1]

X-ray radiation makes the sub-micron beam diagnostics possible, because for such short waves the Rayleigh limitation is not a problem.

The spectral-angular distribution of the energy of radiation from a beam can be obtained as the distribution of energy for radiation from a single particle $d^2W_1/d\omega d\Omega$ multiplied by form-factor F [2]:

$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{d^2W_1(\mathbf{n},\omega)}{d\Omega d\omega} F. \quad (1)$$

The form-factor has two terms, corresponding to the coherent and incoherent radiation:[2-6]

$$F = NF_{inc} + N(N-1)F_{coh}, \quad (2)$$

with N being the number of the particles in the bunch.

Usually the incoherent form-factor is supposed to be equal to unity, like it occurs for synchrotron radiation or transition radiation from an infinite media of an infinite slab. For polarization radiation from the target edge (DR, SPR, TR from a finite slab) the incoherent form-factor does not equal unity:

$$F_{inc} \neq 1. \quad (3)$$

This fact was explained in detail in the paper [2].

One of the main features of the spectral-angular distribution of the polarization radiation is its dependence on the impact-parameter, i.e. the shortest distance between the moving charge and the target surface; see the parameter h in Fig. 1. This dependence is:

$$\frac{d^2W_1(\mathbf{n},\omega)}{d\Omega d\omega} \propto \exp(-2\rho h), \quad (4)$$

where ρ is some function which will be defined below. From Eq. (4) it is clear that the far the particle from the target surface is, the less intensive the radiation is:
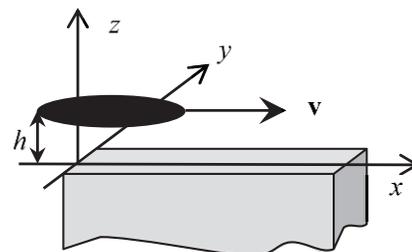

Figure 1: Generation of the radiation by moving the bunch near the target.

FORM-FACTOR

The way to obtain the formula for the coherent and incoherent form-factor was described in [7] for the Diffraction radiation and Smith-Purcell radiation:

$$F_{inc} = \int_V d^3r \left| e^{-i\mathbf{r}\mathbf{q}} \right|^2 f(\mathbf{r}),$$

$$F_{coh} = \left| \int_V d^3r\, e^{-i\mathbf{r}\mathbf{q}} f(\mathbf{r}) \right|^2, \quad (5)$$

where the integral is over the bunch volume V, $f(\mathbf{r})$ is the function of distribution of the particles in the bunch written in the system where the bunch is at rest.

For the polarization radiation (diffraction radiation, Smith-Purcell radiation) the vector **q** has the form [7]:

$$\mathbf{q} = \left(\frac{\omega}{c\beta}, k_y, -i\frac{\omega}{\gamma\beta c}\sqrt{1+\gamma^2\beta^2 n_y^2} \right), \qquad (6)$$

where $\beta = v/c$, v is the particle velocity, c is the speed of light, γ is the Lorentz factor, $\mathbf{k} = \mathbf{n}\omega/c$ is the wave-vector of the radiation. The coordinate system is shown in Fig. 1.

If the distribution in the bunch does not depend on the relative position of the particles, then the transversal and longitudinal parts of the form-factor can be written as the separate terms:

$$\begin{aligned} F_{inc} &= F_{inc}(r_0), \\ F_{coh} &= F_{tr}(r_0)F_l(l), \end{aligned} \qquad (7)$$

where r_0 is the transversal bunch size, l is the length of the bunch.

For example, for the cylindrical bunch of the length l and the radius r_0 with the uniform distribution of the particles it is easy to find [2]:

$$\begin{aligned} F_{inc} &= 2\frac{I_1(2\rho r_0)}{2\rho r_0}, \\ F_{coh} &= 4\frac{\sin^2(\omega l/2v)}{(\omega l/2v)^2}\frac{I_1^2(r_0\omega/c\beta\gamma)}{(r_0\omega/c\beta\gamma)^2}, \end{aligned} \qquad (8)$$

where $\rho = \frac{\omega}{\gamma\beta c}\sqrt{1+\gamma^2\beta^2 n_y^2}$, and $I_1(x)$ is the modified Bessel function of the first order.

For the Gaussian distribution:

$$f(\mathbf{r}) = \frac{1}{\sigma_x\sigma_y\sigma_z}\frac{1}{\left(\sqrt{\pi}\right)^3}\exp\left[-\frac{x^2}{\sigma_x^2}-\frac{y^2}{\sigma_y^2}-\frac{z^2}{\sigma_z^2}\right] \qquad (9)$$

the calculations are more difficult and the result can be seen in the paper [8]. In a brief form these formulae can be written as:

$$\begin{aligned} F_{inc} &= \frac{1}{2}\exp\left[\rho^2\sigma_z^2\right]\left(1-\Phi\left[\rho\sigma_z-\frac{h}{\sigma_z}\right]\right), \\ F_{coh} &= \frac{1}{2}\exp\left[\frac{\rho^2\sigma_z^2}{4}\right]\left(1-\Phi\left[\frac{\rho\sigma_z}{2}-\frac{h}{\sigma_z}\right]\right)e^{-\frac{\omega^2\sigma_x^2}{4c^2\beta^2}-\frac{k_y^2\sigma_y^2}{4}}, \end{aligned} \qquad (10)$$

where $\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ is the Laplace function. It may seem that these exponents increase indefinitely with

$\omega \to \infty$, which is contained in ρ. However, it is known that for $x \to \infty$ the asymptotic form of Laplace function is

$$\Phi(x \to \infty) \approx 1-\frac{e^{-x^2}}{x\sqrt{\pi}}. \qquad (11)$$

Thus, F_{coh} and F_{inc} decrease with growing of frequency from Eqs. (8) and (10) it is seen that the transversal size of the beam can be detected from both the coherent and incoherent radiation.

ANALYSIS OF THE FORM-FACTORS

Below we will consider both the incoherent and coherent form-factors taking into account the exponent in Eq.(4), because it is the term which can strongly influence the radiation intensity and properties, while all other terms, contained in the distribution of energy of radiation from a single particle $d^2W_1/d\omega d\Omega$, do not.

For the analysis let us define the polar angle θ and the azimuthal angle φ of the radiation as:

$$\begin{aligned} n_x &= \cos\theta\cos\varphi, \\ n_y &= \cos\theta\sin\varphi, \\ n_z &= \sin\theta. \end{aligned} \qquad (12)$$

Comparison of Eq. (8) and (10) shows that coherent form-factor of the bunch with the Gaussian distribution depend on the angles of radiation observation, while the coherent form-factor factor of the bunch with the uniform distribution dos not depend on.

In Figs. 2 and 3 the form-factors multiplied by the exponent from Eq. (4) both for the uniform distribution (black curves) and for the Gaussian distribution (red dashed curves) are shown in dependence on the wavelength of radiation.

The dependence of the form-factors on the transversal size of the bunch $r_0 = \sigma_z = \sigma_y$ is shown in Fig. 4. Here the black curve corresponds to the uniform distribution, and the red dashed curve corresponds to the Gaussian distribution.

It should be noted that the constructed theory is valid in X-ray frequency domain, i.e. at frequencies

$$\omega \gg \omega_p \qquad (13)$$

where the responses of the dielectric and metal to the external field are the same. Here ω_p is the plasma frequency. That is why the behavior of the curves at long wavelength can be incorrect.
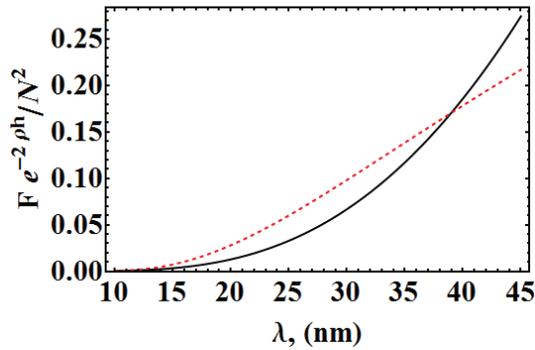
Figure 2: Form-factors normalized to N^2 for coherent radiation for the uniform distribution (black curves) and for the Gaussian distribution (red dashed curves) of the particles in the bunch. Here $\gamma = 10^4$, $h = 50\ \mu m$, $\theta = \gamma^{-1}$, $\phi = 0$, $\sigma_y = \sigma_z = 40\ \mu m$, $r_0 = \sigma_z$, $l = 2\sigma_x$, $\sigma_x = 5\ nm$.
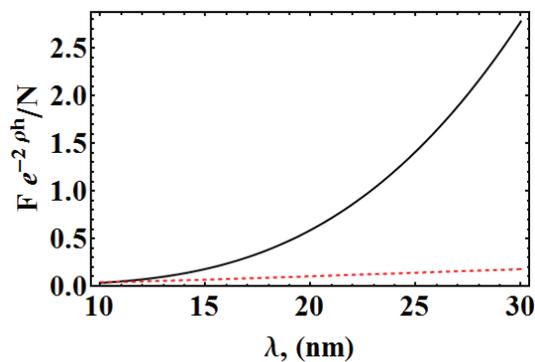

Figure 3: Form-factors normalized to N for incoherent radiation for the uniform distribution (black curves) and for the Gaussian distribution (red dashed curves) of the particles in the bunch. All the parameters are the same as in Fig. 2, but $\sigma_x = 80\ \mu m$.
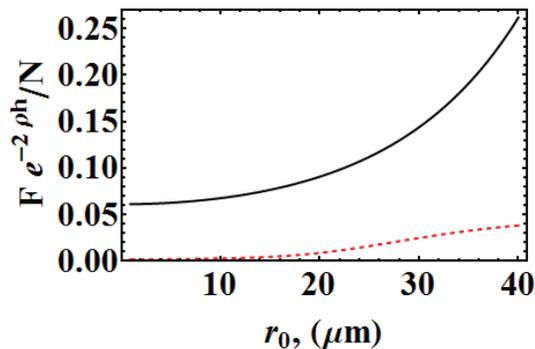

Figure 4: Form-factors normalized to M for the uniform distribution (black curves) and for the Gaussian distribution (red dashed curves) of the particles in the bunch. All the parameters are the same as in Fig. 2, but $\lambda = 10\ nm$.

CONCLUSION

Thus, the theory developed describes the diffraction radiation of electron ultra-relativistic beams in X-ray frequency domain. Speaking of Diffraction radiation we mean, of course, also Smith-Purcell radiation, which can be considered as DR in case the target is a periodical grating.

Using short wavelength radiation, one can define the beam size with a good accuracy, which is enough even for submicron beam diagnostics. The obtained analytical expressions give us the intensity of radiation as a function of its size, i.e., measuring the intensity one can retrieve the information about the bunch.

What is interesting is that the dependence on the bunch size is contained not only in the coherent radiation, but also in the incoherent radiation, which opens new horizons in the beam diagnostics technique.

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