Simulations of single bunch collective effects using HEADTAIL

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Abstract

The HEADTAIL code has developed into a very versatile tool which can be used for simulations of electron cloud induced instabilities as well as for Transverse Mode Coupling Instability and space charge studies. The effect of electron cloud and/or a conventional impedance (resonator or resistive wall) on a single bunch is modeled using a wake field approach. The code naturally allows either for dedicated studies of one single effect or for more complex studies of the interplay between different effects.
Sample results from electron cloud studies on coherent effects as well as TMCI and coherent tune shift studies will be discussed and compared with results from other codes having similar features and/or with existing machine data.

INTRODUCTION AND MOTIVATIONS

The electromagnetic interaction of a high intensity beam with the surrounding environment inside an accelerator ring is recognized to be responsible for unstable collective motion and unwanted beam loss. When the beam intensity is sufficiently high the electromagnetic field self-generated by the beam perturbs the external prescribed fields and acts back on the beam, perturbing in turn its motion. Under unfavourable conditions, the perturbation on the beam further enhances the perturbation on the fields, and an unstable mechanism is initiated. The subject of collective instabilities in accelerators has been studied since the early 1960s [1, 2]. The impact of the understanding of collective instability mechanisms in determining the ultimate performance of an accelerator defines the importance of the subject. Each accelerator, when pushed for performance, encounters some intensity limit, which needs to be understood and cured before moving on to the next limit.
The concepts of wake field and impedance [3, 4] have been introduced and are used to describe this class of phenomena. Every kind of interaction of the beam with itself and with the environment as defined by its geometrical and physical properties defines the classical space charge, resistive wall and resonator-like wake fields and impedances. Furthermore, the interaction of the particle beam with a medium (e.g., non-neutral one component electron plasma, magnetized or not, such as an electron cooler [5, 6] or an electron cloud [7, 8]) and with the electromagnetic field radiated by the beam itself in the arcs (Coherent Synchrotron Radiation) [9] also cause collective phenomena that may limit the performance of a machine. Simple models in terms of conventional wake fields and impedances are often used to describe also these effects and get quick estimates of their importance, even if some times a more complicated modeling is required in order not to miss the more complex physics that they usually involve.
Several codes have been developed over the years to study this class of phenomena, aiming both at calculating wake fields and impedances and at studying their interaction with beam through tracking. An interesting overview of methods, functions and challenges of multi-particle tracking can be found in Ref. [10], as well as Ref. [11] contains an exhaustive repository of all accelerator codes, including those dealing with collective effects. The HEADTAIL code was originally developed in order to study the interaction of a single bunch with a pre-existing electron cloud and therefore predict the instability threshold due to electron cloud [8]. The code was subsequently extended to include also space charge and "conventional" wake fields, so as to turn it into a comprehensive and more flexible tool which can be used to study different collective effects separately, or the interplay between them. In the next section we will thoroughly describe the HEADTAIL code and its functions, and we will put special emphasis on the latest upgrades. Subsequently, some examples of application will be shown in Section III, and conclusions will be drawn in Section IV.

DESCRIPTION OF THE HEADTAIL CODE

In this section we describe the model that we employed to simulate single-bunch effects due to an electron cloud or to an impedance. All the relevant bunch and lattice parameters, as well as the electron distribution and/or the impedance type and parameters, are basic inputs for the tracking simulation of a full particle bunch. The kick approximation is used for the action of the impedance or the electron cloud on the bunch, that means that the action is lumped in one or more points along the ring. The bunch is modeled as an ensemble of N_p macro-particles and it is also sub-divided into N_{sl} slices, which at a given interaction section successively interact with the localized impedance or electron cloud. In the case of an impedance, each bunch slice feels the effect of the transverse and longitudinal wake fields left behind by all the preceding slices. In the case of an electron cloud, the bunch slices interact with the electrons (also modeled as N_e macro-particles and uniformly distributed in the cross-section of the pipe) after one another and each slice sees the electron cloud as deformed by the interaction with the preceding slices. The distortion of the cloud distribution induced by the

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bunch that goes through it, is the mechanism that couples body/tail motion of the bunch with the head motion. The principle of e-cloud simulations is explained in the illustrative flux diagram of Fig. 1.

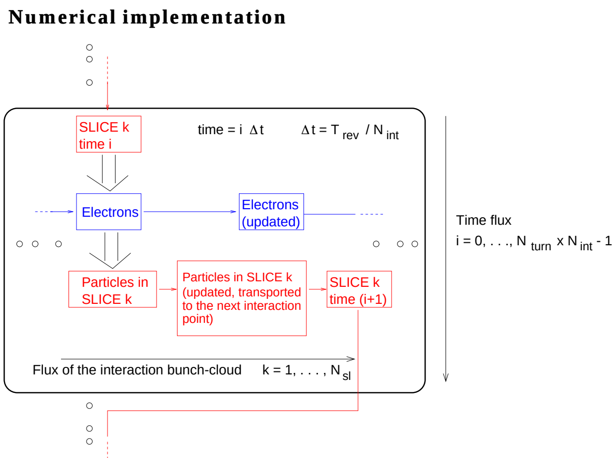

Figure 1: Schematic of the HEADTAIL code when used for electron cloud simulations.

Over one interaction, the momentum changes of the bunch macro-particles (and macro-electrons in the e-cloud case) are computed in time steps that correspond to the longitudinal slices into which the bunch is divided. Between two interactions, the bunch is propagated around the arcs of the ring, where the betatron motion in both transverse planes is modeled by a rotation matrix. Chromaticity and amplitude detuning are taken into account giving to the single particle momentum or amplitude depending tunes. In the longitudinal plane two options are available: the bunch can be longitudinally frozen, or in a bucket. In the latter case, the resulting synchrotron motion can be sinusoidal, linearized, or can be in a barrier bucket with particles streaming freely within the bucket walls. In all these cases the macroparticles move longitudinally and can therefore move across slices over different turns. A new bining of the bunch is therefore necessary at each turn to redistribute particles in the slices. This operation would be required in any case, because the bunch can execute longitudinally synchrotron oscillations (dipolar or quadrupolar) or evolve to different shapes than the initial one due to the action of a longitudinal impedance.
The impedances with which the bunch can interact are broad-band or resistive-wall, and are modeled through their wake fields acting on the different bunch slices, as explained above. The dipole and quadrupole components of the wake fields are weighed by the Yokoya coefficients to account for flat sources of impedances [12]. Space charge can also be included in the simulation both by applying a rotation of the transverse coordinates around the local centroid with a tune shifted by the amount given by the Laslett formula, or by applying one or more kicks per turn coming from the 2D self-generated field. Though more time consuming, the latter option has the advantage of taking into account the nonlinearity of the space charge force.
The output files of HEADTAIL give:

- Bunch centroid positions, rms-sizes and emittances (horizontal, vertical and longitudinal) as a function of time.
- Slice by slice centroid positions and rms-sizes. Coherent intra-bunch patterns can be resolved using this piece of information.
- Transverse and longitudinal phase space of the bunch.

Off line analysis of the HEADTAIL output allows evaluating tune shifts, growth rates, mode spectra. Instability thresholds can be determined through massive simulation campaigns with different bunch intensities, lengths or emittances. A full description of the code updated to November 2002 can be found in the user guide [13]. More recently, a number of new features have been added to HEADTAIL, some of which will be reviewed in the next subsections.

Recent upgrades: electron cloud

HEADTAIL always used a uniform initial distribution of electrons (all having zero initial speed) to interact with the bunch at each interaction section. The value of the density was the saturation value from build up simulations run beforehand. Alternative initial distribution were added at the end of 2002, like with one or two central denser stripes, to better model the real distributions in dipole fields (as resulting both from build up simulations and from measurements at the CERN-SPS). The necessity of a more self-consistent model to gain more confidence in the predictions was evident, because the average electron density over the full pipe cross section can differ by a lot from the local density around the bunch, which is most probably more directly related to the development of instabilities. Therefore, HEADTAIL has been upgraded to load the electron distribution directly from the build up code ECLOUD [14] and use it for the instability simulation. This has required a few changes both in ECLOUD and HEADTAIL.
ECLOUD has been modified to save to file the electron distribution snapshot at the time when a bunch starts going through the cloud. The reason why we chose to take the distribution at the beginning of a bunch passage rather than at the end of the interbunch gap lies in that ECLOUD runs a *clean* routine at the end of each interbunch, with which all macroelectrons with very low charge are suppressed and the number of macroelectrons is about halved.
HEADTAIL has been modified to read the electron distribution in the 4D transverse phase space from another input file. The macroelectrons from ECLOUD have different charges, therefore all the subroutines for field calculation had to be updated to deal with macroparticles having different charges. Upon being loaded, the charges are also rescaled to model an electron cloud spread all over the ring, or over a known fraction of it that can also be specified in the second line of the new input file. It is assumed that the build up simulation that generated the distribution file had been run for a 1 m accelerator segment.

The integration ECLOUD-HEADTAIL, though not completely self-consistent, is certainly a significant step forward with respect to the old model, which only interfaced the two codes through the value of the average density over the pipe section.

Recent upgrades: impedances

Some new features have been also added for the HEADTAIL simulations of a bunch interacting with a conventional impedance. The most interesting and worth discussing here are:

- The possibility of having a beam interaction with several resonators placed at locations with different beta functions has been introduced. This requires setting a new flag in the usual input file and then loading all the values describing the resonators and their locations in a separated file with a given structure.
- The resistive wall model has been extended to include the inductive by-pass effect, which can turn out to be important even for short-range simulations. These wake fields model well the impedance of an LHC collimator, for instance. The wake field in presence of inductive by-pass for thick wall can be very well approximated by the expression [15]:

$$W_1^{\perp}(z) = \frac{cZ_0L}{\pi g^3}\sqrt{\frac{\lambda\mu_r}{\pi|z|}} - \frac{2cL\mu_r}{\pi g^4\sigma}\cdot$$

$$\cdot \exp\left(\frac{4\lambda\mu_r|z|}{g^2}\right)\cdot\left[1-\mathrm{Erf}\left(\sqrt{\frac{4\lambda\mu_r|z|}{g^2}}\right)\right] \quad (1)$$

  where $\lambda = (Z_0\sigma)^{-1}$, σ and μ_r are the conductivity and relative magnetic permeability of the collimator material, L and g the length and the gap of the collimator, respectively. This expression tends to significantly diverge from the classical resistive wall wake for large $|z|$, because it decays more rapidly. For example, using some typical LHC parameters as quoted in Ref. [15], the two expressions give significantly different values on the $|z|$ scale of 3×10^5 km, corresponding to 10^4 turns. Nevertheless, for low conductivity material (like graphite) and small gaps (i.e. for a collimator), the situation looks pretty different and Eq. (1) can differ from the classical resistive wall wake by up to a factor 2 even over one bunch length.
- Nonlinear geometric coefficients have been implemented to take into account of near-wall effects, which may become important in collimators (especially in collimator tests), because a significant part of the beam gets very close to the jaws. These coefficients can be expressed through lengthy summations of cosine and sine functions (trigonometric and hyperbolic) and can be computed from Refs. [16, 17]. The exact expressions can be found in Ref. [18]. It is to be noted that the use of all the sine and cosine functions to calculate the kick on each bunch particle can significantly slow down the execution of the program when the number of bunch macroparticles and the number of bunch slices are chosen to be very high. A compromise between a requirement for low noise tracking and computational speed is certainly necessary for this type of calculations.

SOME SIMULATION RESULTS

Dependence of the e-cloud instability threshold on energy

The dependence of the Electron Cloud Instability (ECI) on energy has been studied using the HEADTAIL code with the parameters in Table 1.

Table 1: SPS Parameters used in the ECI study

Name	**Symbol (Unit)**	**Value**
Momentum	p_0 (GeV/c)	14 to 270
Norm. trans. emitt. $(1\cdot\sigma)$	$\epsilon_{x,y}$ (μm)	2.8
Long. emitt. $(2\cdot\sigma)$	ϵ_z (eVs)	0.35
Bunch length	σ_z (m)	0.3
Bunch population	N	1.1×10^{11}
Vertical tune	Q_y	26.13
Momentum comp.	α	0.00192
E-cloud density	ρ_e (m^{-3})	10^{12}

To gain an insight into the physical mechanism that determines the dependence of the instability threshold on energy, we have first looked for thresholds at different energies with a fixed electron cloud density and with the following assumptions:

- Fixed bunch length
- Fixed longitudinal emittance and transverse normalized emittances
- Cavity voltages adjusted such that the bunch would always be matched to its bucket
- The electron cloud builds up in the dipole regions, therefore the electron motion is bound along the vertical direction.

This study was done in the framework of the upgrade of PS to PS2 or PS2+, which would allow injecting into the SPS with a higher injection energy [19, 20, 21]. In fact, the assumptions above come from having considered the different energies as possible new injection energies into the SPS (same production scheme upstream and bunch into matched bucket injection).
Figure 2 shows that the ECI threshold scales unfavourably with energy in the given assumptions. The reason is that, even if the bunch becomes more rigid at a higher energy, and therefore less sensitive to collective effects, it also becomes transversely smaller, which enhances the effect of the electron cloud pinch. Besides, the matched voltage decreases like $|\eta|/\gamma$, which causes a decrease of the synchrotron tune with consequent less mixing in the longitudinal plane.

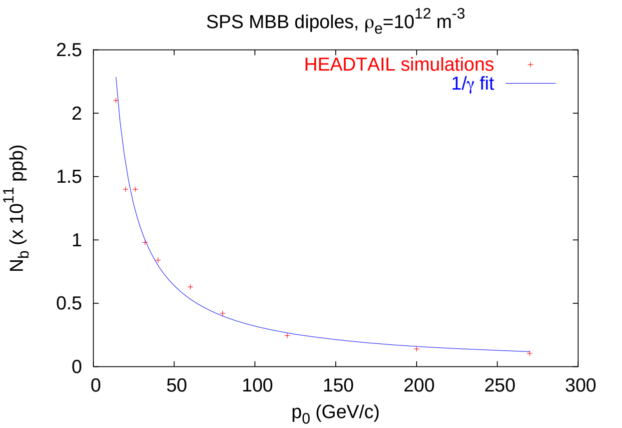

Figure 2: Simulated ECI threholds at different energies, study done with fixed e-cloud density.

A full self-consistent scan would be much more CPU time consuming. For a "coarse" intensity scan we would at least scan 10 bunch intensity values for each energy value (10 x 10 runs if we are taking 10 different energy values). As many (100) ECLOUD runs are needed beforehand to get the electron distributions that have to be input into HEADTAIL. The number of macroelectrons N_{el} comes from ECLOUD and ranges usually between 5×10^4 and 10^5. N_p and N_{sl} need to be chosen as a balance between:

- The bunch slicing still assures a good resolution of the electron motion: $N_{sl} \gg n_{e,osc}$, with n_{e_osc} number of oscillations performed by the electrons during one bunch passage.
- All slices are enough populated ($> 10^3$) , even those in the tails.

Typical numbers are $N_p = 3 \times 10^5$ and $N_{sl} = 80$ and CPU times amount to about 10h per run (512 turns). A sample result of ECI as coherent centroid motion for different bunch intensities at 50 GeV/c is shown in Fig. 3.

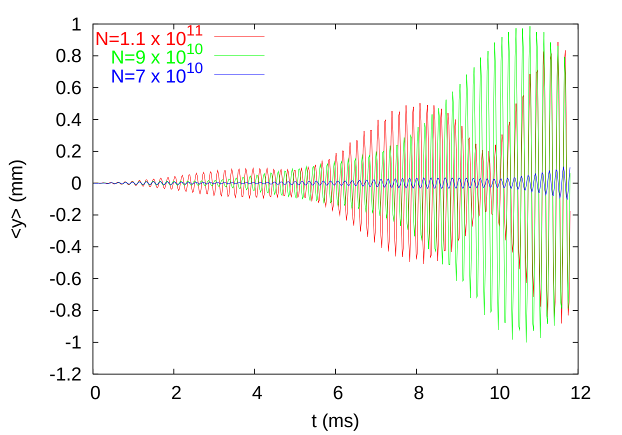

Figure 3: Vertical centroid motion with self-consistent electron cloud for different bunch intensities.

TMCI studies in the CERN-PS

We have used the HEADTAIL code to do instability simulations of a single bunch interacting with a broad-band impedance and find the TMCI instability threshold in different working conditions. TMCI and instability thresholds can also be predicted with analytical formulae [22], but the main advantages of the macroparticle simulation are:

- Simulations can be run for particles in a sinusoidal bucket, whereas the analytical formula is only valid in the linear approximation of longitudinal motion. Besides, the stability of an unmatched bunch can also be studied because the longitudinal dynamics is correctly modeled also when the bunch is not matched to the bucket and executes quadrupole oscillations.
- Both the effect of dipole and quadrupole wake fields for flat pipe can be included.
- Space charge can be included and its effect disentangled [23].
- From macroparticle simulation we can obtain the full unstable bunch evolution and use it to compare data from head-tail monitors with the simulated intra-bunch motion.

The excellent agreement between theory and macroparticle simulation was already shown in Ref. [24], where results from HEADTAIL and MOSES (code based on the solution of the analytical mode coupling equations) were favourably compared. The instability observed at the CERN-PS with the nTOF beam in 2003 when crossing transition has been simulated with HEADTAIL using the PS broad-band impedance model. The parameters used in the simulation are those of the experiment in which the instability was observed, and they are listed in Table 2.

Table 2: PS parameters used in the TMCI study

Name	Symbol (Unit)	Value
Momentum	p_0 (GeV/c)	6
Norm. trans. emitt. $(1 \cdot \sigma)$	$\epsilon_{x,y}$ (μm)	70,16
Bunch length	σ_z (m)	2.22
Mom. spread	$\delta p/p_0$	0.0025
Cavity voltage	V (MV)	0.2
Bunch population	N	$4. \times 10^{12}$
Tunes	$Q_{x,y}$	6.25,6.25
Momentum comp.	α	0.027
Shunt impedance	R_T (MΩ/m)	3
Quality factor	Q	1
Resonance frequency	$\omega_r/2\pi$ (GHz)	1

Figures 4 show snapshots taken at turn 149 of the sum and difference (horizontal and vertical) BPM signals as resulting from the simulations done without (top) and with space charge (bottom). The influence of space charge is not essential to explain the observed instability, but it is clear that taking it into account a small shift in the bunch delta signal frequency appears and the shape of the loss distribution along the bunch is also affected. HEADTAIL simulations of the PS instability compare very well with the measured profiles, Fig. 5.

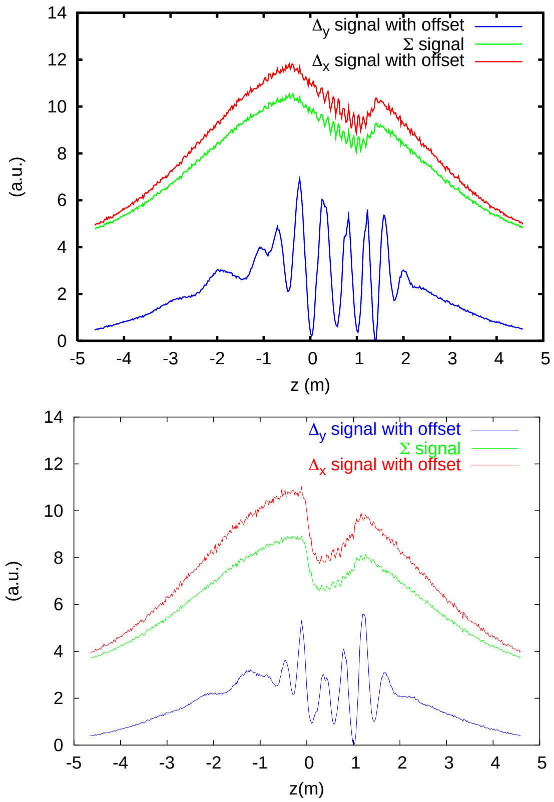

Figure 4: Simulated BPM signal of the PS instability at transition without space charge (top) and with space charge (bottom).

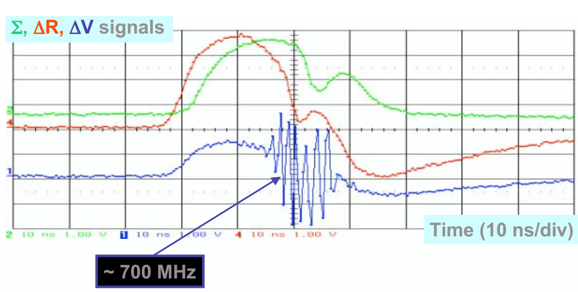

Figure 5: Measured BPM signal at the PS.

Tune shift induced by a collimator impedance

In 2004 a prototype of the 1 m long LHC collimator was installed in the SPS in order to study experimentally its effects on the beam. Tune shift measurements could be used to probe the imaginary part of the collimator impedance and assess the most reliable model to be applied for predictions [25]. The parameters of the SPS experiment are summarized in Table 3. These measurements have been then simulated with the HEADTAIL code using different models and approximations of the collimator wake field. The tracking study follows step by step the procedure used in the analytical approach [26], but it is based on a single bunch wake field description. We started from a classical resistive wall in a flat chamber. This model yields a tune shift which is larger by over a factor of 2 (at the smallest gap) than the one that was measured at the SPS. Next we considered the wake field from resistive wall with inductive by-pass and still used the linear approximation for the factors of flat chamber. The tune shift at 1 mm gap decreases, but not enough as to explain the low values measured at the SPS. The next two steps allowed us to fully recover the SPS experimental results from 2004. First, a transverse distribution cut was introduced for the protons, which seemed necessary since the collimator gap at 1 mm is hardly twice the rms-size of the distribution at the collimator location. While this did not cause major changes in the simulation results at 1.5 and 2 mm, probably due to the more significant intensity loss (experimentally a loss by about 30% was observed in the BCT signal when the collimator jaws were closed [25]), the tune shift for the 1 mm case got lower. Lastly, the geometric nonlinear terms explained in the previous section were introduced to correct the wakes at the large amplitudes in the simulation. A summary overview on the tune shifts predicted with the different models is shown in Fig. 6. It is evident that, going from the classical resistive wall theory to the inductive by-pass with distribution cut and geometric nonlinear terms for the wake, the strongly nonlinear g^{-3} dependence of the tune shift becomes linear in the considered range of gap values.

Table 3: SPS experiment parameters

Name	**Symbol (unit)**	**Value**
Momentum	p_0 (GeV/c)	270
Bunch population	N	10^{11}
Long. emitt. $(2 \cdot \sigma)$	ϵ_z (eVs)	0.35
Bunch length	σ_z (m)	0.21
Mom. compact.	α	0.00192
Norm. trans. emitt. $(1 \cdot \sigma)$	$\epsilon_{x,y}$ (μm)	2.8/2.8
Tunes	$Q_{x,y}$	26.14/26.18
Chromaticities	$\xi_{x,y}$	0., 0.
Collimator gap	g (mm)	1, 1.5, 2

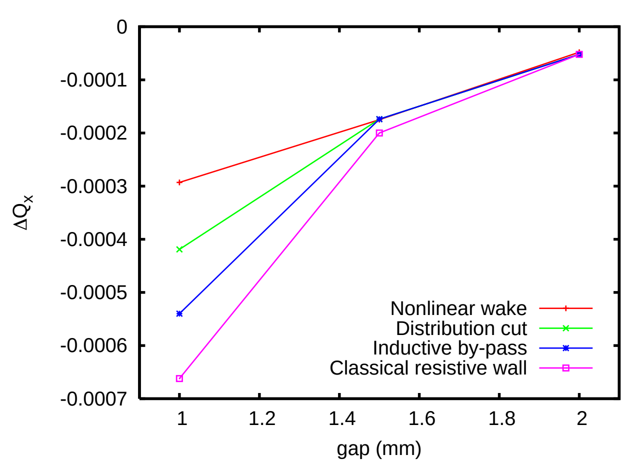

Figure 6: Overview on all the models: tune shift as a function of the collimator gap.

HEADTAIL tracking using resistive wall with inductive by-pass wake field and nonlinear wakes requires a compromise between accuracy and computational speed. The use of external libraries for the special functions can significantly slow down the execution of the program. Therefore, a frozen wake field option has been implemented, which computes the slice-to-slice wakes at the beginning of the execution, stores the values and then uses them turn by turn. Obviously, this approximation can only be applied for matched bunches. Furthermore, too frequent calls of sine and cosine functions for the nonlinear wakes ($6\times$ slice-particle pair) need to be avoided. Fortunately, the non-oscillatory character of the wake allows for a coarser slicing of the bunch with respect to resonator runs ($N_p = 10^5$ and $N_{sl} = 50$). CPU times per run are about 8h.

CONCLUSIONS

HEADTAIL has developed into a versatile tool that can be used to do particle tracking with a variety of collective single-bunch interactions (electron cloud, broad-band impedances, resistive wall, space charge). The upgrade of HEADTAIL over the last year includes:

- Use of self-consistent electron distribution for the e-cloud simulations imported from build-up code ECLOUD
- Use of an arbitrary number of resonators interacting with the beam
- An improved model of resistive wall and nonlinear wakes

HEADTAIL performances are satisfactory in terms of computational speed with an appropriate choice of modeling and numeric parameters because CPU times never exceed 1 day/run. The applications that we have chosen to present in this paper make it evident that the benchmark of HEADTAIL against experimental results (where possible) is successful.

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