Numerical Minimization of Longitudinal Emittance in Linac Structures

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Abstract

The design of linear colliders is based on manipulations in the longitudinal phase space. The longitudinal phase space single bunch tracking code LiTrack [2] allows to simulate bunch-compression up to 3rd order and RF acceleration with wake fields. In this paper a numerical approach is proposed to achieve an optimal bunch compression design with respect to an exact bunch energy, peak current and a vanishing skewness in the current profile.

A minimal longitudinal emittance is considered as a quality factor of the bunch, where the bunch energy, peak current and a symmetric charge distribution are represented as constraints. An optimization problem is defined to minimize the longitudinal emittance with a predetermined bunch-energy and peak-current with respect to the charge distribution symmetry. The actual design is based on LiTrack with an optimization solver extension to find an optimal bunch corresponding to the newly introduced constraints.

INTRODUCTION

The design of linac - based accelerators requires a high - precision system set up of the longitudinal phase space. It is protracted to perform such a task manually so that a computer program is needed. LiTrack, a fast longitudinal phase space tracking code, is proposed in [2], which simulates both bunch compression and linac acceleration. In this code the acceleration is described as a sinusoidal variation and the bunch compression systems are characterized by the 'longitudinal dispersion' and the nominal energy.

In this paper, a numerical optimizer for longitudinal phase space tracking using a Sequential Quadratic Programming (SQP) solver is developed. A fast optimization routine demands a new organization in the tracker. It is inevitable to subdivide the actual tracker design into two sections: a particle distribution calculator and a fast lattice tracker which is called by optimization routine. The tracker needs an additional function code, called FIT-point, which calculates the constraint function values and the objective function value.

In order to improve the ability to simulate a two-stage bunch compression system, which consists of a RF accelerating section, a higher harmonic RF section and two dipole magnet chicanes, an extension to the tracker is required. An analytical model of this two-stage bunch compression system is defined using the energy and the momentum derivatives up to 3rd order of the combined accelerator voltages of the injector linac and the 3rd harmonic linac. The tracker is also extended with 'multiknobs' as control parameter,

to describe bunch compression using higher harmonic RF systems [3][1]. A convenient control environment with respect to the optimization expansion is realized by a new Graphical User Interface (GUI), which supports the control of the new functions.

The paper is organized as follows: Section 2 is dedicated to the new GUI and the Tracking Control. The section 3 describes the optimization routine and its usage by GUI. Numerical results are presented in section 4.

THE OPTIMIZATION ROUTINE

A minimal longitudinal emittance is generally considered as a quality factor of the particle bunch, where the bunch energy, peak current and a symmetric charge distribution are represented as constraints. Under these conditions, a constrained optimization problem is defined to minimize the longitudinal emittance with a predetermined bunch-energy and peak-current with respect to the charge distribution symmetry. For the solution of this problem, the tracker is now extended now with an optimization solver based on a SQP formulation to determine an optimal bunch corresponding to the newly introduced constraints.

With respect to the nature of the tracker, a statistical definition of the longitudinal emittance is needed. The rms emittance ϵ was first given in [6] as

$$\epsilon(x) = 4\sigma_z(x)\sigma_\delta(x)\sqrt{1 - r(x)^2}. \tag{1}$$

Where the index z corresponds to the longitudinal position, δ to the relative energy deviation and the variable r to the correlation coefficient of both distributions. With vector x as the design variable. The longitudinal rms emittance is objective function. Futhermore the sign of the correlation coefficient is used as constraint. The general minimization problem is specified as follows:

$$\min_x \{\epsilon(x)\} \tag{2}$$

subject to the constrains:

$$c(x) \leq 0, \tag{3}$$
$$c_{\text{eq}}(x) = 0, \tag{4}$$
$$l_{\text{b}} \leq x \leq u_{\text{b}}. \tag{5}$$

The nonlinear equality constraint function $c_{\text{eq}}(x)$ consists of the beam energies, the peak currents and the skewness values of z in the related FIT-point. The nonlinear inequality constraint function $c(x)$ contains the skewness values, too. Other components are the correlation coefficients. Both functions contain the skewness values: the first one

forces full symmetry γ_0 and the second one allows a predefined deviation. The vectors l_b and u_b define a set of lower and upper bounds on the design variable in x. The solution is always in the range given in equation (5).

The nonlinear equality constraint consist of three general cases

$$
\begin{aligned}
c_\mathrm{eq}(x) &:= I_\mathrm{pk}(x) - I_{\mathrm{pk}\,\mathrm{t}}, & (6) \\
c_\mathrm{eq}(x) &:= \bar{E}(x) - E_\mathrm{t}, & (7) \\
c_\mathrm{eq}(x) &:= \gamma(x) & (8)
\end{aligned}
$$

and their combinations, where I_pk means the peak current, $\bar{E}$ the mean bunch energy and γ the 3rd standardized moment (skewness). The target values $I_{\mathrm{pk}\,\mathrm{t}}$ and E_t are defined by the user corresponding to bunch compression system under considerations.

The nonlinear inequality constraint consist of

$$
\begin{aligned}
c(x) &:= r(x) \cdot s. & (9) \\
c_1(x) &:= \gamma(x) - \gamma_\mathrm{max} & (10) \\
c_2(x) &:= -\gamma(x) - \gamma_\mathrm{max}. & (11)
\end{aligned}
$$

Here, s is the user defined correlation sign constraint. This allows to optimize with respect to a positive or negative correlated phase space. The nonlinear ineqality constraint could be expanded by 10 and 11. Here γ_max is defined as user input and denotes the absolute maximum value of the allowed skewness.

GRAPHICAL USER INTERFACE AND TRACKING CONTROL

The GUI panel is shown in Figure 1. It is subdivided into eight sub-panels by their function. Furthermore the GUI features a convenient menu bar to manage the project. The file management handles the project files, wake files and external particle files by corresponding dialogs.

Beamline Panel

The beamline parameters are edited by the related panel. Each component is a column which is organized as follows:

Table 1: Component Definition Tools

Object	Function
Check Box	Enables plot
Buttons(+/-)	Subtract, add or insert component
Pictogram	Shows actual component
Pop-up menu	Dial particles distributions or wakes
Edit Boxes	Parameter input

The first component is a particle source by default. The particle source may be chosen by the related popup menu. The scroll bar allows to leaf through the beamline. When an entry is changed a brief caption appears to the left of the

table. Table 2 shows the most commonly used components and their pictograms, see [2]. The blank slots are not used

Table 2: List of pictograms

Pic	Descr.					
Chicane$_2$	Chicane	R_{56}	E_0			
Chicane$_4$	Compr.	R_{56}	T_{566}	U_{5666}	E_0	
9-cell	Linac	V_0	φ	λ	i_w	L
Lens	FIT	I_pk	E	γ_max	s	
Brick	End					

by the tracker. The **FIT-point** defines the target values, which are used in the optimization routine as constraints only. The 3rd parameter describes the absolute value of the permitted skewness. The 4th parameter gives the sign of the correlation coefficient in the optimized phase space. The phase space is not manipulated by the FIT-point (denoted by the lens-pictogram in Fig 1).

Knob Panel

The knob panel (in Fig 1) allows a convenient handle of bunch compression systems with a higher harmonic RF section. The four 'knobs' are related to the beam momentum and its first three derivatives, this feature is developed in [3]. The first scroll bar regulates the bunch energy, the second manipulates momentum chirp and determines the peak current. The third scroll bar adjusts the second derivative and balances the beam distribution in the centre region. The third derivative allows to adjust the tails. The knobs are independent to each other. The usage of the knobs has been introduced in [1].

Other Panels

The start values and constraints of the optimization are defined by the **Constraints Panel**. Each column is related to a component in beamline panel. The upper group per column is dedicated to the first parameter in the beamline panel, the lower group is dedicated to the second one.

The attributes of the **initial bunch** are adjustable using the related sub-panel. The **Plotting Control** is assumed from [2]. The **Optimization Tolerances** are adjustable by their panel near the bottom of the GUI. These tolerances terminate the optimization routine. The value TolFun specifies the termination tolerance for the objective function. TolX describes the tolerance for the design variable. Furthermore, the termination tolerance for the maximum nonlinear constraint violation is specified by TolCon, see [5].

NUMERICAL RESULTS

The two stage bunch compression system of the XFEL is used as an example for optimization. This system consists of two bunch compressors BC1 and BC2 and their upstream accelerating RF sections. The 3rd harmonic section
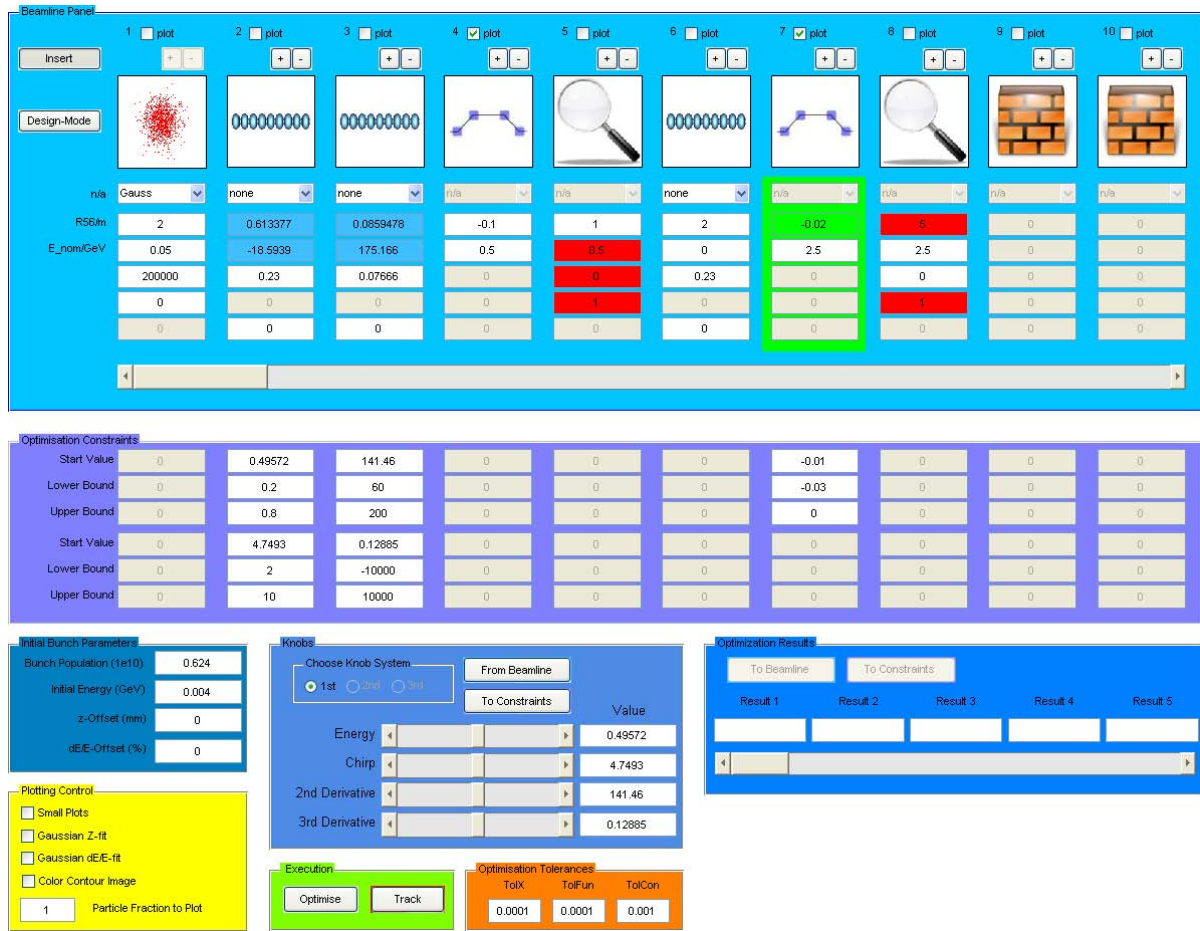
Figure 1: Graphical User Interface

is placed upstream of BC1. A FIT-point is placed downstream to each bunch compressor. As a design goal, the total charge symmetry, a bunch energy of $0.5\,\mathrm{GeV}$ and a peak current of $1\,\mathrm{kA}$ are required after BC1. Furthermore an energy of $2.5\,\mathrm{GeV}$ and a peak current of $5\,\mathrm{kA}$ is required after BC2.
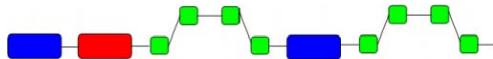
Figure 2: Two Stage Bunch Compression System

In the first bunch compressor R_{56} is set to $0.1\,\mathrm{m}$. The R_{56} of the second bunch compressor allows to reach a peak current of $5\,\mathrm{kA}$. For this reason it is used as a design variable in the optimization. An energy of $2\,\mathrm{GeV}$ is added by the second accelerating section. The first accelerating section and the 3rd harmonic section are used to optimize the bunch. The 'multiknobs' are used to describe and optimize these sections.

The start values x_0 are determined approximitly. In this example the usage of the knobs allows to adjust the beam energy almost exactly. The start values of the other knobs are chosen roughly with respect to the peak current and skewness. The peak current in the second FIT-point is used

to indicate the R_{56} start value of bunch compressor two. The initial particle coordinates are inputted from an external file. The external distribution tesla_6MeV_200k.zd is used.

The lower and upper bounds are defined empirically with respect to x_0. The chosen constraints are centralized in Table 3.

Table 3: Component Definition Tools

	FIT-1	FIT-2
Peak Current (kA)	1	5
Energy (GeV)	0.5	
Skewness	0	
Correlation Sign	1	1

The blank target slots are not used in optimization. The chosen initial values are given in Table 4. Using these start values tracking yields Figure 3. Where in the first (top in 3) and second (bottom in 3) FIT-point with the energy profiles (left), longitudinal phase spaces (center) and the current profiles (right). The results of the optimization procedure are depicted in Table 5. The results of the optimization are plotted in Figure 4

Table 4: Start values in example

Parameter		Value
Energy	(GeV)	0.49
Chirp	(GeV m^{-1})	4.4
1st derivative	(GeV m^{-2})	140
2nd derivative	(GeV m^{-3})	0
BC2 R_{56}	(m)	0.0175

Table 5: Optimization Results

Parameter		Value
Energy	(GeV)	0.4957
Chirp	(GeV m^{-1})	4.7486
1st derivative	(GeV m^{-2})	141.51
2nd derivative	(GeV m^{-3})	828.856
V_1	(GV)	0.6150
φ_1	(deg)	-19.0393
V_n	(GV)	0.0861
φ_n	(deg)	174.0446
BC2 R_{56}	(m)	0.019335
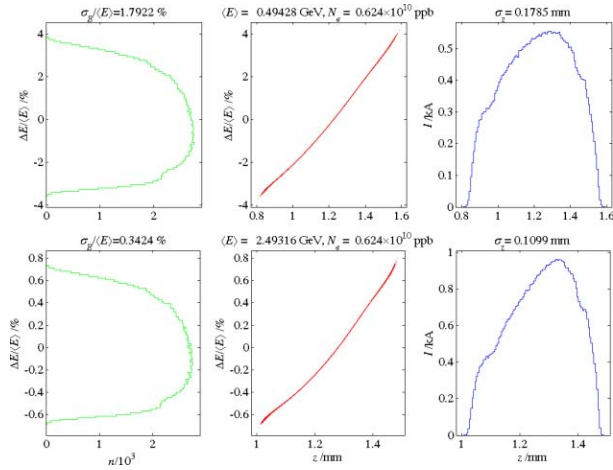
Figure 3: Start Values (in FIT-points)
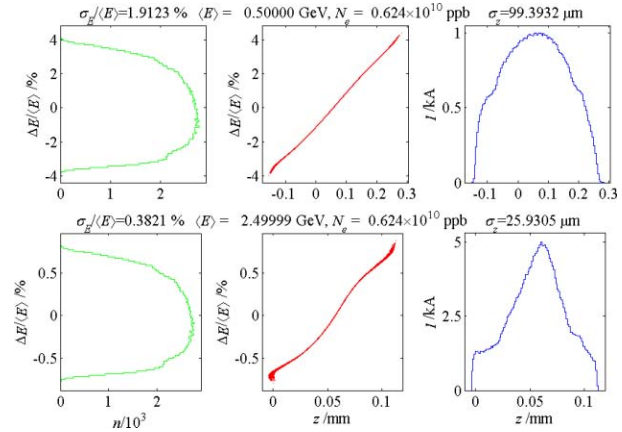
Figure 4: Optimization Results (in FIT-points)

All target values of Table 3 are achieved by the optimizer automatically. The present example needs a computational time of $35\,\mathrm{s}$ in 11 iterations and 76 function counts with 200000 macro-particles. The newly introduced usage of the 'multiknobs' allows a convenient initial value approximation, but a manually set up of the bunch compression system with this accuracy in the constraints is not possible at that rate. The use of the nonlinear parameter optimization yields the intended improved values for the system design within the given error margin tolerance.

It is beneficial to choose the start values conscientiously. The present minimization routine finds local minima, therfore it is possible that the optimizer iterates to a minimum which does not meet the constraints and does not converge. The design variable bound should be defined carefully. It is possible to define the bounds inapplicably, so that the solution space will vanish.

The new optimization routine requires Matlab as well as the Optimization Toolbox. Because of the optimization is available exclusively from GUI Matlab 7.0 or higher is needed [5].

CONCLUSION

In this paper the system set up of bunch compression systems was improved using a constrained optimization, realized by a SQP - based formulation. It was shown, that such a formulation in connection with a graphical user interface allows a convenient system setup of arbitrary bunch compression systems.

Most of the computation time was needed for tracking. The wake field calculation combined with the optimization increases this effect and yields a greater time need. For this reason the wake field calculation time need is in the focus of the future work.

In the future work a higher robustness with respect to the start values is intended by an extension with globally convergent genetic optimization algorithms.

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