

# A METHOD OF CORRECTING THE BEAM TRANSVERSE OFFSET FOR THE CAVITY BUNCH LENGTH MONITOR\*

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## Abstract

Cavity bunch length monitor uses monopole modes excited by bunches within the cavities to measure the bunch longitudinal root mean square (rms) length. It can provide a very high accuracy and high resolution. However, when the bunch passes through the cavities with transverse offset (that is, the bunch moves off the cavity axis), the amplitude of the monopole modes will change and cannot reflect the bunch length precisely. In this paper, a method of correcting the beam transverse offset is proposed. Simulation results show that the method can reduce the error of the bunch length measurement significantly.

## BACKGROUND

The cavity bunch length monitor was widely used for the past few years because of its high accuracy, high resolution and nondestructive measurement [1-4].

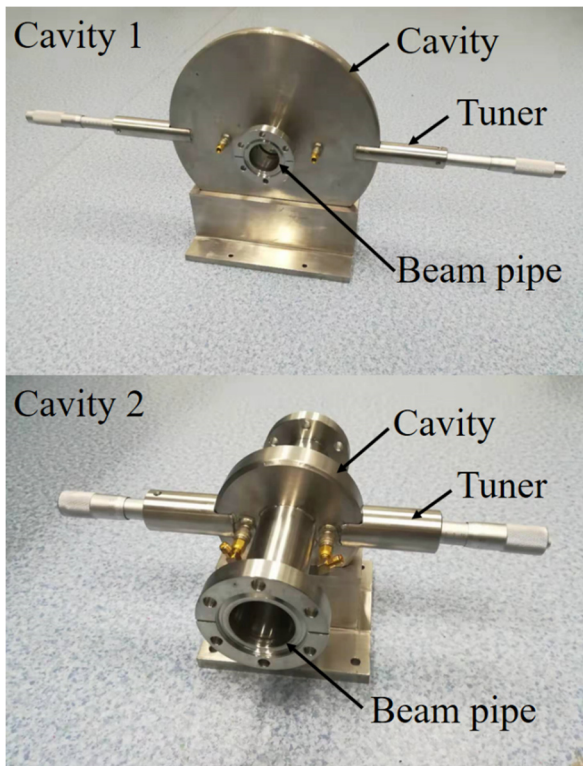


Figure 1: The cavity bunch length monitor.

When a bunch passes through the cavity, the monopole modes will be excited. At least two monopole modes in different frequencies are needed, and the bunch length  $\sigma$  can be obtained by solving the equation [5]

$$\sigma = \sqrt{\frac{2}{\omega_2^2 - \omega_1^2} \ln \frac{k_2 V_1}{k_1 V_2}} \quad (1)$$

Where  $k_1$  and  $k_2$  are constants,  $\omega_1$  and  $\omega_2$  are the working frequencies, and  $V_1$  and  $V_2$  are the output voltage of the two monopole modes respectively. As shown in Fig. 1, the cavity bunch length monitor in the National Synchrotron Radiation Laboratory infrared free-electron laser facility (FELiChEM) is composed of two cavities. The  $TM_{010}$  mode resonating at 0.9515 GHz in the cavity 1 and the  $TM_{020}$  mode resonating at 6.1847 GHz in the cavity 2 are used to measure the bunch length. The radii of the two cavities are 123.3 mm and 46.1 mm respectively. Each cavity has two coaxial probes with axial symmetry that are used to extract the signal of the monopole modes.

## ERROR OF BEAM OFFSET

The output voltage of the monopole mode excited by a bunch within a cavity can be written as [6,7]

$$V = \frac{1}{2} \omega q \sqrt{\frac{Z(R/Q_0)}{Q_{ext}}} \exp\left(-\frac{\omega^2 \sigma^2}{2}\right) \quad (2)$$

Where  $q$  is the bunch charge,  $Z$  is the impedance of the detector, and  $Q_{ext}$  is the external quality factor of the mode.  $(R/Q_0)$  is the normalized shunt impedance, which can be described as

$$\frac{R}{Q_0} = \frac{\left| \int \mathbf{E} ds \right|^2}{\omega U} \quad (3)$$

Where  $U$  is the stored energy of the mode in the cavity, and the numerator indicates integration of the electric field of the mode along the beam orbit. The bunch will "see" different electric fields when it passes through the cavity in different transverse positions, so the monopole modes amplitudes excited by the bunch are different. Monopole modes are axially symmetric and have a field maximum on the cavity axis. The electric field intensity distributions of  $TM_{010}$  mode within cavity 1 and  $TM_{020}$  mode within cavity 2 along the diameter are shown in Fig. 2 and Fig. 3, respectively.

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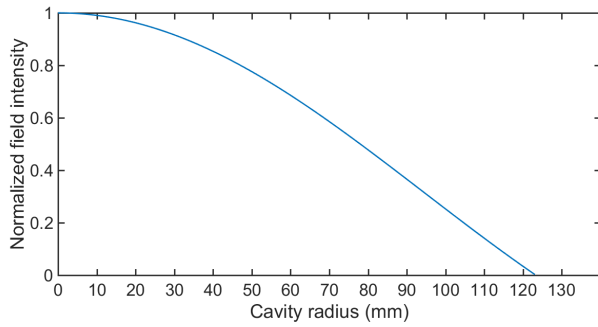


Figure 2: The electric field intensity distributions of  $TM_{010}$  mode within cavity 1 along the diameter.

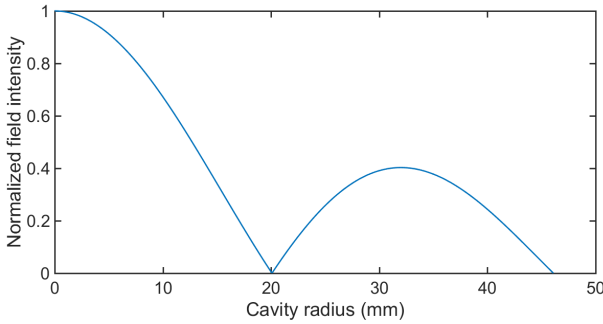


Figure 3: The electric field intensity distributions of  $TM_{020}$  mode within cavity 2 along the diameter.

It can be seen that the monopole modes will weaken slightly when the bunch passes through the cavity with transverse offset, which leads to the error in the bunch length measurement.

At the same time, the dipole modes will be excited when the bunch moves off the cavity axis, which can disturb the monopole mode. The larger the beam offset value is, the stronger the dipole modes become. So this is also one of the error sources.

Reference [5] analyzes quantitatively the influence of beam transverse position on the cavity bunch length measurement. It can be seen that the error increases with increasing beam offset. Therefore, the influence brought by the offset must be considered and the error need to be corrected.

## ERROR CORRECTION

The longitudinal electric field of the  $TM_{0n0}$  mode in a cylindrical cavity can be described as [8]

$$E_z = E_0 J_0\left(\frac{y}{r} u_{0n}\right) e^{-j\beta z} \quad (4)$$

Where  $E_0$  is a constant,  $J_0(x)$  is zero order Bessel function,  $y$  is the beam transverse offset,  $r$  is the cavity radius,  $u_{0n}$  is the  $n$ th root of zero order Bessel function, and the exponential term represents vibration. According to Eq. (3), it can be seen that the amplitude of the monopole mode depends on the Bessel function term. The correction coefficient  $\eta$  can be defined as

$$\eta = \frac{E_{z,axis}}{E_{z,offset}} = \frac{J_0(0)}{J_0\left(\frac{y}{r} u_{0n}\right)} = \frac{1}{J_0\left(\frac{y}{r} u_{0n}\right)} \quad (5)$$

It represents the ratio of the longitudinal electric field of the  $TM_{0n0}$  mode excited by the bunch moving along axis to that excited by the bunch moving off axis. According to Eq. (2) and Eq. (3), the following equation holds

$$V_{offset} \eta = V_{axis} \quad (6)$$

Specific correction method is as follows. First, according to the cavity dimension and the beam position obtained from BPMs, calculate the correction coefficient  $\eta$ . Then, multiply the output voltages of the monopole modes by  $\eta$ . Finally, take the output voltages revised into Eq. (1) and calculate the bunch length.

For cavity 1 working at  $TM_{010}$  mode,  $r$  is 123.3 mm,  $n$  is 1, and  $u_{01}$  is 2.405, while for cavity 2 working at  $TM_{020}$  mode,  $r$  is 46.1 mm,  $n$  is 2, and  $u_{02}$  is 5.520. The correction coefficients for cavity 1 and cavity 2 in different beam offsets are calculated and listed in Table 1 and Table 2.

Table 1: Partial Correction Coefficients for Cavity 1

Offset (mm)	$y/r$	$\eta$
0	0	1.000000
1	0.0081	1.000016
2	0.0162	1.000066
3	0.0243	1.000148
4	0.0324	1.000263
5	0.0406	1.000411

Table 2: Partial Correction Coefficients for Cavity 2

Offset (mm)	$y/r$	$\eta$
0	0	1.000000
1	0.0217	1.000118
2	0.0434	1.000471
3	0.0651	1.001060
4	0.0868	1.001885
5	0.1085	1.002947

## SIMULATION RESULTS

To verify this error correction method, the simulations were performed using the simulation software CST Particle Studio. The two cavities were modeled. The virtual bunches with length of 2 ps and 5 ps at different transverse positions were adopted in the simulation, and the bunch charge was set to 1 nC. Set ports at the actual coaxial antenna positions to obtain the output signals. Simulation results of the bunch length were calculated by taking the output voltage into Eq. (1). The simulation results are shown in Fig. 4, Fig. 5, and Table 3.

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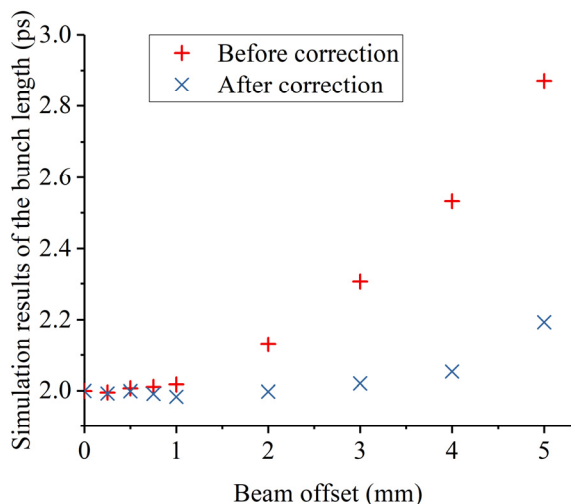


Figure 4: Simulation results of two-picosecond bunch length measurement.

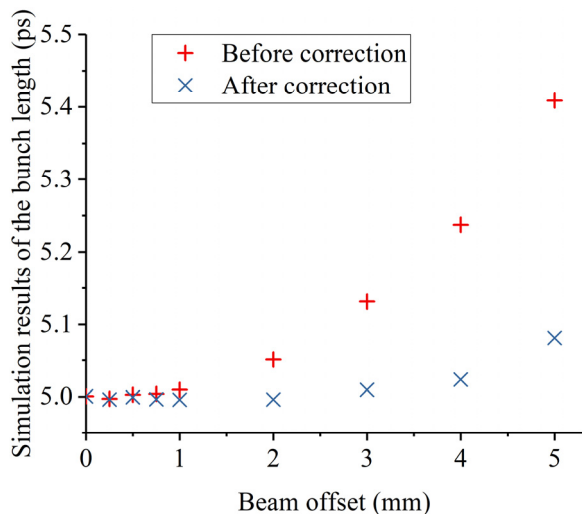


Figure 5: Simulation results of five-picosecond bunch length measurement.

Table 3: Simulation Results of Relative Error

Bunch Length (ps)	Offset (mm)	Relative Error Before Correction (%)	Relative Error After Correction (%)
2	1	0.861	0.853
2	2	6.516	0.128
2	3	15.276	1.009
2	4	26.610	2.678
2	5	43.542	9.671
5	1	0.193	0.082
5	2	1.017	0.075
5	3	2.625	0.190
5	4	4.751	0.473
5	5	8.173	1.632

It can be seen that the relative errors of the bunch length measurements brought by the beam transverse offset reduce significantly after correction. For large offset, this error correction effect is more obvious.

## DISCUSSION AND SUMMARY

In the cavity bunch length monitor using monopole modes, beam transverse offset is one of the important error sources. Generally, it is assumed the bunch passes on the cavity axis because the monopole mode is deemed to be independent of the beam transverse position. For bunch charge measurement, the offset can be ignored. But for bunch length measurement, the influence brought by the offset must be considered. In this paper, taking the cavity monitor at FELiChEM as an example, the reason of error is analyzed, and a correction method is proposed. The simulation results verify the feasibility of this method. Relevant beam experiments will be performed in the future.

However, this method can only correct the errors brought by the beam offset. For the interference from dipole modes, it can do nothing.

In addition, this correction method depends on the information of the beam position obtained from BPM. As an improvement, the cavity itself can also be used to evaluate the beam transverse position roughly, because the different signal of the coaxial probes with axial symmetry only contain the dipole modes. Although this kind of simple "BPM" cannot provide a high resolution, it is enough for the error correction.

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