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BEAM BASED ALIGNMENT OF SOLENOID*

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Abstract

In this paper, we present the procedure and experimental results of measuring electron beam position and angle relative to the solenoid axis.

INTRODUCTION

Preserving low emittance of the beam in the transport line require good alignment of the beam trajectory with axis of the focusing element [1]. Axis of a quadrupole can be easily found either buy using array of the Hall probes or vibrating wire measurements [2, 3]. Location of the solenoid axis is much less straightforward due to quadratic dependence of file on the deviation from axis. Situation becomes more complicated because beam coming through a center but with angle with respect to the axis is also deflected by solenoid. Therefore, method of determining beam trajectory vs. solenoid axis with beam itself becomes a necessity.

In [1] the solenoid current was scanned and downstream beam position was recorded. The obtained data were fit using the transfer matrix of solenoid calculated by a special script [4].

We utilize similar approach but since we do not have overlapping magnetic and electrical fields the transfer matrix can be calculated from the magnetic measurement data and beam energy. We also generalized the method to an arbitrary transfer function between solenoid and beam position monitor.

METHODE DESCRIPTION

Calculation of Solenoid Transfer Function

The transfer function of a hard edge solenoid (uniform field of certain length) is well known [5]:

$$\begin{pmatrix} \tilde{x} \\ \tilde{x}' \\ \tilde{y} \\ \tilde{y}' \end{pmatrix} = M_{rot} M_f \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} \quad (1)$$

where rotation matrix is:

$$M_{rot} = \begin{pmatrix} cos\theta & 0 & sin\theta & 0 \\ 0 & cos\theta & 0 & sin\theta \\ -sin\theta & 0 & cos\theta & 0 \\ 0 & -sin\theta & 0 & cos\theta \end{pmatrix} \quad (2)$$

and $\theta = \int eB_{\parallel}(s)/2p\, ds$, where e is electron charge, p is its momentum, and $B_{\parallel}(s)$ is axial field. The focusing matrix can be calculated from the following equation:

$$M_f = \begin{pmatrix} cos\theta & sin\theta/k & 0 & 0 \\ -ksin\theta & cos\theta & 0 & 0 \\ 0 & 0 & cos\theta & sin\theta/k \\ 0 & 0 & -ksin\theta & cos\theta \end{pmatrix} \quad (3)$$

where k is $eB_{\parallel}(s)/2p$.

The real solenoid can be represented as series of hard edge solenoids with different magnetic fields.

The rotation of beam trajectory was utilized for beam energy measurement.

Transfer Matrix Calculation

We treated focusing elements as infinitesimally short, separated by known drift space. In this case transfer matrix from the solenoid under test to BPM can be found as:

$$M_{tr} = M_{driftBPM} \prod \left(M_{foci} M_{drifti} \right) M_{sol} \quad (4)$$

where $M_{driftBPM}$ is matrix o drift space between the last focusing element and beam position monitor (BPM) used for position observation, M_{foc} is matrix of the focusing element (either solenoid or quadrupole), and M_{sol} is matrix of the solenoid being scanned.

Transfer matrix of the solenoid was calculated using the magnetic measurement data:

$$M_{sol} = M_{drift2} \prod \left(M_{rot} M_f \right) M_{drift1} \quad (5)$$

where M_{drift1} and M_{drift2} are transfer matrices of drifts with negative length transporting beam from the solenoid center to the start of the data and from the end of the data to the solenoid center, respectively.

Data Processing

For each solenoid current the transfer matrix was calculated according to the Eq. (4). Then two $4\times N$ matrices $\boldsymbol{R}_x$ and $\boldsymbol{R}_y$ were formed:

$$X = \begin{bmatrix} x_1 \\ \cdots \\ x_N \end{bmatrix} = R_x \begin{bmatrix} x_0 \\ x'_o \\ y_0 \\ y'_0 \end{bmatrix} \quad (6)$$

$$Y = \begin{bmatrix} y_1 \\ \cdots \\ y_N \end{bmatrix} = R_y \begin{bmatrix} x_0 \\ x'_o \\ y_0 \\ y'_0 \end{bmatrix} \quad (7)$$

where X and Y are $1\times N$ vectors of recorded positions, and vector W, appearing on the right side, has offsets and angles of the beam with respect to solenoid axis (X and Y are also relative to this axis). The matrices are formed

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from the respective elements of transfer matrix corresponding to the solenoid current.

$$R_x = \begin{pmatrix} M_{tr11\,1} & M_{tr12\,1} & M_{tr13\,1} & M_{tr14\,1} \\ M_{tr11\,2} & M_{tr12\,2} & M_{tr13\,2} & M_{tr14\,2} \\ \cdots & \cdots & \cdots & \cdots \\ M_{tr11\,N} & M_{tr12\,N} & M_{tr13\,N} & M_{tr14\,N} \end{pmatrix} \quad (8)$$

$$R_y = \begin{pmatrix} M_{tr31\,1} & M_{tr32\,1} & M_{tr33\,1} & M_{tr34\,1} \\ M_{tr31\,2} & M_{tr32\,2} & M_{tr33\,2} & M_{tr34\,2} \\ \cdots & \cdots & \cdots & \cdots \\ M_{tr31\,N} & M_{tr32\,N} & M_{tr33\,N} & M_{tr34\,N} \end{pmatrix} \quad (9)$$

We need to find values for the right-hand vector W minimizing deviations from the observed values $\delta X = X - R_x W$ and $\delta Y = Y - R_y W$.

$$\sum_{i=1}^{N}(\delta x_i^2 + \delta y_i^2) = \delta X^T \delta X + \delta Y^T \delta Y \quad (10)$$

Minimum of the Eq. (10) is reached when its partial derivatives over W are equal zero or in the vector form

$$\left(R_x^T R_x + R_y^T R_y\right) \cdot W - R_x^T X - R_y^T Y = 0 \quad (11)$$

or

$$W = \left(R_x^T R_x + R_y^T R_y\right)^{-1}\left(R_x^T X - R_y^T Y\right) \quad (12)$$

Actual readings have offsets due to the misalignments, BPM errors, deflection of the beam during transport. To account for these offsets, we employed two methods. In the first one of the measurement was chosen as reference and data were processed in the incremental form $X = X - x_m$ and $Y = Y - y_m$. The matrices were modified accord-ingly. Such approach was used during the first measurements but it artificially assigns too much weight for one set of data. Therefore, we decided to include into the W two offsets

$$W = \begin{bmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ x_{offs} \\ y_{offs} \end{bmatrix} \quad (13)$$

$$R_x = \begin{pmatrix} M_{11\,1} & M_{12\,1} & M_{13\,1} & M_{14\,1} & 1 & 0 \\ M_{11\,2} & M_{12\,2} & M_{13\,2} & M_{14\,2} & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \\ M_{11\,N} & M_{12\,N} & M_{13\,N} & M_{14\,N} & 1 & 0 \end{pmatrix} \quad (14)$$

$$R_y = \begin{pmatrix} M_{31\,1} & M_{32\,1} & M_{33\,1} & M_{34\,1} & 0 & 1 \\ M_{31\,2} & M_{32\,2} & M_{33\,2} & M_{34\,2} & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & & \\ M_{31\,N} & M_{32\,N} & M_{33\,N} & M_{34\,N} & 0 & 1 \end{pmatrix} \quad (15)$$

The offsets correspond to the expected beam position if beam is injected directly on the solenoid axis.

EXPERIMENTAL RESULTS

For the real-time application, we developed a MATLAB script. Initially we used the first approach (no calculation of the offsets) and part of GUI is shown in Fig. 1. The script was setting the solenoid current, waited for the predefined period to the end of the transient process and measured beam position. At the end of the scan fit was performed and data were displayed.

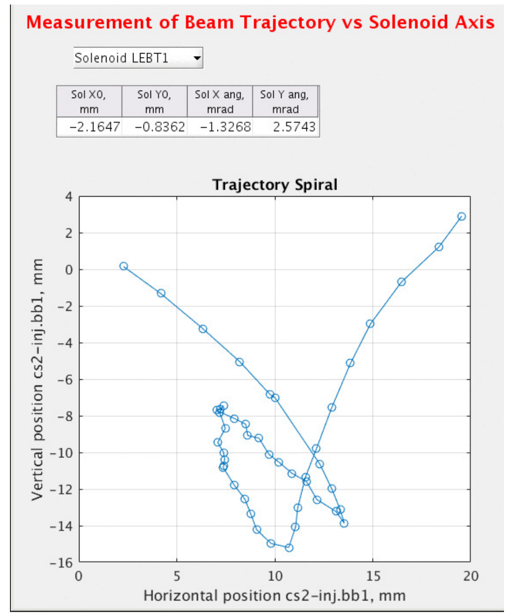

Figure 1: Partial view of a MATLAB GUI for performing solenoid beam based alignment. Not show ae controls for the number of points and scan range.

We tested the accuracy of the procedure by changing the incoming angle of the beam with a horizontal trim in front of the solenoid. The results of the test are shown in Table 1. As one can see the agreement is very good. The effect on the vertical plane is most likely due to the roll of the trim.

Table 1: Results of the Procedure Test with Beam

I_{trim}, A	**Bend Angle, mrad**	**X', mrad**	**Y', mrad**
-2.6	10.1	0.23	4.09
0.0	0.0	-10.59	3.29
1.0	-3.9	-14.69	3.05

After the verification, the procedure was routinely used for orbit measurement and correction. The data of the scan were saved for the later analysis.

Later we have developed script for the analysis of the saved using the second method with fitting of BPM offsets. The results of the analyses are shown in Fig. 2. The measurements were performed at another accelerator with much lower beam energy.

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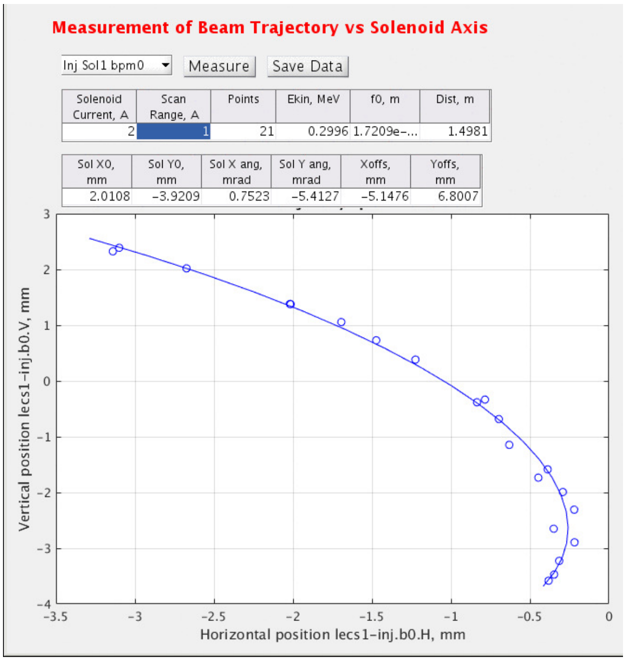

Figure 2: The results of beam based alignment with fitting of the BPM offset.

CONCLUSIONS

We have developed method of the beam based alignment of a solenoid in generalized beam transport line with multiple focusing elements. The method requires knowledge of the axial field of solenoid and beam energy.

The method was tested experimentally on two accelerators and three types of solenoids, and successfully used for the routine measurement of beam position and angle in solenoids.

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