TRACKING CODE WITH 3D SPACE CHARGE CALCULATIONS TAKING INTO ACCOUNT THE ELLIPTICAL SHAPE OF THE BEAM PIPE*

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Abstract

The determination of electron cloud instability thresholds is a high priority task in the ILC [1] damping rings research and development objectives. Simulations of electron cloud instabilities are therefore essential to determine the positron damping ring design. Recently, perfectly conducting beam pipes with arbitrary elliptical shapes have been implemented as boundary conditions in the Poisson solver package MOEVE [7]. The 3D space charge algorithm taking into account a beam pipe of elliptical shape is presented here. The results for the electric field are compared with results simulated with different boundary conditions. In this paper, we also present the first results from our new particle tracking program which includes the Poisson solver MOEVE for space charge calculations.

INTRODUCTION

Damping rings are necessary to reduce the emittances delivered by particle sources to the values required for the linear collider. In positron storage rings, electrons produced by photoemission, ionization and secondary emission accumulate in the vacuum chamber forming an "electron cloud" which can act back on the stored bunches. The single bunch emittance of the positron bunches can be inflated through the interaction with the electron cloud. Particle tracking programs which model the interactions of a single bunch and the electron clouds require the calculation of the space charge fields at each discrete time step. Space charge fields along with other applied EM fields determine the Lorentz force which impacts the particle trajectory.

A common method to calculate the space charge fields from spatially distributed charges is the particle mesh method (see [5]). It requires the solution of the Poisson equation and this solution is strongly influenced by the applied boundary conditions (b. c.). In [2] conducting b. c. were applied to a rectangular shaped beam pipe together with an FFT-based Poisson solver. However, the rectangular cross section is not the best approximation to the true geometry of the beam pipe.

In this paper we present an algorithm for the solution of Poisson's equation with conducting b. c. on an elliptical cross section of the beam pipe. Further, we present 3D simulation results for open (free space) and conducting b. c. Conducting b. c. were applied to the walls of a rectangular and elliptical pipe. Finally, we tracked a Gaussian bunch along a drift with a new tracking program, which is still under development. The differences in some bunch parameters are shown for a bunch tracked within a rectangular and an elliptical beam pipe, respectively.

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MATHEMATICAL MODEL

A 3D domain in which the Poisson equation is usually solved resembles a rectangular box Γ with $Dirichlet$ b. c. on $\partial\Gamma_1$ (transversal boundary planes) and $open$ boundary conditions on $\partial\Gamma_2$ (boundary planes of the domain in longitudinal direction). For this case Poisson's equation reads as

$$\begin{aligned} -\Delta\varphi &= \frac{\varrho}{\varepsilon_0} && \text{in } \Gamma \subset \mathbb{R}^3, \\ \varphi &= g && \text{on } \partial\Gamma_1, \\ \frac{\partial\varphi}{\partial n} + \frac{1}{r}\varphi &= 0 && \text{on } \partial\Gamma_2, \end{aligned} \tag{1}$$

where $\Gamma = [-a,a]\times[-b,b]\times[-c,c]$ and $\partial\Gamma = \partial\Gamma_1 \bigcup \partial\Gamma_2$.

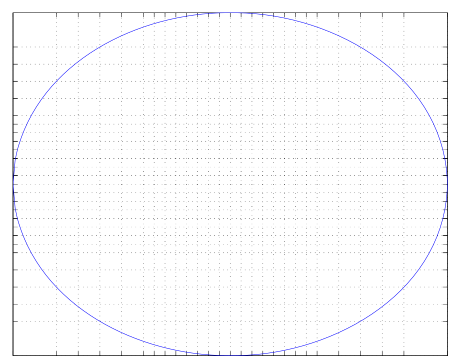

Figure 1: Cross section of the discretization domain.

For beam pipes with an elliptical cross section we consider the Poisson equation on the cylindrical domain Ω (Figure 1) with

$$\begin{aligned} -\Delta\varphi &= \frac{\varrho}{\varepsilon_0} && \text{in } \Omega \subset \mathbb{R}^3, \\ \varphi &= 0 && \text{on } \partial\Omega_1, \\ \frac{\partial\varphi}{\partial n} + \frac{1}{r}\varphi &= 0 && \text{on } \partial\Omega_2, \end{aligned} \tag{2}$$

where $\partial\Omega_1$ is the side surface of the cylinder with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } -c < z < c,$$

$\partial\Omega_2$ are the two elliptical bases of the cylinder satisfying

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \text{ and } z = \pm c$$

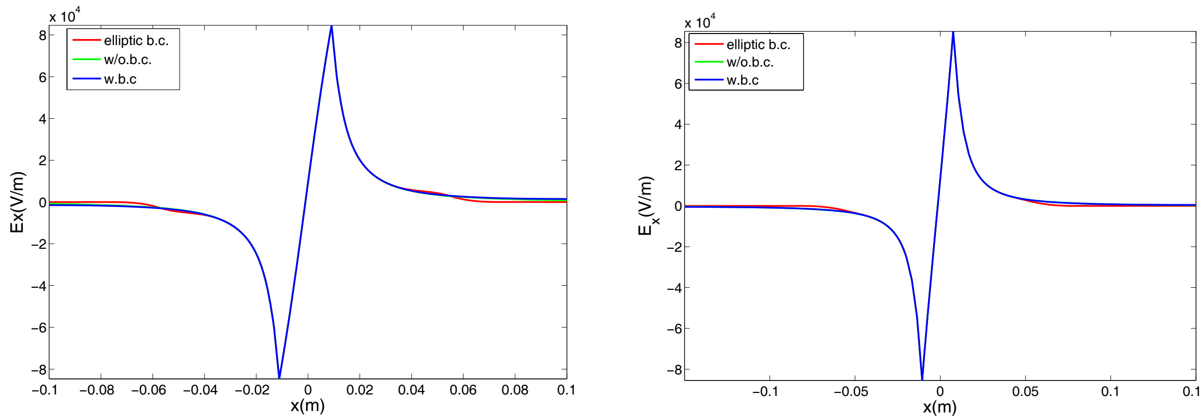

Figure 2: Electric field E_x along x-axis of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right) computed with open (w/o. b. c.) and conducting b. c. on a rectangular (w. b. c.) and elliptic (elliptic b. c.) pipe.

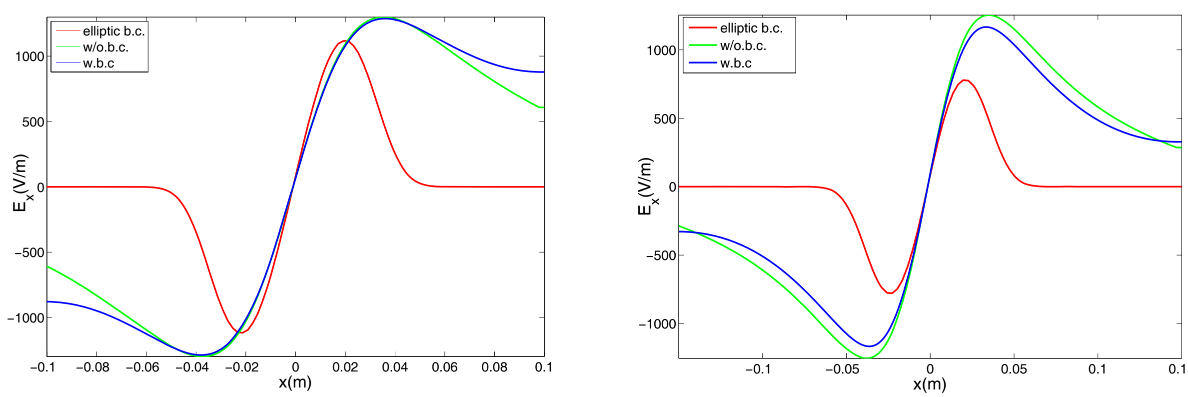

Figure 3: Electric field E_x along $y = \pm b/2$ of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right) computed with open (w/o. b. c.) and conducting b. c. on a rectangular (w. b. c.) and elliptic (elliptic b. c.) pipe.

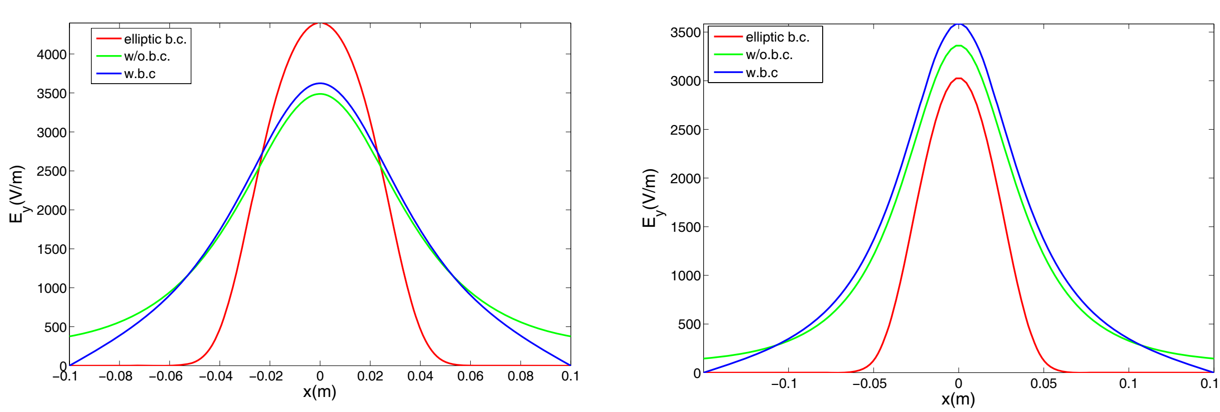

Figure 4: Electric field E_y along $y = \pm b/2$ of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right) computed with open (w/o. b. c.) and conducting b. c. on a rectangular (w. b. c.) and elliptic (elliptic b. c.) pipe.

and being perpendicular to the z-axis. The boundary condition $\varphi = 0$ on $\partial\Omega_1$ means that the surface of the beam pipe is assumed to be an ideal electric conductor. The discretization volume in which the cylindrical computational domain Ω is embedded (Figure 1) is the same rectangular box Γ as in (1). The domain Γ is discretized along the x-, y- and z-axis in N_x, N_y and N_z (in general case non-equidistant) steps, respectively. So, the discretization of (2) with second order finite differences leads to a linear system of equations $\boldsymbol{Au} = \boldsymbol{b}$, where $\boldsymbol{u}$ is the vector with the potential values and $\boldsymbol{b}$ the vector with the space charge density at the grid points. The above linear system of equations for the domain Ω contains only the equations for the points which are inside of Ω. The number of unknowns is considerably smaller because in each (x, y)-plane all grid points outside the ellipse are skipped. The system matrix $\boldsymbol{A}$ is block structured; however the blocks will have different dimensions and the symmetry of $\boldsymbol{A}$ is lost in contrast to the system we get for the rectangular domain Γ (see [6] for explanation). Although the system matrix $\boldsymbol{A}$ is non-symmetric it is positive definite. Therefore the BiCGSTAB[1] algorithm can be applied to solve the linear system of equations. In [6] a detailed description of the matrix properties and the algorithm can be found.

SIMULATION RESULTS

Space Charge Fields

In order to compare the calculated fields with open and conducting b. c on a rectangular and elliptical pipe, we choose a spherical bunch with a uniformly distributed charge of -1 nC. It is situated in the center of the beam pipe. The bunch radius r is 10 mm ($r << a, b$). The electric field depends significantly on the b. c., especially in the proximity of the boundary. Figure 2 shows the component E_x of the field along the x-axis in a square box ($a = b$) and in a rectangular box with $a = 1.5b$.

The difference between the electric field in open space, in a rectangular box, and in an elliptical cylinder becomes more evident as we move closer to the boundaries in both directions. Figure 3 and 4 show the two transversal components E_x and E_y along the line $y = \pm b/2$.

Tracking Results

Recently, a tracking program has been implemented including the new Poisson solver for space charge calculations in elliptic beam tubes. The tracking program is based on the leap frog time integration scheme which is described, for instance, in [3].

As an example we present here a Gaussian bunch with $\sigma_x = \sigma_y = \sigma_z = 1$ mm, which has been modeled with 1,000 macro particles. The bunch has a total charge of -1 nC and an average kinetic energy of 5 MeV. The initial particle distribution was adopted from Astra's generator program [4]. Figure 5 represents the initial particle distribution.

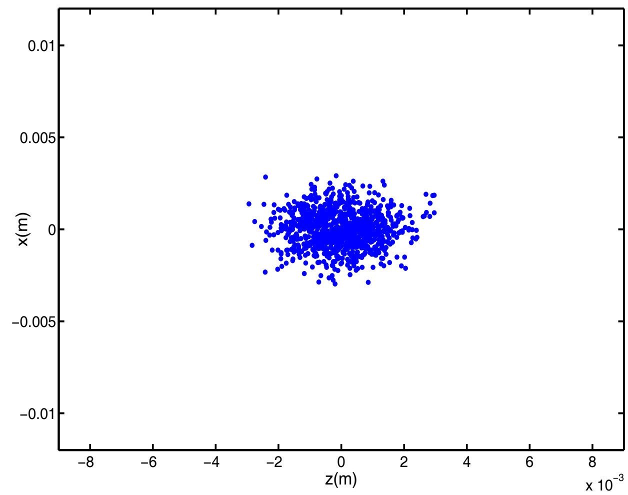

Figure 5: Initial particle distribution in longitudinal direction $z = 0.0$ m.

The macro particles were tracked over a time of 11 ns which is related to a drift over a distance of $z = 3.29$ m. Figure 6 and 7 show the longitudinal particle distribution at the end of the drift. The related space charge calculations were applied with open b. c. (Figure 6) and a circular beam pipe with 10 mm diameter (Figure 7), respectively. After this drift the bunch has a size of $\sigma_x = \sigma_y = 1.4$ mm and $\sigma_z = 1.1$ mm with the space charge calculation in open space. The tracking in the beam pipe results in a bunch size of $\sigma_x = \sigma_y = 3.3$ mm and $\sigma_z = 1.1$ mm.

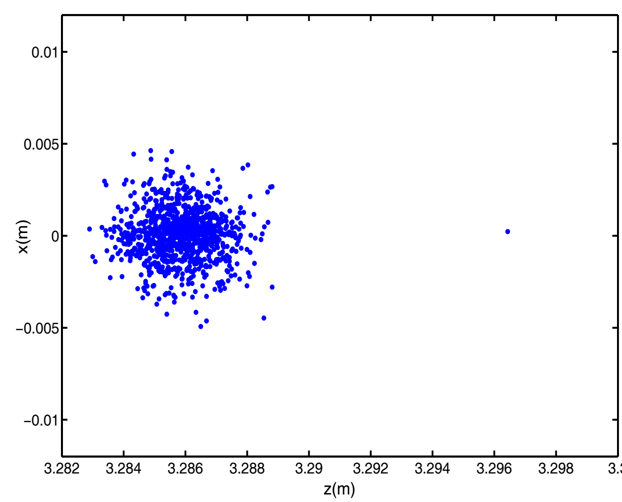

Figure 6: Particle distribution in longitudinal direction after tracking with open boundary conditions ($z = 3.29$ m).

CONCLUSION

In general we have found that electric space charge fields inside an elliptical beam pipe found by a 3D Poisson solver

[1] BiConjugate Gradient STABilized algorithm

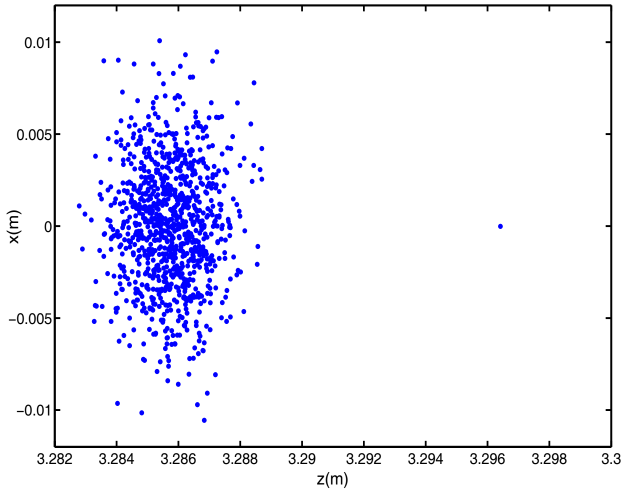

Figure 7: Particle distribution in longitudinal direction after tracking in a circular beam pipe ($z = 3.29$ m).

differ significantly from approximations which use solutions of the Poisson equation in free space or in a rectangular box. The largest differences are found close to the elliptical boundary. The first tracking results show that a bunch tracked with different b. c. has a different transversal expansion (Figures 6 and 7) The larger transversal dimensions of the bunch tracked in the beam pipe compared with the bunch tracked with open b. c. are in accordance with Figure 4 (the left part - for circular beam pipes). Namely, the transversal electric field simulated in the circular pipe is considerably larger than the same field simulated with open b. c..

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