PRECISE BETATRON TUNE MEASUREMENT IN TPS STORAGE RING

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Abstract

To acquire precise beam orbits from beam position monitors (BPMs) in circular accelerators is one of the most significant diagnosis to measure beam parameters. However, the precise spectrum analyses from BPM data acquisitions such as betatron tune, dynamics aperture and frequency map etc. that are depended on discrete Fourier transform (DFT) or the fast Fourier transform (FFT). A method of the fast Fourier transform correction (FFTc) was employed for the more accurate spectrum analyses in Taiwan Photon Source (TPS) [1] storage ring. We performed the numerical model of FFTc, the frequency error analyses of FFTc by using two window functions in some spectral lines. And the precise betatron tune measurements in TPS storage ring will be presented.

INTRODUCTION

DFT and FFT are generally applied in spectrum analysis of harmonics signals. These methods are convenient and fast to analyse spectrum peaks, and their resolutions or frequency errors are proportional to the inverse of the spectral sampling point *N*. In circular accelerators, the spectrum of betatron oscillations must be resolved in limited turns, and their amplitude of oscillations be decayed with beam turns because of particle nonlinear decoherence motion [2], radiation damping… etc. The conventional method as FFT could not be satisfied with the precise spectrum analyses. The points-limited signals in spectral domain is difficult to solve an accurate spectrum response. To overcome these limits and recover real parameters for the harmonic signals. Some different methods were proposed.

A computing method called average phase advance (APA) [3, 4] is the one of the advanced frequency approaches to obtain more precise spectrum than those of the plain FFT applying in CERN SPS. Numerical analysis of the fundamental frequencies (NAFF) [5] in the chaotic signal measurements is another method for the accurate frequency transform. Both of above-mentioned methods are based on iterative routines to get the precise frequency for the betatron tune measurements. We employed an advanced frequency transform technique of the corrected FFT [6] to decrease the frequency errors by correcting the value of the spectrum peak. This technique be verified in a known spectral signal, and further be applied in the betatron tune measurement in TPS storage ring.

FAST FOURIER TRANSFORM CORRECTION

This method of FFT correction (FFTc) could obtain not only the accurate frequency but also amplitude and phase of the spectrum peak. The correction of spectrum analysis with a rectangular and a hanning window function induced the different shape of spectrum lobes, the results of simulation and measurement will be presented in the following sections. In the frequency domain, the corresponding frequency is shown as $f = K \cdot (f_s/N)$, where f_s is sampling frequency, K is spectral line number ($K = 0 \sim N/2 - 1$), and N is the sampling points also beam turns in the circular accelerator. The frequency correction equations of the rectangular window and hanning window are characterized as eq. (1) and (2), respectively.

$$\Delta K_{rect} = \begin{cases} \frac{Y_{k+1}}{Y_{k+1}+Y_k} \ (Y_{k+1} \geq Y_{k-1}), 0 \leq \Delta K \leq 1 \\ \frac{-Y_{k-1}}{Y_k+Y_{k-1}} \ (Y_{k+1} \leq Y_{k-1}), -1 \geq \Delta K \geq 0 \end{cases} \tag{1}$$

$$\Delta K_{hann} = \begin{cases} \frac{2Y_{k+1}-Y_k}{Y_{k+1}+Y_k} \ (Y_{k+1} \geq Y_{k-1}), 0 \leq \Delta K \leq 1 \\ \frac{Y_k-2Y_{k-1}}{Y_k+Y_{k-1}} \ (Y_{k+1} < Y_{k-1}), -1 \geq \Delta K \geq 0 \end{cases} \tag{2}$$

The corrected frequency is $X_0 = K + \Delta K$ in Fig. 1, Y_k is the highest amplitude at corresponding frequency *k*, Y_{k+1} and Y_{k-1} are the amplitude of the corresponding frequencies *k*+1 and *k*-1 near the spectrum peak.

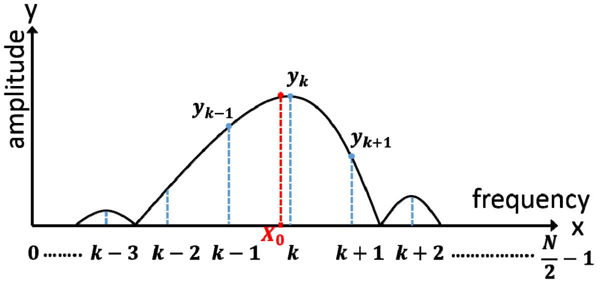

Figure 1: Scheme of the frequency correction spectrum in the rectangular window.

FFTC SIMULATION IN KNOWN SPECTRAL LINES

A known spectral signal is sufficient to ensure the frequency errors with noises by using FFTc method in different window functions. We produced a sinusoidal function including three main frequencies as the simulated signals of betatron tune as the below formula Eq. (3).

$$f(t) = \sqrt{2} \begin{bmatrix} \cos\left(2\pi f_1 t + \frac{10\pi}{180}\right) + cos\left(2\pi f_2 t + \frac{20\pi}{180}\right) \\ +cos\left(2\pi f_3 t + \frac{30\pi}{180}\right) \end{bmatrix},$$

$$f_1 = 0.123245\,, f_2 = 0.143426, f_3 = 0.385572 \tag{3}$$

This signal is described by sampling points $N = 1000$ to simulate the beam propagation turns in the circular accelerator, and the transformed frequencies are represented as the betatron tunes. Figure 2 shows the spectrum of Eq. (3) from FFT with the rectangular window adding 0 and 20 % white Gaussian noise, respectively.

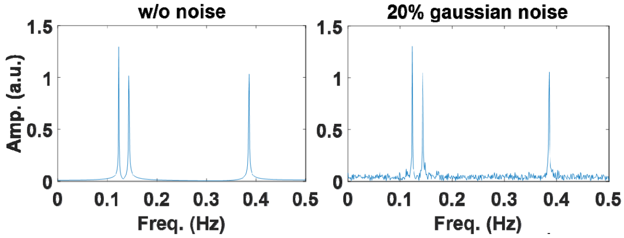

Figure 2: Spectrum of the simulated spectral lines with 0 and 20 % noise, respectively.

The simulated frequency errors were defined as $error = \left|f_{1\sim3} - FFTc_{1\sim3}\right|$ from $N = 100$ to 2000. The theoretical frequency error of FFT is proportional to N^{-1} as the black dotted line in figure 3(a) ~ (d). The red, blue and black circles mean the absolute errors of three frequencies after FFTc approach. The errors in different frequencies are compared following with turns by using rectangular window, without noise in Fig. 3(a) and with noise in 3(b). Similarly, fig. 3(c) and 3(d) show the errors in hanning widow without and with noise.

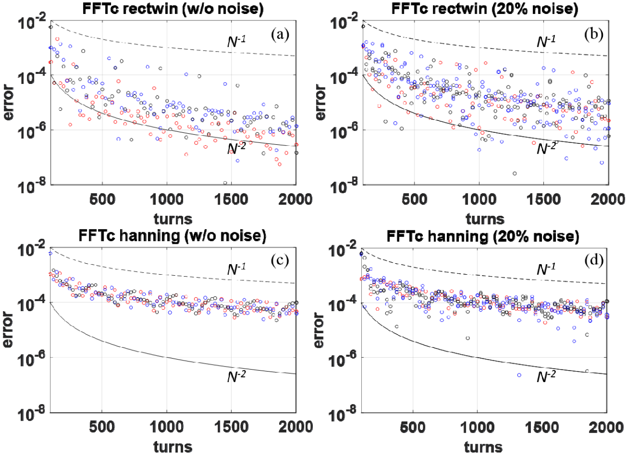

Figure 3: The frequency errors in rectangular and hanning duction with 0 and 20 % noise, respectively.

The errors from FFT approach identically distribute near the curve of N^{-1}, it means that the frequency error accuracy is limited in 10^{-3} within 1000 sampling points. In fig.3, overall the tune errors of FFTc approach are 10^{-4} ~10^{-5} in the both of windows without adding noises. And, the errors of FFTc were advanced to one order than that from FFT under noisy fluctuations. Table. 1 summarizes the error values of FFTc in 1000 sampling points, the absolute frequency errors are less than 10^{-4} comparing with the known three frequencies (f_1 ~ f_3). The comparison of the errors by using different window function in FFTc, although the fig. 3(a), (b) using the rectangular window function could take the more exact transformed frequencies, the fig. 3(c), (d) using hanning window function show a better numerical convergence than fig. 3(a), (b) in these frequencies. The corrected frequency approach by FFTc is exactly to advance the accuracy of frequency errors in the simulation, and could be applied on the real machine to analyse a precise betatron tune in TPS.

Table 1: The Comparison of the Errors in the FFTc Approach

	target frequency ($f_1 \sim f_3$)	
$f_1 = 0.123245$	$f_2 = 0.143426$	$f_3 = 0.385572$
Window function	Correction frequency $f_{c1\sim3}$	Absolute frequency error $\lvert f_{c1\sim3}-f_{1\sim3}\rvert$
rectangular	$f_{c1} = 0.123239$	5.8016e-06
	$f_{c2} = 0.143431$	5.4641e-06
	$f_{c3} = 0.385556$	1.5997e-05
hanning	$f_{c1} = 0.123368$	1.2253e-04
	$f_{c2} = 0.143624$	1.98066e-04
	$f_{c3} = 0.385658$	8.62722e-05

BETATRON TUNE MEASUREMENT

The betatron tune analyses from the beam orbit changes are widely employed to understand the stability of betatron motion in the circular accelerator. The betatron tunes of horizontal and vertical are $\nu_x = 26.1802$ and $\nu_y = 14.2601$ in the model of Taiwan Photon Source (TPS) with Double-Minimum-Betay (DMB) design [7] in the insertion devices (ID) section. However, the beam energy loss and electrons incoherence motion leads to the attenuation of betatron oscillations, the characteristics of betatron behaviour be observed weaker and weaker in the limited turns in the storage ring of TPS. The resolution frequency of TPS is 578- kHz, and betatron oscillation error will be 578 times at betatron tune error 0.001. The accurate analysis of betatron tune less than 10^{-4} is necessary for further diagnoses about betatron tune such as dynamics aperture, resonant frequency map etc.

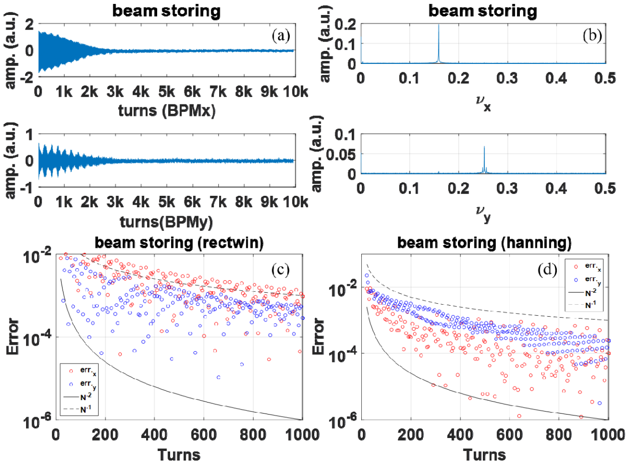

Figure 4: when beam storing, turn the kick-magnet on (a) turn-by-turn (TBT) betatron oscillations in horizontal and vertical plane, (b) the betatron tune spectrum, (c) and (d) the errors of betatron tune in horizontal and vertical plane by using rectangular and hanning window.

When electron bunch beam injected into the TPS storage ring, and stored the beam charge stably. A general betatron tune measurement is to perturb the bunch beam deviating its transverse orbit shifts a little offsets in the storage ring via a pulsed magnet. The magnitude of beam orbit oscillations decay following with the beam propagating turns measured by a specific BPM. Figure 4(a) shows the clearly transverse beam orbit oscillations in a limited 2500 ~ 3000 turns. 4(b) is the FFT spectrum of betatron oscillations in the 10000 turns. In fig. 4(a) and 4(b), the red and blue circles symbol the absolute tune errors between the FFT and FFTc. The tune errors are defined as $error = |FFT_{10000\,turns} - FFTc_{1\sim10000\,turns}|$. The tune ν_x and ν_x are shown the transverse betatron tune in horizontial and vertical plane, and the black dotted line and black line are the curve of inverse of beam turns (N^{1}) and inverse with square of beam turns (N^{2}), respectively. Figure 4(c) and 4(d) show the errors analysed by using rectangular and hanning windows in FFTc. A tune accurate of 10^{-4} could be in 1000 turns by using hanning window, FFTc method in fig 4(d).

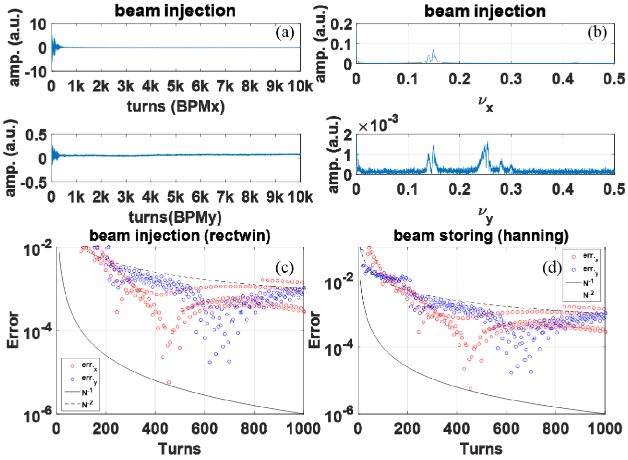

Figure 5: when beam injection, turn the kick-magnet off (a) turn-by-turn (TBT) betatron oscillations in horizontal and vertical plane, (b) the betatron tune spectrum, (c) and (d) the errors of betatron tune in horizontal and vertical plane by using rectangular and hanning window.

However, considering the status of ID commissioning, the beam orbits will be modulated and kept at the specific trajectory for providing the maximum and narrow-band spectrum synchrotron radiation sources to experimental users. Utilizing a pulse-magnet for kicking a transverse beam offset does not allowed because of inducing the unstable power and broad-band spectrum of the synchrotron radiation form ID. The temporary beam orbit distortion occurs at the moment of beam injection from transfer line, there are few kick-magnets for transferring bunch-by-bunch beam into the storage ring. The betatron tune measurement is based on the beam offset while beam injection. This tune measurement will not excite beam offset periodically in the storage ring to influence beam emission. Figure 5(a) displays the slight betatron oscillations that are decayed quicker than that of fig. 4(a) due to the operation duration of pulse magnet is shorter. The efficient tune was transformed in front of few hundred turns in fig. 5(b). Although the transformed data are the same in 10000 turns, the tune spectrum is noisier than that of fig4 (b) because of the significant betatron oscillations are limited. Figure 5(c) and 5(d) both show that the characteristic turn values at 400 and 700 turns in horizontal and vertical direction. The tune errors are symbolled the same as above-mentioned. Less than 10^{-4} even 10^{-5} betatron tune accurate could be acquired within a limited 1000 beam turns. The numerical method of FFTc approach is one kind of technique to obtain the precise frequency analyses in the real machine of circular accelerators

CONCLUSION

We presented a numerical method -FFTc for solving the precise spectrum. The two window functions, rectangular and hanning window functions as the numerical filters approach to find the corresponding frequencies. We proof this mechanism in a sample sinusoidal formula with three known frequencies as the simulation of betatron tune. The frequency errors of FFTc are much less than the inverse of the sampling points that the limited accurate of the FFT. The exact frequency errors 10^{-5} and 10^{-4} are obtained under 1000 sampling points in rectangular and hanning window, respectively. Based on the simulation results, we further applied this technique to find more precise betatron tunes in TPS storage ring from the BPM TBT acquisitions. We analysed the precise betatron tunes and their errors are near 10^{-4} when beam storing, BPM acquirements in 1000 beam turns. Furthermore, the accuracy betatron tune errors are less than 10^{-4} within 700 beam turns.

Advanced frequency transform are generally applied in the precise betatron tunes analyses. We demonstrated one kind of these numerical methods FFTc, introduced the mathematic model, simulated the possibility and applied this method in real machine. FFTc is a precise, convenient and simple tool to solve the spectrum analyses.

ACKNOWLEDGMENTS

Authors highly appreciate the technological supports from Instrumentation & Control Group at NSRRC.

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