# VIRTUAL PEPPER-POT TECHNIQUE FOR 4D PHASE SPACE MEASUREMENTS 

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## Abstract

A novel method for 4-dimensional transverse beam phase space measurement is proposed at the Photo Injector Test facility at DESY in Zeuthen (PITZ) for ongoing beam coupling studies. This method is called Virtual Pepper-Pot (VPP), because key principles of the pepper-pot mask scheme are applied. The latter approach is of limited use in high-brightness photo injectors, because of technical reasons. At PITZ a slit scan method instead is the standard tool for reconstruction of horizontal and vertical phase spaces. The VPP method extends the slit scan technique with a special postprocessing. The 4D transverse phase space is reconstructed from a pepper-pot like pattern that is generated by crossing each measured horizontal slit beamlet with all measured vertical slit beamlets. All elements of the 4D transverse beam matrix are calculated and applied to obtain the 4D transverse emittance, 4D kinematic beam invariant and coupling factors. The proposed technique has been applied to experimental data from the PITZ photo injector optimization for 0.5 nC bunch charge. Details of the VPP technique and results of its application will be discussed.

## INTRODUCTION

Good knowledge of particle beam properties is required in many scientific experiments. For example, of particular importance for FEL electron sources is the optimization of the beam emittance. Transverse beam phase space is under ongoing studies at the Photo Injector Test Facility at DESY in Zeuthen (PITZ) towards improving the electron source for the European XFEL and detailed understanding of beam dynamics in photo injectors.

Electron beam asymmetries have been observed at PITZ and gun quadrupoles are installed to correct them to a large extend [1]. Slit scans are used to provide 2-dimensional phase space information, but 4-dimensional phase space characterization is needed to understand beam asymmetries and transverse coupling in details. More sophisticated methods are required, e.g. imaging with a pepper-pot mask [2]. While the aforementioned method should provide the desired information, a pepper-pot mask method has limitations in its applicability to the PITZ setup.
A new technique called Virtual-Pepper Pot is proposed at PITZ. It is extension of the slit scan analysis by using a pair of complementary slit scan measurements. Despite it is inspired by the pepper-pot mask principles, most limitations of the latter are not present. The results of the application of the Virtual-Pepper Pot technique on data of experimental studies using gun quadrupoles provide a view on its usability in practice.

## TRANSVERSE BEAM PHASE SPACE

The particle motion in the transverse plane is described by the 4D transverse phase space of two position coordinates $x$ and $y$ and their corresponding angles $x^{\prime}$ and $y^{\prime}$. The 4D transverse beam matrix (4D TBM) $\sigma^{4 \mathrm{D}}$ is defined as [2, 3]

$$
\sigma^{4 \mathrm{D}}=\left(\begin{array}{cccc}
\langle x x\rangle & \left\langle x^{\prime} x\right\rangle & \langle y x\rangle & \left\langle y^{\prime} x\right\rangle  \tag{1}\\
\left\langle x x^{\prime}\right\rangle & \left\langle x^{\prime} x^{\prime}\right\rangle & \left\langle y x^{\prime}\right\rangle & \left\langle y^{\prime} x^{\prime}\right\rangle \\
\langle x y\rangle & \left\langle x^{\prime} y\right\rangle & \langle y y\rangle & \left\langle y^{\prime} y\right\rangle \\
\left\langle x y^{\prime}\right\rangle & \left\langle x^{\prime} y^{\prime}\right\rangle & \left\langle y y^{\prime}\right\rangle & \left\langle y^{\prime} y^{\prime}\right\rangle
\end{array}\right)
$$

with the corresponding variances and covariances. The beam matrices of the horizontal and vertical phase spaces are respectively the top-left and bottom-right two by two submatrices of the 4D TBM. The 4D transverse emittance of the particle beam is in relation with the determinant of the 4D TBM:

$$
\begin{equation*}
\epsilon_{4 \mathrm{D}}^{2}=\operatorname{det}\left(\sigma^{4 \mathrm{D}}\right)=\epsilon_{x}^{2} \epsilon_{y}^{2}-C_{x y}^{4} \tag{2}
\end{equation*}
$$

where $\epsilon_{x}$ and $\epsilon_{y}$ are the horizontal and vertical emittances and $C_{x y}$ is the coupling term, which is related to the correlations between the horizontal and vertical phase spaces of the beam.

The measured scaled normalized RMS emittance is defined as $\epsilon_{x, n}=f_{\text {scaling }} \beta \gamma \epsilon_{x}$, where $\gamma$ is the relativistic factor of the beam defined from the mean beam energy $\gamma=1 / \sqrt{1-\beta^{2}}$ and $\beta=v / c$ with $v$ as the beam velocity. The scaling factor $f_{\text {scaling }}$ is introduced to correct systematic errors of slit scan measurements. It is described in [4].

## RELATED METHODS

Before introducing the VPP technique it is of benefit to present key points of the slit scan technique that are extended by the VPP technique and a few other methods of performing 4D beam measurements.

A standard procedure for measuring projected emittance at PITZ is the single slit scan method [4,5]. A detailed description of this approach and the experimental setup used at PITZ can be found in Chapter 5 of [6]. Briefly, a space charge dominated electron beam is masked with a horizontal or vertical movable slit. After the slit an emittance dominated beamlet continues its propagation. Downstream of the slit is a scintillating screen that images the beamlet. The beam profile at the slit position is also measured with a scintillating screen for reference, yielding an information on the local beam size. A reconstruction of the phase space of the beam is possible from multiple shots of the beam scanned with a movable slit. The sets of horizontal and vertical shots are separated and require a change of the slit orientation and scanning direction. It should be also noted, that the slice emittance can also be measured by slit scan with a time deflecting structure [7].

Imaging of the beam with a pepper-pot mask is a technique for measuring the 4 D transverse phase space of particle beams [2, 8]. The pepper-pot mask has an ordered cluster of small aperture openings. There are known technical considerations for a pepper-pot mask design [9]. As an example, lets consider the spacing between the holes of the mask. The beam to be measured must a cover sufficient number of holes to allow good accuracy. Respectively, for compact beams the distance between neighboring holes must be accordingly small and positions should be well defined. However, there must be enough separation of the imaged beamlet spots. A typical RMS beam size for 1 nC beams measured at PITZ is 0.2-0.3 mm [5]. In addition, a large number of precisely positioned holes in strong scattering materials of sufficient thickness to mask the major part of the beam are costly to obtain. A similar approach worth mentioning is the use of a TEM grid in the place of the PP mask, that is applicable to low-charge beams [2].

Other methods for measuring the characteristics of the 4D transverse dynamics of a particle beam are quadrupole scan [10] and multiple screens imaging. The beam coupling is obtainable from both methods, but for space-charge dominated beams these methods have limited use $[2,6]$.

## VIRTUAL-PEPPER POT

The Virtual Pepper-Pot (VPP) technique is a novel method for data analysis that addresses PP mask issues, while it preserves the ability to perform 4D phase space measurements. There is no physical PP mask. Instead, the VPP technique is based on the data from slit scans in both transverse directions by pairing horizontal with vertical slit scan measurements. An analog pattern of a PP measurement is constructed, that is referred to as the VPP technique. The idea is illustrated in Fig. 1.

Data from both horizontal and vertical slit scan measurements must be available for studying the electron beam. The process of combining pairs of horizontal and vertical beamlet images to generate VPP images is called beamlet crossing. The idea behind beamlet crossing is that for a given slit pair there is an imaged signal that corresponds to the opening of the crossed slits and that signal is in the common region where the beamlets overlap, as shown in Fig. 1. This is correct for laminar beams. By crossing beamlets of multiple slit positions the images of a cluster of virtual apertures is generated, that is an analog to a pepper-pot mask. As the crossed beamlet images are generated separately, there is no overlap and therefore the virtual openings are not subject to many of the concerns of a physical pepper-pot mask.
There are few notable characteristics of the VPP procedure. First of all, the presented VPP method is a strictly multi-shot method to measure beam emittance. It relies on stable operation of the accelerator. At PITZ the photo injector satisfies this condition [11]. Secondly, the beamlet images have to be crossed with a carefully chosen procedure, as the resulting phase space properties are greatly dependent
on the correctness of the crossing of the beamlets. The last topic is discussed later in this section.

The full beam image at the location of the slit mask is used as a reference for checking the performance of the beamlet collector screen and deriving the scaling factor. By matching with fit the sequence of beamlet image total intensities over the slit scan with the corresponding projection of the full beam several essential parameters are estimated, including what is referred to as baseline level difference. It is representation of the baseline cut observed in the beamlet profile. Due to the presence of noise and much weaker beamlet signals at the beam tails some part of the beamlet image intensity is lost and is missing as baseline intensity level on the beamlet profile when compared to the full beam projection. Following the analysis a value referred to as a charge cut is defined with $(1-$ charge cut $)=\frac{\text { beamlet intensity }}{\text { full beam intensity }}$. The charge cut value reveals what fraction of the beam is not included in the later analysis. Generally there is a difference in the charge cut values of the horizontal and the vertical slit scans.


Figure 1: Illustration of the general idea of beamlet crossing Beam masks with openings (colored strips) and beamlet collector screens (gray screens) are shown. The cyan and yellow areas represent beamlet images from horizontal and vertical slits. The green area is the crossed image of the two beamlets (left) and is assumed to represent the beamlet from a single hole in a pepper-pot (right).

For strongly non-laminar beam there is an issue in beamlet crossing that is the correct separation of beamlet signal into belonging to a virtual opening and other that does not. Beamlet electrons passing outside of the defined virtually crossed opening are still able to land within the region of electrons from the opening.

A method to perform beamlet crossing is using the pixelwise minimum operation. In this method the minimum pixel value of both beamlets is selected as the final pixel value, $Q_{\text {cross }}=\min \left(Q_{x}, Q_{y}\right)$, where $Q_{x}$ and $Q_{y}$ are the horizontal and vertical beamlet signal in any arbitrary region of the beamlet screen. Before applying the above beamlet crossing operation all beamlet images have to be normalized. Due to slight differences in the conditions for the measurements, e.g. number of pulses used and the resulting charge cut, there is to be expected a normalization difference between different slit scans. For unequal normalization of the initial pixel values the following is valid

$$
\min \left(A Q_{x}, B Q_{y}\right) \neq \min \left(Q_{x}, Q_{y}\right)
$$

if $A$ and $B$ are different normalization factors numbers. Only in the case $A=B$ the minimum value crossing produces the desired VPP images with consistent normalization across all pixels.

The ratio of the horizontal to vertical emittance from the slit scan analysis is used as a reference for renormalization. The VPP analysis is expected to produce the same pair of emittance numbers with the same ratio. With systematic errors the pair of emittances from the VPP procedure do not have exactly the same values, but are very close if the renormalization is chosen to keep the ratio as the reference from the slit scan. Therefore this renormalization strategy is used at the moment.

## RESULTS

The results presented in this section are computed from data of projected emittance measurements at PITZ [1]. The photocathode laser pulse has 1.2 mm diameter (beam shaping aperture) on the photocathode and Gaussian temporal profile with a FWHM of about 11 ps . The electron beam charge is 0.5 nC . The photocathode gun accelerates the beam to $6.5 \mathrm{MeV} / \mathrm{c}$ mean momentum and after the booster the beam mean momentum is $22.3 \mathrm{MeV} / \mathrm{c}$. At the gun exit there are normal and skew quadrupoles made from air coils and independently powered by two power supplies. These coils are referred as the compensating gun quadrupoles. Two cases are studied: correcting quadrupoles turned off, that is coil currents of 0 and correcting quadrupoles turned on with -0.5 A for the normal coils and -0.6 A for the skew. The projected emittance in the two cases is measured by horizontal and vertical slit scan as a function of the main gun solenoid current. Typically 100-200 slit positions for a single scan are used with steps of $20-40 \mu \mathrm{~m}$. The slit opening is $10 \mu \mathrm{~m}$.

The 2D emittance values from the slit scan and VPP analysis from a solenoid scan measurements are presented in Fig. 2. The VPP emittance values are slightly higher in comparison to the slit scan results. This could be related to the above mentioned non-laminarity issue.

Generalized emittance invariants [12] in 4D phase space can be studied with the VPP technique. These invariants remain constant when the beam travels in linear accelerator optics. The invariant plotted in Fig. 3 has the equation:

$$
\begin{equation*}
I_{2}^{(2)}=\epsilon_{x}^{2}+\epsilon_{y}^{2}+2 e_{x y}^{2} \tag{3}
\end{equation*}
$$

where $e_{x y}$ is defined with a 2 x 2 submatrix of the 4 D TBM by

$$
e_{x y}^{2}=\left|\begin{array}{cc}
\langle x y\rangle & \left\langle x^{\prime} y\right\rangle  \tag{4}\\
\left\langle x y^{\prime}\right\rangle & \left\langle x^{\prime} y^{\prime}\right\rangle
\end{array}\right|
$$

The 4D emittance is shown as well.
Gun quadrupoles do not have a strong effect on the values of the invariant close to the emittance minimum, as it is expected from linear optics. But the invariant changes under the presence of non-linear space charge effects for high main solenoid current or due to systematic errors in the analysis. The exact reason is still to be understood. The decreasing



Figure 2: Solenoid scan and VPP technique compared to 2D emittance values. Data from measurements with correct-ing qudrupoles (top plot) and without (bottom).


Figure 3: The emittance invariant $I_{2}^{(2)}$ and $\epsilon_{4 \mathrm{D}}$ over the solenoid scan with and without correcting quadrupoles.
effect of the correlations on the 4D emittance relative to the product of the 2D emittances is rather small - it is less than $8 \%$ in all data points.

A correlation value is introduced with the equation

$$
\begin{equation*}
\rho_{4 \mathrm{D}}=\sqrt{1-\left(\frac{\epsilon_{4 \mathrm{D}}}{\epsilon_{x} \epsilon_{y}}\right)^{2}} \tag{5}
\end{equation*}
$$

for correlations between the $x$ and $y$ phase spaces in analogy to the Pearson coefficient [13]. Its values are presented in Fig. 4 with blue lines. From the VPP results the correlation value is in the range $0.15-0.4$, which indicates that there is a correlation between horizontal and vertical phase spaces.


Figure 4: The correlation from Eq. (5) over the solenoid scan with and without correcting quadrupoles (left axis). It is compared with two definitions for the beam coupling factor for the same data points (right axis).

Additionally, the $\rho_{4 \mathrm{D}}$ is higher for the beam with compensating gun quadupoles. Therefore the gun quadrupoles modify the phase spaces, but in this setup seem to add to the correlations, instead of compensating them.

Additionally two definitions for the beam coupling factor are used here. The earlier definition proposed in [10] is

$$
\begin{equation*}
C=\sqrt{\frac{\epsilon_{x} \epsilon_{y}}{\epsilon_{4 \mathrm{D}}}}-1 \tag{6}
\end{equation*}
$$

The later definition is used at PITZ formulated in [14] is

$$
\begin{equation*}
t=\frac{\epsilon_{x} \epsilon_{y}}{\epsilon_{4 \mathrm{D}}}-1 \tag{7}
\end{equation*}
$$

When $C=t=0$ there is no coupling. Both coupling factor values from the VPP analysis are presented in Fig. 4. Both coupling factors and the correlation value $\rho_{4 \mathrm{D}}$ present similar trends over the solenoid scan.

According to these results one can conclude that application of gun quadrupoles optimized for round and symmetric beam transverse distributions does not necessarily remove transverse coupling and in this case even slightly increases $x-y$ correlation. Nevertheless, the 4D emittance is getting reduced by applying gun quads. This could be related to a modification of the magnetic field configuration (solenoid, RF field and gun quads), which serves for a better emittance compensation [15].

## CONCLUSION

The proposed Virtual-Pepper Pot technique reveals a notable difference between slit scan emittance and VPP emittance. On the other hand the corresponding solenoid scan curves have consistent shape. The difference is assumed to be a systematic error of the VPP analysis. To better understand the systematic error contribution of different factors in slit scan and VPP measurements a simulation study is foreseen. Beam correlations were discovered between the
horizontal and vertical phase-spaces. Three ways to evaluate them were compared. All of them deliver similar and consistent results.

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## REFERENCES

[1] M. Krasilnikov et al., "Electron beam asymmetry compensation with gun quadrupoles at PITZ," in 38th Int. Free Electron Laser Conf.(FEL'17), Santa Fe, NM, USA, August 20-25, 2017, JACOW, Geneva, Switzerland, 2018, pp. 429-431.
[2] D. Marx et al., "Single-shot reconstruction of core 4d phase space of high-brightness electron beams using metal grids," Phys. Rev. Accel. Beams, vol. 21, p. 102 802, 10 Oct. 2018 Dor: 10.1103/PhysRevAccelBeams.21.102802.
[3] M. Hernandez, "Four dimensional trace space measurement," Feb. 2005. DoI: 10.2172/839744.
[4] S. Rimjaem et al., "Optimizations of transverse projected emittance at the photo-injector test facility at desy, location zeuthen," Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, vol. 671, pp. 62-75, 2012.
[5] M. Krasilnikov et al., "Experimentally minimized beam emittance from an $l$-band photoinjector," Phys. Rev. ST Accel. Beams, vol. 15, p. 100701 , 10 Oct. 2012. Doi: 10.1103/ PhysRevSTAB.15.100701.
[6] L. Staykov, "Characterization of the transverse phase space at the photo-injector test facility in DESY, Zeuthen," Universität Hamburg, Dr. DESY-THESIS-2012-041, 2012, Universität Hamburg, Diss., 2008, p. 139.
[7] R. Niemczyk et al., "Slit-based slice emittance measurements optimization at PITZ," presented at the 8th Int. Beam Instrumentation Conf. (IBIC'19), Malmö, Sweden, Sep. 2019, paper TUPP013, this conference.
[8] O. Klemperer and W. D. Wright, "The investigation of electron lenses," Proceedings of the Physical Society, vol. 51, no. 2, pp. 296-317, Mar. 1939. DoI: 10.1088/0959-5309/ 51/2/308.
[9] Y. Yamazaki, T. Kurihara, H. Kobayashi, I. Sato, and A. Asami, "High-precision pepper-pot technique for a lowemittance electron beam," Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, vol. 322, no. 2, pp. 139145, 1992.
[10] E. Prat and M. Aiba, "Four-dimensional transverse beam matrix measurement using the multiple-quadrupole scan technique," Phys. Rev. ST Accel. Beams, vol. 17, p. 052 801, 5 May 2014. Doi: 10.1103/PhysRevSTAB.17.052801.
[11] I. Isaev, "Stability and performance studies of the pitz photoelectron gun," PhD thesis, Universität Hamburg, 2017.
[12] G. Rangarajan, F. Neri, and A. Dragt, "Generalized emittance invariants," in Particle Accelerator Conference, 1989. Accelerator Science and Technology., Proceedings of the 1989 IEEE, IEEE, 1989, pp. 1280-1282.
[13] R. Falk and A. D. Well, "Many faces of the correlation coefficient," Journal of Statistics Education, vol. 5, no. 3, 1997.
[14] Q. Zhao, M. Krasilnikov, H. Qian, and F. Stephan, "Beam Transverse Coupling and 4D Emittance Measurement Simulation Studies for PITZ," 29th Linear Accelerator Conference, Beijing (China), 16 Sep 2018-21 Sep 2018, Geneva, Switzerland: JACoW Publishing, Sep. 16, 2018, p. 4. DoI:
10.3204/PUBDB-2018-05140.
[15] L. Serafini and J. B. Rosenzweig, "Envelope analysis of intense relativistic quasilaminar beams in rf photoinjectors:ma theory of emittance compensation," Phys. Rev. E, vol. 55, pp. 7565-7590, 6 Jun. 1997. Dor: 10.1103/PhysRevE. 55. 7565.

