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REVIEW OF BEAM-BASED TECHNIQUES OF IMPEDANCE MEASUREMENT*

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Abstract

Knowledge of a vacuum chamber impedance is necessary to estimate limitations of particle beam intensity. For new accelerator projects, minimization of the impedance is a mandatory requirement. The impedance budgets are computed during the machine design. Beam-based measurements of the impedance are usually carried out at the beginning of the machine commissioning. Comparisons of the impedance computations and measurements often show significant discrepancies, a factor of two or even more is not something unusual. Since the accuracy of impedance computations is not sufficient, the beam-based measurements are important to estimate the machine impedance and to predict stability conditions for high-intensity particle beams.

WAKEFIELDS AND IMPEDANCES

The beam intensity in storage rings is usually limited by its interaction with electromagnetic fields induced in a vacuum chamber by the beam itself (wakefields). The beam-wakefield interaction is described in terms of wake functions defined as normalized integrals of the Lorentz forces that act on a test particle moving behind a leading particle which excites the wakefields. The velocity of both particles is $\mathbf{v}$ ($|\mathbf{v}| = c$). To analyze the beam stability in most practical cases, it is enough to consider only monopole longitudinal $W_\parallel$ and dipole transverse $\mathbf{W}_\perp$ wake functions. The longitudinal wake function is [1]:

$$W_\parallel(\tau) = -\frac{1}{q}\int_{-\infty}^{\infty} E_z(t,\tau)\ \mathrm{d}t\ , \qquad (1)$$

where q is the charge of leading particle, $\tau = s/c$, s is the distance between the leading and the trailing particles, c is the speed of light. The transverse wake function is defined similarly to the longitudinal one but the integral is normalized by the dipole moment qr of the leading particle; $\mathbf{W}_\perp$ is a vector with horizontal and vertical components:

$$\mathbf{W}_\perp(\tau) = -\frac{1}{q\,r}\int_{-\infty}^{\infty} [\mathbf{E}(t,\tau) + \mathbf{v}\times\mathbf{B}(t,\tau)]_\perp\ \mathrm{d}t\ . \qquad (2)$$

The longitudinal and transverse wake functions are related to each other by the Panofsky-Venzel theorem [1, 2].

For a beam with arbitrary charge distribution, its interaction with wakefields is described by the wake potential:

$$V(\tau) = \int_0^{\infty} W(t)\lambda(\tau - t)\mathrm{d}t\ , \qquad (3)$$

where $\lambda(t)$ is the longitudinal charge density normalized as $\int_{-\infty}^{\infty}\lambda(t)\mathrm{d}t = 1$.

In the frequency domain, each part of the vacuum chamber is represented by a frequency-dependent longitudinal $Z_\parallel$ and transverse $Z_\perp$ impedances defined as Fourier transforms of the corresponding wake functions. For any vacuum chamber component, the impedance can be approximated by a set of equivalent resonators plus the resistive-wall impedance. The beam interaction with the narrowband impedance and with the broadband one can be analyzed separately. We can assert that the narrowband impedance leads to the bunch-by-bunch interaction and can result in multi-bunch instabilities, whereas the broadband impedance leads to the intra-bunch particle interaction and can cause single-bunch instabilities.

Computation of the impedance budget is an essential part of accelerator design. The impedance of a vacuum chamber is computed by element-wise wakefield simulations using 3D finite-difference simulation codes solving Maxwell equations with the boundary conditions determined by the chamber geometry. The fields are excited by a bunched beam with pre-defined charge distribution. The code output is a wake potential (3) and the impedance is calculated as $Z(\omega) = \tilde{V}(\omega)/\tilde{\lambda}(\omega)$, where $\tilde{V}$ and $\tilde{\lambda}$ are the Fourier transforms of the wake potential and of the longitudinal charge density, respectively. So the bandwidth of the impedance derived from the simulated wake potential is limited by the bunch spectrum width which is inversely proportional to the bunch length defined for the simulation. The mesh size of the solver is very important, it should be small enough to get reliable results for a given bunch spectrum. For a typical bunch length of few millimeters, full 3D simulation of wakefields in a big and complex structure is quite difficult.

Beam-based measurement of the impedance is an important part of a machine commissioning. Comparisons of impedance computations and beam-based measurements show significant discrepancies for many machines, a factor of two or even more. There are many publications describing thorough calculations of impedance budgets and finally the total impedance is multiplied by a "safety factor" of two. Some accelerator facilities have not achieved their design beam currents because the collective effects nave not been predicted correctly at the design stage. Since the accuracy of impedance computations is not sufficient, the beam-based measurements are important to estimate the machine impedance and to predict stability conditions for high-intensity particle beams.

LONGITUDINAL BROADBAND IMPEDANCE

For the longitudinal broadband impedance, the measurable single-bunch effects are: current-dependent bunch lengthening, synchronous phase shift, and energy spread

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growth due to microwave instability. These effects are dependent on integral parameters combining the impedance and the bunch power spectrum: the effective impedance and the loss factor. If the bunch length is much shorter than the ring average radius, the normalized effective impedance $(Z_\parallel/n)_{\rm eff}$ is defined as

$$\left(\frac{Z_\parallel}{n}\right)_{\rm eff} = \frac{\int_{-\infty}^{\infty} Z_\parallel(\omega)\frac{\omega_0}{\omega}h(\omega)\mathrm{d}\omega}{\int_{-\infty}^{\infty} h(\omega)\mathrm{d}\omega}\,, \tag{4}$$

where $Z_\parallel(\omega)$ is the frequency-dependent longitudinal impedance, $n = \omega/\omega_0$ is the revolution harmonic number, $\omega_0 = 2\pi f_0$ is the revolution frequency, $h(\omega) = \tilde{\lambda}(\omega)\tilde{\lambda}^*(\omega)$ is the bunch power spectrum, $\tilde{\lambda}(\omega)$ is the Fourier transform of the longitudinal charge density $\lambda(t)$. For a Gaussian bunch, $h(\omega) = e^{-\omega^2\sigma_t^2}$, where $\sigma_t = \sigma_z/c$, σ_z is the bunch length. If the low-frequency longitudinal impedance is assumed to be inductive, the normalized impedance $Z_\parallel/n$ is frequency-independent.

The loss factor $k_\parallel$ determines the coherent loss $\Delta E = k_\parallel q^2$ of the beam energy caused by the beam-impedance interaction; q is the bunch charge. The loss factor can be expressed in terms of the wake potential $V_\parallel$ or of the impedance $Z_\parallel$:

$$k_\parallel = \int_{-\infty}^{\infty} V_\parallel(t)\,\lambda(t)\mathrm{d}t = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_\parallel(\omega)\,h(\omega)\mathrm{d}\omega\,. \tag{5}$$

If the bunch is not very short (few millimeters is a typical bunch length for electron/positron storage rings), the measurable single-bunch effects resulted from the beam interaction with a complex impedance produced by computer simulations can be described with reasonable accuracy using a simple broadband resonator model. For longer bunches, even the simplest inductive model is acceptable.

The longitudinal broadband impedance can be estimated by direct measurement of the bunch profile as a function of beam intensity using a streak-camera or a dissector tube, or by indirect measurement of the bunch length using the bunch spectrum width from a button-type pickup electrode.

Interaction of a beam with a broadband impedance deforms the longitudinal bunch profile $\lambda(t)$. A zero-intensity bunch profile $\lambda_0(t)$ is Gaussian. Below the microwave instability threshold, $\lambda(t)$ as a function of the bunch current $I_b = qf_0$ can be described by the Haissinski equation [3]:

$$\lambda(t) = K\lambda_0(t)\exp\left(-\frac{\alpha I_b}{\omega_s^2\sigma_0^2 E/e}\int_{-\infty}^{t} S(t+\tau)\lambda(\tau)\mathrm{d}\tau\right)\,, \tag{6}$$

where $S(t) = \int_0^t W_\parallel(\tau)\mathrm{d}\tau$, α is the momentum compaction, ω_s is the synchrotron frequency, E is the beam energy. The normalizing factor K is determined by the condition $\int_{-\infty}^{\infty}\lambda(t)\mathrm{d}t = 1$. The Haissinski equation can be solved numerically for a certain impedance model (e.g. broadband resonator) and the model parameters can be find by fitting the measured bunch profile with the equation solution [4]. The intensity-dependent bunch lengthening ($\alpha > 0$) can be approximately described by a cubic equation [5]:

$$\left(\frac{\sigma_t}{\sigma_{t0}}\right)^3 - \frac{\sigma_t}{\sigma_{t0}} = \frac{I_b\,\alpha}{\sqrt{2\pi}\,\nu_s^2\,(\omega_0\sigma_{t0})^3\,E/e}\,\mathrm{Im}\left(\frac{Z_\parallel}{n}\right)_{\rm eff}\,, \tag{7}$$

where $\nu_s = \omega_s/\omega_0$ is the synchrotron tune, σ_{t0} is the r.m.s. bunch length at zero intensity.

Above the microwave instability threshold, the longitudinal beam dynamics is characterized by the intensity-dependent energy spread growth and a turbulent bunch lengthening. The relative energy spread $\delta \equiv \sigma_E/E$ can be estimated from a measured r.m.s. horizontal size σ_x determined by the combination of betatron and synchrotron contributions: $\sigma_x^2 = \beta_x\varepsilon_x + (\eta_x\delta)^2$, where β_x is the beta function, ε_x is the emittance, η_x is the dispersion. The transverse beam size is usually measured by a visible light monitor or a pin-hole X-ray camera located in a dispersive section. The beam dynamics above the threshold is very complex, and comprehensive numerical simulations are needed to fit the measured intensity-dependent energy spread with the model impedance.

Note, it is practically impossible to find the microwave instability threshold from the measured bunch lengthening although it is clearly visible on a graph of the measured intensity-dependent energy spread [6], this is also confirmed by numerical simulations [7]. Thus, formula (7) could be useful to fit the bunch lengthening even if the beam current exceeds the microwave instability threshold.

The loss factor can be estimated from the measured intensity-dependent shift $\Delta\phi_s$ of the beam synchronous phase. This phase shift is caused by the coherent energy loss, which is compensated in the accelerating RF cavities every beam turn, as well as the energy loss caused by synchrotron radiation. The formula of synchronous phase shift is derived from the energy balance of the beam:

$$\Delta\phi_s = \frac{I_b k_\parallel}{f_0 V_{\rm RF}\cos\phi_{s0}}\,, \tag{8}$$

where $V_{\rm RF}$ is the RF voltage, ϕ_{s0} is the synchronous phase at zero current. The loss factor $k_\parallel$ depends on the bunch length growing with the beam current, so the phase shift as a function of the beam current is non-linear. For a low-current beam, we can neglect the bunch lengthening and, with this approximation, the phase shift $\Delta\phi_s$ can be assumed proportional to the zero-current loss factor.

The current-dependent shift of synchronous phase can be measured directly using synchrotron light diagnostics (streak-camera, dissector tube) or RF system diagnostics. To reduce the systematic error resulted from the drift or jitter of the diagnostic instruments, the two-bunch technique is useful. The longitudinal profiles of two bunches are measured simultaneously, one bunch has variable intensity, whereas the other bunch with a fixed intensity is the reference, so the systematic error caused by the instrument drift is eliminated.

The other technique is based on measurement of the closed orbit distortion caused by the coherent energy loss [8]. If the dispersion and its derivative is zero in the accelerating

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RF cavities, the orbit deviation can be assumed proportional to the dispersion $\eta(s)$. The loss factor $k_\parallel$ can be estimated by measuring the orbit distortion $\Delta x(s)$ as a function of the beam current variation ΔI_b [9]:

$$k_\parallel = \frac{f_0}{\Delta I_b}\,\frac{E}{e}\,\frac{\Delta x(s)}{\eta(s)}\,. \tag{9}$$

If the RF cavities are located in several places, this method can be used to measure the longitudinal loss factor of a section between the cavities. High-precision beam position monitors (BPMs) are now a standard component of beam diagnostics, so the beam orbit can be measured precisely [10].

TRANSVERSE BROADBAND IMPEDANCE

For the transverse broadband impedance, the measurable effects are current-dependent shift of betatron frequencies and rise/damping time of the chromatic head-tail effect. Similar to the longitudinal case, these effects are dependent on integral parameters combining the impedance and the bunch spectrum: transverse effective impedance $Z_{\perp\mathrm{eff}}$ and dipole kick factor $k_\perp$. If the bunch length is much shorter than the ring average radius, the effective impedance is defined as

$$Z_{\perp\mathrm{eff}} = \frac{\int_{-\infty}^{\infty} Z_\perp(\omega) h(\omega-\omega_\xi)\mathrm{d}\omega}{\int_{-\infty}^{\infty} h(\omega-\omega_\xi)\mathrm{d}\omega}\,, \tag{10}$$

where $Z_\perp(\omega)$ is the frequency-dependent transverse impedance, $\omega_\xi = \xi\omega_0/\alpha$, $\xi = d\nu_\beta/(dE/E)$ is the chromaticity, $\omega_\beta = \nu_\beta\omega_0$ is the betatron frequency.

The transverse dipole kick $\Delta x'$ caused by the beam-impedance interaction is

$$\Delta x' = \frac{q}{E/e} k_\perp x\,, \tag{11}$$

where x is the beam transverse offset, $k_\perp$ is the kick factor

$$k_\perp = \int_{-\infty}^{\infty} V_\perp(t)\,\lambda(t)\mathrm{d}t = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_\perp(\omega)\,h(\omega)\mathrm{d}\omega\,. \tag{12}$$

If the bunch is not very short, the transverse single-bunch effects can be analyzed using the simplified impedance models such as broadband resonator or pure inductive impedance for longer bunches, similarly to the longitudinal impedance.

Interaction of a bunched beam with the broadband impedance results in the transverse mode coupling [11]. If the chromaticity is zero, a fast head-tail instability occurs above the threshold beam current when the coherent (0-th) mode is coupled with the lowest (−1-st) head-tail mode. If the chromaticity is non-zero, the coherent mode damps upon the positive chromaticity and becomes unstable when the latter is negative, and the higher-order head-tail modes behave oppositely. The rise/damping rates decrease rapidly with the mode number, so only few lowest modes are essential and the eigenmode analysis is efficient. The complex frequency Ω_l of l-th head-tail mode can be found solving the eigenvalue problem [11], $\mathrm{Re}\Omega_l$ is the intensity-dependent shift of the mode frequency, $\mathrm{Im}\Omega_l$ is the rise/damping rate.

For the coherent mode, the intensity-dependent tune shift $\Delta\nu_\beta$ and the chromatic damping rate τ_ξ^{-1} can be obtained by spectral analysis of beam oscillations measured by a turn-by-turn BPM. The impedance model parameters are estimated by fitting the measured data with a solution of the eigenvalue problem [4]. If the frequency shift of 0-th mode is small compared with the synchrotron frequency ω_s, the linear approximations [11] for $\Delta\nu_\beta$ and τ_ξ^{-1} are applicable:

$$\Delta\nu_\beta = -\frac{I_b}{2\omega_0 E/e}\sum_i \beta_i k_{\perp i}\,,\quad \tau_\xi^{-1} = \frac{I_b\xi\omega_0}{2\pi\alpha E/e}\sum_i \beta_i \mathrm{Re}Z_{\perp i}\,. \tag{13}$$

Here the summation is over the whole ring, β_i is the amplitude betatron function at the location of i-th local impedance.

LOCAL IMPEDANCE

A local transverse impedance acts on the beam as a defocusing quadrupole, strength of which depends on the beam intensity. The wakefield kick (11) is proportional to the beam charge and its transverse offset at the impedance location.

Measurement of the intensity-dependent betatron phase advance $\mu(s)$ along the ring allows determining the contributions of different sections of the vacuum chamber into the coherent shift of betatron tune. In such a way, one can obtain the azimuthal distribution of the transverse impedance [9].

$$\Delta\mu(s) = -\frac{\Delta I_p}{8\pi CE/e}\int_0^s \beta(\zeta)\,\mathrm{Im}Z_\perp(\zeta)\,\mathrm{d}\zeta\,, \tag{14}$$

where I_p is the peak bunch current ($I_p = \frac{\sqrt{2\pi}}{\omega_0\sigma_t} I_b$ for a Gaussian bunch), C is the ring circumference. Accuracy of this technique is determined by the turn-by-turn resolution of the BPMs, the signals of which are used to calculate the betatron phase. Typically, the intensity-dependent change of the BPM-to-BPM phase advance is rather small, so this technique requires very good turn-by-turn BPM resolution.

The orbit bump method [12, 13] is more sensitive because the BPMs are used in the narrowband orbit mode rather than in the broadband turn-by-turn mode and the noise is much smaller. This method is based on the measurement of a wave-like orbit distortion created by the local wakefield kick (11). If a local orbit bump is created at the impedance location s_0 , the intensity-dependent orbit distortion is:

$$\Delta x(s) = \frac{\Delta q}{E/e} k_\perp x_0 \frac{\sqrt{\beta(s)\beta(s_0)}}{2\sin\pi\nu_\beta}\cos\left(|\mu(s)-\mu(s_0)| - \pi\nu_\beta\right)\,, \tag{15}$$

where x_0 is the orbit bump height. This wave-like orbit distortion can be measured using BPMs, and the wave amplitude is proportional to the kick factor at the bump location. For better accuracy, the systematic error caused by intensity-dependent behavior of the BPM electronics is also measured and then subtracted.

Rapid evolution of BPM electronics allows us to improve much the method accuracy. Now we can measure the orbit distortion of the order of several micrometers [14] compared to the 100-micrometer orbit distortion measured at the very

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beginning of the bump method development [12]. Further improvement looks problematic because of the systematic errors caused by hysteresis effects of orbit correctors and by the orbit drifts during the measurement.

A technique, which significantly improves the accuracy of the bump method has been recently developed and tested [15]. This technique is based on an AC orbit bump created by sine-wave excitation of four fast correctors adjacent to the section, impedance of which is measured. The narrowband sine-wave signals provide better signal-to-noise ratio. Use of fast correctors eliminates the systematic error caused by hysteresis. The error caused by orbit drifts is also suppressed because the measured signal is not affected by the orbit motion outside the excitation frequency range. The resolution is good enough to measure the orbit distortion of the order of 0.1 μm, which is an order of magnitude smaller than the sensitivity of the conventional bump method.

To measure the impedance of a vacuum chamber component with variable geometry such as beam scrapers or in-vacuum undulators, both orbit and turn-by-turn techniques are effective [10, 16]. Using the reference bunch technique and precise BPMs, a contribution of the movable element to the total betatron tune shift can be accurately measured.

TRANSVERSE NARROWBAND IMPEDANCE

The transverse coupled-bunch instability (CBI) is driven by long-range wakefields (narrowband impedance), usually by trapped modes in cavity-like strictures in the vacuum chamber and by resistive-wall impedance. For M equally-spaced rigid bunches, the complex frequency shift $\Delta\Omega = \Omega - \omega_\beta$ of the n-th CBI mode is [11]:

$$\Delta\Omega_n = -\frac{i}{4\pi}\frac{\omega_0\beta}{E/e} M I_b \sum_{p=-\infty}^{\infty} Z_\perp\left((pM+n)\omega_0 + \omega_\beta\right) , \tag{16}$$

The frequency shift and rise/damping rate are equal to $\mathrm{Re}\Delta\Omega$ and $\mathrm{Im}\Delta\Omega$, respectively.

Using a transverse multi-bunch feedback system, we can individually excite each CBI mode, then stop the excitation and measure free oscillations (damped or anti-damped) [17]. The model of narrowband transverse impedance includes the resistive-wall impedance [18] and a set of narrowband resonators representing the trapped modes. The parameters of the impedance model are estimated by fitting the measured rise/damping rates of each mode with the values calculated using formula (16). Comparison of the model resonators fitting the measured data with the impedance computed by wakefield simulations is helpful to identify the sources of the resonances.

CONCLUSION

The beam-based measurements are important to estimate the machine impedance and to predict stability conditions for a high-intensity particle beam because the accuracy of impedance budget computations is not sufficient. Comparisons of the impedance computations and beam-based measurements show significant discrepancies for many machines, a factor of two or even more. Integral parameters combining the impedance and the bunch spectrum can be measured: effective impedance, longitudinal loss factor and transverse kick factor. For the longitudinal broadband impedance, the measurable effects are: bunch lengthening, synchronous phase shift, dispersive orbit distortion, and energy spread growth above the microwave instability threshold. These effects can be measured using streak cameras, dissector tubes, RF system diagnostics, BPMs, pin-hole X-ray cameras and synchrotron light monitors. For the transverse broadband impedance, the measurable effects are: coherent betatron tune shift, chromatic head-tail damping, and intensity-dependent orbit distortion. To measure these effects, turn-by-turn BPMs are used. The transverse narrowband impedance can be analyzed using the modal rise/damping times of the transverse coupled-bunch instability measured by bunch-by-bunch feedback systems.

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