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TIME-RESOLVED ELECTRON-BUNCH DIAGNOSTICS USING TRANSVERSE WAKEFIELDS*

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Abstract

The development of future free electron lasers (FELs) requires reliable time-resolved measurement of variable ultrashort electron-bunchs characteristics. A possible technique is to streak the bunch in the transverse direction by means of time-dependent external fields. In this paper we explore the possible use of self-generated electromagnetic fields. A passive deflector, consisting of a dielectric-lined waveguide, is used to produce wakefields that impart a time-dependent transverse kick to the relativistic electron bunch passing off-axis. We investigate the technique and its performances and explore its possible application at the Fermilab Accelerator Science and Technology (FAST) facility.

INTRODUCTION

Modern physics accelerators sever a variety of science from high-energy physics and nuclear physics to radiation sources and electron microscopes. Conventional methods used in the preparation of beams for accelerator application often cannot keep pace with the new demands, thus, new approaches continue to emerge. Techniques to tailor the electron beam phase space distribution by means of external and internal fields have come to play an increasingly important role in linear accelerators over the last decade. Precise control of beam phase space distribution is foreseen for beam-driven advanced acceleration techniques and for novel radiation sources including free-electron lasers and THz radiators. A wide of techniques has been developed to utilize the fields to influence the beam distribution. One of the manipulations operates within one degree of freedom, e.g., those based on the use of external and internal fields to control the distribution in one of three 2D phase-spce planes: $(x - p_x)$, $(y - p_y)$, $(z - p_z)$.

In this paper, the self-generated wakfields, as the internal fields, is used as a tool to provide the transverse kick on the beam so as to introduce a correlation between time and the transverse beam distribution. The first part of this paper is to review the transverse equations of motion in the presence of wakefield and explore the use of a passive deflector to provide time-dependent deflecting kick to a relativistic electron bunch. Such a capability could enable the development of new passive (and cheap) beam diagnostics [1]. The passive deflector does not need to be powered and it is easier to be manufactured compared to a rf transverse deflecting structure, thus, resulting in a considerable cost saving. The passive deflector is self-synchronized with the beam by design, being the wakefield excited by the bunch itself when it travels through a dielectric tube. Thus, we could use it to perform time-resolved measurements of a relativistic electron bunch based on the self-transverse wakefield interaction of the beam itself passing off-axis through a dielectric-lined tube and reconstruct the beam profile from the resulting image of the streaked beam on the downstream profile monitor. The second part of this paper is to explore some possible ways to reconstruct the profile of the beam as a way of beam diagnostics.

BASIC EQUATIONS

We first introduce the coordinate system under consideration and take an electron propagating along an accelerator beam line with applied external fields. The transverse coordinates are x and y while the longitudinal laboratory coordinate along the straight beamline is z. In order to quantify the bunch dynamics, it is often convenient to introduced $\zeta(t) \equiv z(t) - c\int_0^t \beta(t')dt'$ where ζ represents the axial position of an electron with respect to the bunch centre ($\zeta = 0$) at the time t. Since the beam dynamics also involves the momenta we introduce p_i the conjugate momenta associated to the spatial coordinates $i = x, y, \zeta$ and note that for a bunch $p_\zeta \gg (p_x, p_y)$. For convenience we also introduce the angular divergence as $x' \equiv \frac{p_x}{p_z}$ and $y' \equiv \frac{p_y}{p_z}$. Finally we introduce the relative momentum spread as $\delta \equiv \frac{p}{\langle p \rangle}$ where $p^2 = p_x^2 + p_y^2 + p_z^2$.

In order to describe the dynamics of a bunch in presence of transverse wakefield it is often useful to describe the bunch as an ensemble of axial slices. The transverse position of these slices at a given position z along the beamline is a function of ζ and parameterized as $\boldsymbol{x}(\zeta, z)$ where the vector $\boldsymbol{x} \equiv (x, y)$. Considering the case of a transverse wakefield giving rise to the transverse Green's function $w_\perp(\zeta)$ along, e.g., the x direction we can write the corresponding transverse horizontal force as

$$F_x(\zeta, z) = e^2 \int_\zeta^\infty d\zeta' \rho(\zeta') w_\perp(\zeta - \zeta') x(\zeta', z) d\zeta' \qquad (1)$$

where e is the electronic charge. Consequently the transverse equation of motion can be written as [2,3]

$$\begin{aligned} \frac{d}{dz}\left[\gamma(z)\frac{d}{dz}x(\zeta, z)\right] + K^2\gamma(z)x(\zeta, z) = \\ r_0 \int_\zeta^\infty d\zeta' \rho(\zeta') w_\perp(\zeta - \zeta') x(\zeta', z) d\zeta' \end{aligned} \qquad (2)$$

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where $r_0 \equiv \frac{e}{mc^2}$ is the classical radius of the electron, $\gamma \equiv \frac{\mathcal{E}}{mc^2}$ is the relativistic Lorentz factor (here $\mathcal{E}^2 \equiv p^2c^2 + m^2c^4$ is the total energy), and K describes external focusing fields.

In a drift space ($K = 0$) and assuming the beam energy remains unchanged $\gamma(s) = \gamma$ and $\frac{d\gamma}{dz} = 0$, the latter equation simplifies to

$$\frac{d^2x(\zeta,z)}{dz^2} = \frac{r_0}{\gamma}\int_\zeta^\infty d\zeta'\rho(\zeta')w_\perp(\zeta-\zeta')x(\zeta',z)d\zeta'. \quad (3)$$

Taking the wakefield to be constantly applied over a length L the previous equation can be integrated to yield

$$\begin{aligned} x'(\zeta,z) &= \frac{dx(\zeta,z)}{dz} \\ &= \frac{Lr_0}{\gamma}\int_\zeta^\infty d\zeta'\rho(\zeta')w_\perp(\zeta-\zeta')x(\zeta',z)d\zeta'. \end{aligned} \quad (4)$$

The most-left equality is valid under the ultra-relativistic approximation $\gamma \gg 1$. Assuming that the slice position does not change during the interaction but only its divergence is affected (this is the so-called "impulse approximation") we can further simplify the previous equation into

$$\begin{aligned} x'(\zeta,z) &= \frac{dx(\zeta,z)}{dz} \\ &= x(\zeta,z)\frac{Lr_0}{\gamma}\int_\zeta^\infty d\zeta'\rho(\zeta')w_\perp(\zeta-\zeta')d\zeta'. \end{aligned} \quad (5)$$

This equation is the basis of transverse-wakefield calculation: knowing the longitudinal charge distribution $\rho(\zeta)$ and the transverse Green's function describing the electromagnetic wake, one can infer the transverse displacement of longitudinal slices.

BEAM DIAGNOSTICS

We now consider the possible use of transverse wake to streak the beam aka to what is commonly done with a transverse-deflecting cavity. This possibility was explored in Ref. [1] where it was pointed out that one could in principle reconstruct the longitudinal distribution and some preliminary results were presented. In this Section we first remind the principle of operation of an active transverse-deflecting cavity and then derive the equation for a passive deflector. We finally explore various ways of retrieving the longitudinal charge distribution.

Analysis of the Active Transverse Deflector

A common time-domain diagnostic method to infer the duration of sub-picosecond employs a transverse-deflecting resonant radiofrequency (RF) cavity [4]. The cavity usually operates on the TM_{110} mode and therefore sustains a transverse time-dependent magnetic field B. As the bunch travels through the cavity [5]. Resulting in a transverse kick (e.g. in the x direction) of the form

$$x'(\zeta,z) \simeq \frac{Lr_0E_0}{\gamma}\sin(k\zeta+\varphi), \quad (6)$$

where L is now the length of the cavity, E_0 the peak electric field provided by the cavity and φ an arbitrary phase shift we henceforth take to be $\varphi = 0$, and $k = \frac{2\pi}{\lambda_{rf}}$ is the wavevector associated to the wave supported by the RF cavity. In practice the bunch length σ_ζ is such that $\sigma_\zeta \ll \lambda_{rf}$ so that the sin() function can be approximated by its first-order Taylor expansion. In such a case we have

$$x'(\zeta,z) = \frac{Lr_0E_0}{\gamma}k\zeta \equiv \kappa\zeta, \quad (7)$$

and the kick is linearly dependent on the bunch longitudinal coordinate. In the previous equation κ is referred to as the normalized kicking strength. A typical experimental setup for measuring the longitudinal bunch distribution consists in recoding the transverse distribution $f_x(x)$ downstream the deflecting cavity. To analyze such a measurement we recall that the transverse phase-space coordinate $\boldsymbol{x} \equiv (x, x')$ downstream of a beamline with transfer matrix R is given by $\boldsymbol{x} = R\boldsymbol{x}_0$ where x_0 is the initial coordinate upstream of the beamline. Taking R to be the transfer matrix from the cavity exit to the observation point we can write for the position of one electron

$$x = R_{11}x_0 + R_{12}x_0', \quad (8)$$

where $x_0' = \kappa\zeta + x_{0,-}'$ with $x_{0,-}'$ understood as the electron's initial angle prior to receiving the deflecting kick. Under such an assumption the horizontal position of an electron at the observation point reduces to

$$\begin{aligned} x &= R_{11}x_0 + R_{12}(\kappa\zeta + x_{0,-}'). \\ &= R_{11}x_0 + R_{12}x_{0,-}' + R_{12}\kappa\zeta \equiv x_\beta + R_{12}\kappa\zeta, \end{aligned} \quad (9)$$

where x_β is position due to the betatronic motion. The latter equation can be rewritten as

$$x = x_\beta + x_\zeta, \quad (10)$$

which simplifies to $x = x_\zeta$ when the deflector is turned off. Introduction the probability distribution for x_β and x_ζ to be respectively $f_\beta(x_\beta)$ and $f_\zeta(x_\zeta)$ and further considering the variables to be independent, the probability distribution associated to x is given by the convolution

$$f(x) = \int_{-\infty}^{+\infty} f_\beta(x_\beta)f_\zeta(x-x_\beta)dx_\beta. \quad (11)$$

The longitudinal distribution is related to $f_\zeta(x)$ via the charge conservation relation $f_\zeta(x)dx = \rho(\zeta)d\zeta$ that is $\rho(\zeta) = |R_{12}\,\kappa|\,f_\zeta\,(R_{12}\,\kappa\zeta)$. Therefore we need to extract the function $f_\zeta(x)$ from Eq. (11). We note that $f(x)$ and f_β can be directly measured by recording the distribution at the observation point respectively with and without powering the deflecting cavity. One can then performed a deconvolution [6] to retrieve $f(x)$. Another possibility is to ensure the beta function at the observation point is very small so that $f_\beta(x_\beta) \simeq \delta(x_\beta)$ [where $\delta()$ is the Dirac's function] consequently simplifying Eq. (11) to $f(x) \simeq f_\zeta(x)$.

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Equations for a Passive Deflector

Given the description of the active deflection scheme, we can now modify the previous equations to apply them to the passive-deflection technique. Equation (9) is especially modified as

$$x = x_\beta + R_{12} x_0'(\zeta), \tag{12}$$

where $x_0'(\zeta) \equiv x'(\zeta, z = 0)$ with $z = 0$ corresponding to the position where the kick is applied (i.e. the center of the deflecting structure in the impulse approximation); see Eq. (5). We point out that $x_\zeta \equiv R_{12} x_0{}'(\zeta)$ is now a nonlinear function of ζ.

Also, using the charge conservation relation $f_\zeta(x)dx = \rho(\zeta)d\zeta$, that is

$$\rho(\zeta) = |R_{12}\frac{dx_0'(\zeta)}{d\zeta}| f_\zeta(x) \tag{13}$$

Here we can get the derivative $\frac{dx_0'(\zeta)}{d\zeta}$ from Eq. (5). The wake function then can be obtained from Eq. [7], here the transverse wake function is the numerical result along ζ when the beam travels through the waveguide with offside $r_0 = b$. Here, the inner radius of the wave guide is $b = 4.50 \times 10^{-4}$ m and outer radius $a = 5.50 \times 10^{-4}$ m and the dielectric constant of the medium is $\epsilon = 4.41$. The result is shown in Fig. 1.

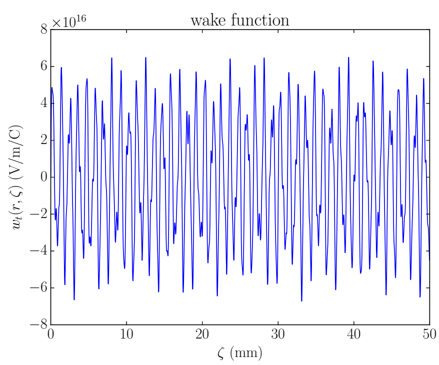

Figure 1: Transverse wake function along ζ.

Thus, through substituting the already known wake function in Eq. (5), we can obtain the derivative $\frac{dx'_0(\zeta)}{d\zeta}$. After we extract $f_\zeta(x)$ from Eq. (11), the the rest of the problem is to solve the self-consistent equation for the probability distribution $\rho(\zeta)$ along ζ. Since we can measure the probability distribution on the monitor, we can directly obtain the values of $f(x)$ and the probability distribution $f_\beta(x)$ when the deflecting is turned off in Eq. (11). Then we can use the deconvolution method to extract the longitudinal distribution $f_\zeta(x)$. Finally, we can get the longitudinal distribution $\rho(\zeta)$ through Eq. (13).

The algorithm implemented to to retrieved the longitudinal bunch distribution $\rho(\zeta)$ given the observed distribution $f_\zeta(x)$ consists of an iterative method summarized in the pseudo code 1. The algorithm we selected is a simple adaptive loop commonly used in feedback control systems. Specifically, we first make a guess of the longitudinal charge density $\rho(\zeta)$ and compute the corresponding projected function $f_\zeta(x)$ from which the incoming charge density is recovered. The adaptive loop consists in readjusting the initial longitudinal charge density given as detailed in the pseudo code 1.

Algorithm 1 Longitudinal charge distribution retrieval

1: **define** $\mathcal{G}$ ▷ gain for the adaptive loop
2: **read** $f_\zeta^m(x)$ ▷ measured beam profile after deconvolution
3: initialize $\rho_0(\zeta)$ ▷ initial (guessed) charge distribution
4: **for** $i \in [0, N]$ **do**
5: $x(\zeta)$=TransWake[Green, $\rho_i(\zeta)$] ▷ compute deflecting kick for a given Green's function
6: $f_\zeta(x)$=Streak[$\rho_i(\zeta)$, $x(\zeta)$] ▷ evaluate streaked profile
7: $\rho_i^e(\zeta) = f_\zeta(x) \times |\frac{dx}{d\zeta}|$ ▷ estimated charge distribution from streaked profile
8: $\rho_{i+1}(\zeta) = \rho_i + \mathcal{G} \times (\rho_i^e(\zeta) - \rho_i)$ ▷ successive approximation
9: $\epsilon_i = \sum_x [|(| f_\zeta(x) - f_\zeta^m(x))]$
10: **end for**
11: **plot** $\rho_N(\zeta)$

The algorithm is proved to be practical by using a generated super gaussian longitudinal distribution as in Fig. 2. The red line is the generated initial distribution and the blue dot line is the reconstructed distribution after enough times of iterations. We can see they fit very well.

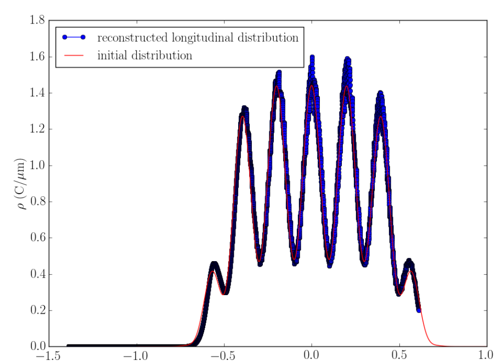

Figure 2: Comparison between the initial longitudinal distribution and the reconstructed distribution

To verify the validation of the method mentioned above, we firstly produce a beam bunch by the ELEGANT [8]. To better verify the this, the beam bunch is designed with some periodic peaks in the direction of x and y. The total charge of

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the beam is $1\times10^{-7}C$ with total particle number of 100, 000. The undeflected beam is shown as in Fig. 3:

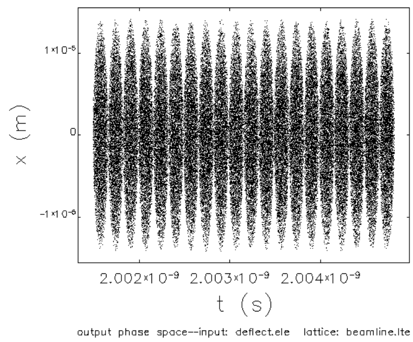

Figure 3: Undeflected beam along t in the longitudinal direction.

After the beam passes through the waveguide with an offside, the beam is deflected by the self-generated wake field, and the shape changes as in Fig. 4 and Fig. 5:

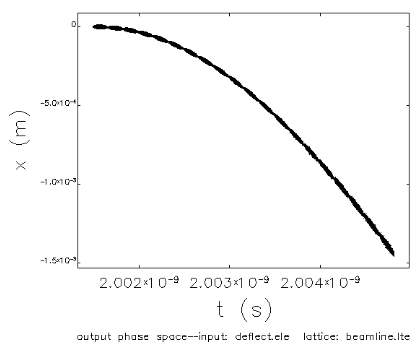

Figure 4: Deflected beam along t in the longitudinal direction.

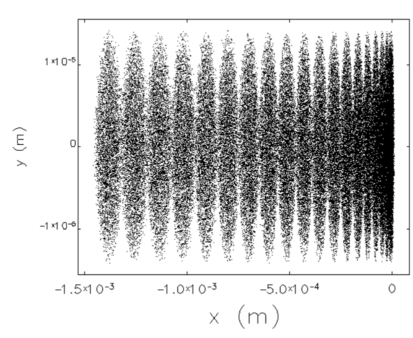

Figure 5: Deflected beam projected on the monitor $x-y$ direction.

We then test the proposed method to reconstruct the longitudinal profile of the beam bunch, using the density distribution of the deflected beam and undeflected beam along x on the $x-y$ monitor. We can see that the result basically fits well with Fig. 3.

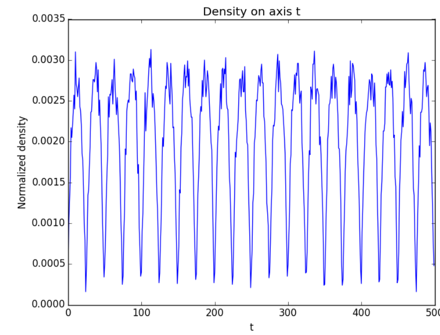

Figure 6: Normalized density along binned t .

CONCLUSION

Thus, in this paper, we proved the deconvolution and iteration method to extract the longitudinal profile of a ultrashort beam in simulation. The result basically agrees with the measurement of the simulation. Thus, the result of the time-resolved measurement based on the passive deflector of a relativistic beam is valid. Also, other methods need to be explored and tested, and further experiment should be verified.

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