THEORETICAL ANALYSIS AND SIMULATION OF A COMPACT FREQUENCY MULTIPLIER FOR HIGH POWER MILLIMETER AND TERAHERTZ SOURCES

A. Vrielink*[1], S. Tantawi , F. Toufexis[1], SLAC National Accelerator Laboratory, Menlo Park, USA
[1]also at Department of Electrical Engineering, Stanford University, Stanford, USA

Abstract

As demands on accelerating gradients and temporal resolution of beam diagnostics and manipulation schemes grow, millimeter-wave and terahertz (THz) accelerator structures may present a natural solution. The recent advent of a radiofrequency undulator [1] and the development of a 0.45 THz accelerator [2] demonstrate growing interest in this frequency regime; however, growth in this area is limited by the lack of efficient, compact high power sources. We present a novel vacuum electronic device featuring an interaction between a radially bunched electron beam and azimuthally traveling waves. The use of an inward traveling radial sheet beam mitigates space charge effects at the low operating energy of 10 keV-30 keV and allows for a high input beam current of approximately 0.5 A-10 A. Based on preliminary calculations, these devices could operate from 50 GHz to 250 GHz with tens of kiloWatts of output power, while the expected efficiency would scale from 60% at 80 GHz to 15% at 230 GHz. Here we present the underlying theory, possible structure design, and preliminary results from analytical calculations and simulation.

INTRODUCTION

Within the mm-wave and THz frequency regime, ranging from 50 GHz to 10 THz, the power of electronic sources falls off as the inverse of the frequency squared while the power of optical sources increases only linearly. In the region bridging the radiofrequency and visible bands of the spectrum, these two trends produce a minimum commonly referred to as the "THz Gap". Particle accelerator applications requiring average power in the kW to MW regime at these frequencies rely primarily on vacuum electronic devices (VEDs).

Unfortunately, VEDs capable of these high average powers, namely gyrotrons, require magnetic fields of several Tesla and typically operate as oscillators. Modifications have also been made to existing devices (klystrons, TWTs, etc.) to extend their frequency range through overmoding, multi-beam, and sheet beam concepts. These have met with limited success due to mode competition in overmoded devices, mechanical difficulties with single-moded devices, and stringent requirements on the electron beam voltage and focusing optics necessary to mitigate space charge effects.

To address these issues, we are developing a series of devices that circumvent traditional scaling laws through the use of concentric spherical shells and a unique beam-wave interaction. In typical klystrons, the electron beam travels along a linear path with the beam-wave interaction induced by a longitudinal density modulation in the beam. Instead, we propose using an electron beam with a radial density modulation which excites azimuthally traveling spherical electromagnetic (EM) modes. Herein we present an analytical treatment of this interaction, including studies to determine optimal spherical geometries for the output cavities and preliminary estimates for the total efficiency of these structures.

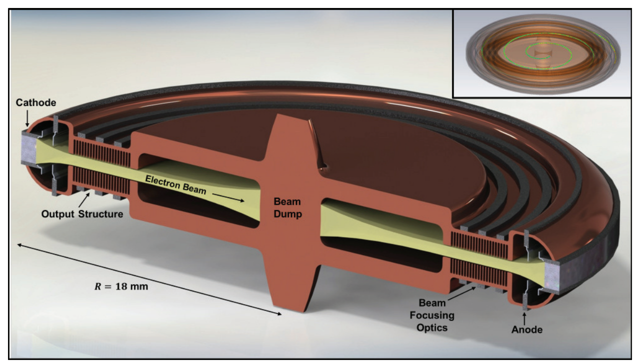

Figure 1: Cross sectional view of the proposed structure. While the beam path is shown in yellow, the actual electron distribution is shown in green from a top down view in the top right inset. The beam appears to spiral due to the relationship between azimuthal position and emission time induced by the RF fields at the cathode yet each individual electron travels radially with negligible azimuthal velocity.

Design Concept

One possible design for this type of interaction involves a continuously deflected CW beam interacting with spherical sector cavities. This concept has been demonstrated experimentally at 57 GHz and is discussed in [3]. Here we focus on the design in Fig. 1, an equatorial configuration involving a radially traveling sheet beam with the emission time of individual electrons determined by their azimuthal position on the cathode. This beam interacts with a series of concentric spherical shell cavities capped at $\pm\theta_{\text{shell}}$.

While out of the scope of this paper, a key challenge in this design is the unique requirements of the electron source, which must produce a high current density radially modulated at a subharmonic of the operating frequency. Our proposed solution to this is a thermionic RF Schottky gun. The mode excited in the input cavity will be a traveling wave in the azimuthal direction, with radial fields that will lower the potential barrier. The barrier, and hence the current

* vrielink@stanford.edu

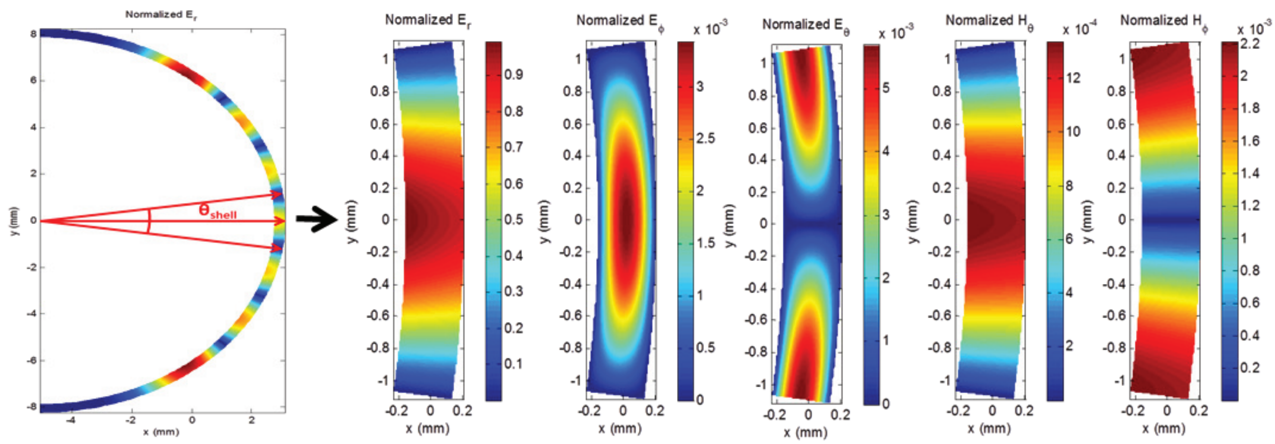

Figure 2: Field distribution for the case of $m = 20$, $n = 28$ and $C1, C2 = 0$ where the radial field is supported by two concentric spherical shells. Note that the same field distribution can be excited on axis if the cavity is capped at θ_{shell}.

emitted from the cathode, will thus be modulated at this subharmonic. The large radius at which the beam is emitted is a significant advantage here, reducing image charge effects at the cathode and space charge effects as the beam travels. This allows for higher input current to the output structure.

THEORY

The modes for the output cavity are transverse magnetic to the radial direction and are traveling in the azimuthal or ϕ direction. The general expression for the radial electric field of these modes is given by equation 1.

$$E_r = \frac{\alpha n(n+1)\sqrt{kr}e^{im\phi}}{r^2 y}[J_{n+\frac{1}{2}}(kr) + \mathrm{C1}\ Y_{n+\frac{1}{2}}(kr)]* \\ [P_n^m(\cos(\theta)) + \mathrm{C2}\ Q_n^m(\cos(\theta))] \quad (1)$$

These modes, supported by spherical boundary conditions, are defined by the parameters $n, m, k, C1$ and $C2$. m determines the number of azimuthal nodes, while n defines the poloidal variations of the field. These together with $k = \frac{\omega}{c}$, where ω is the resonant frequency, determine the radial variation of the field. By choosing a large index number for the field variation in the θ direction, the electric field components are predominantly in the radial direction, (E_θ is 0 where E_r is peaked and E_ϕ is approximately 2-4 orders of magnitude smaller). The field in this case is confined to a small portion of the poloidal angle θ and the spherical shell cavity can be capped so that only a small shell is left without affecting the field profile. This reduces the stored energy of the cavity and increases the interaction efficiency.

The parameter m is selected to ensure the angular phase velocity of the RF fields excited in the cavity matches that of the electron beam, a condition necessary for a sustained interaction. This condition is met when $\omega_{\text{out}} = m\omega_{\text{in}} = 2\pi m/T_{\text{in}}$ where T_{in} is the period of the azimuthal density modulation of the beam entering the output cavity and ω_{out} is the desired output frequency.

$C2$ determines the poloidal angle at which the field maxima are located and is thus selected to align with the electron beam axis, $\theta_{\text{peak}} = \theta_{\text{beam}}$. For the spherical sector cavity, $\theta_{\text{beam}} \approx 10°$, while for the equatorial configuration $\theta_{\text{beam}} = 90° \Rightarrow C2 = 0$. $C1$ can be determined by the positions of the inner and outer conductor, and the value of n selected. Of importance for multiple cavity structures, increasing n allows for spherical shell cavities with ever increasing radius for the same m and ω_{out}.

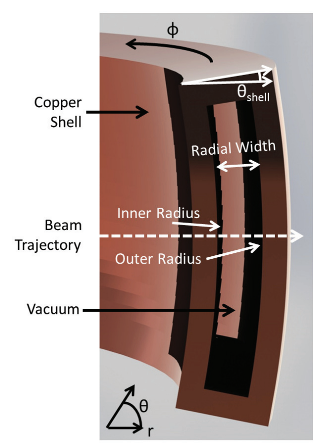

Figure 3: Cross sectional view of a single cavity.

Using the analytical expression for the radial field (we can neglect the other, much weaker field components initially) we can rapidly calculate the cavity efficiency. Starting with the expression for the complex power balance, equation 2, we can derive an expression for the excited field amplitude in terms of several key cavity parameters. [4]

$$\frac{1}{2}\int_{\text{port}} E^* \times H \cdot \mathrm{d}S + \frac{1}{2}\int_{\text{cavity walls}} E^* \times H \cdot \mathrm{d}S = \\ -\frac{1}{2}i\omega \int \mu|H|^2 - \epsilon|E|^2 \mathrm{d}V - \frac{1}{2}\int E^* \cdot J\mathrm{d}V \quad (2)$$

The resulting expression in the case of no cavity input power is given by equation 3, where α_0 is the field amplitude, Q_0 is the cavity quality factor, U_0 is the stored energy in the

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cavity, ω_{out} is the output frequency and β is the coupling coefficient. Here we approximated the beam as a ring of charge of infinitesimal radial and polar width, azimuthally modulated at the input frequency such that the frequency component at m is $J_m = q\delta(r - r_0)\delta(\theta - \theta_0)e^{im\theta}$. We set the output cavity detuning, δ_0=0 for this analysis, and optimized the coupling coefficient β.

$$\alpha_0^2 = -\frac{Q_0}{U_0\omega_{\text{out}}(1 + \beta + i\delta_0 Q_0)} \int_V \tilde{E}^* \tilde{J}(\alpha_0)\text{dV} \quad (3)$$

The total efficiency of the cavity can be found once alpha is known, and simplifies to the expression in equation 4.

$$\frac{\eta}{I_{\text{beam}}} = \frac{P_{\text{out}}}{P_{\text{beam}} I_{\text{beam}}} = 4\frac{\alpha^2 ckU_0}{E_{\text{beam}} Q_0} \quad (4)$$

RESULTS

The basic single cavity geometry, along with pertinent dimensions, is shown in figure 3. Consider the case where E_{beam} = 10keV, m = 7 and ω_{in} =11.4 GHz, implying ω_{out} = 7∗11.4 GHz ≈ 80 GHz.

The maximum efficiency occurs when the radial extent of the cavity corresponds to an electron transit time of half an RF period. Additionally, these cavities operate better at low voltages, when the transit time is longer. These effects are demonstrated in Fig. 4.

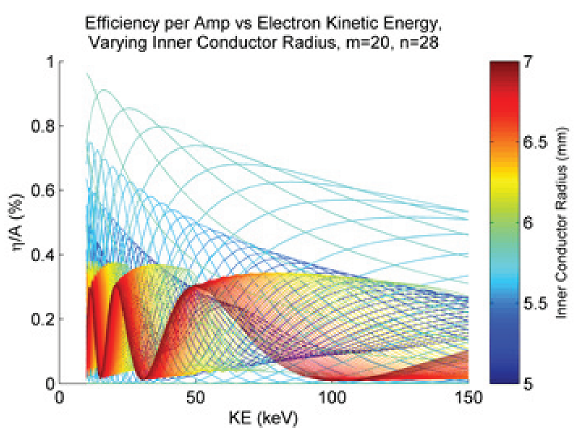

Figure 4: Efficiency as a function of input beam energy and cavity radial width, demonstrating the transit time effect.

If we fix the radial cavity width to this transit distance, the inner cavity radius is fixed by the value of n chosen. As m is fixed, selecting n also sets the required value of θ_{shell}. The approximate efficiency as a function of n is plotted in figure 5, ignoring changes in the beam velocity. This approximation is reasonable for small transverse fields and weak beam-wave interaction as is the case here. While the peak efficiency for our 1A macroparticle occurs at n = 9 and reaches only 4.5%, the true strength of this type of device is the opportunity for extended interaction through multiple cavities. We find that the inner radius increases by a value larger than the radial cavity width as n increases, thus we can easily stack 50-70 of these cavities. The cumulative efficiency is shown in figure 5 and approaches 60% per Ampere for a 10keV beam at an optimal coupling coefficient. A similar analysis at 230 GHz produces a cumulative efficiency of 15% per Ampere.

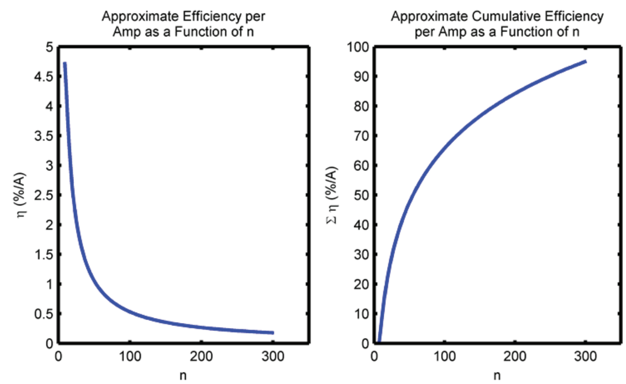

Figure 5: Power extraction efficiency per amp for several values of n, per cavity (left) and cumulative (right).

CONCLUSION

From these studies we obtained optimized geometries for a structure operating at 80 GHz with an input beam modulated at 11.4 GHz (m=7), and a similar one at 228 GHz and m = 20. The cumulative efficiency of these structures for a single macroparticle (ie. ignoring the full beam dynamics), ranged from 60% at 80 GHz to 15% at 230 GHz.

To obtain this efficiency in practice will require adjusting the radial width of each cavity to the actual transit time of the beam, which will be decreasing as the beam loses energy to the fields. A coupling structure will need to be designed to couple the power out of each cavity and match the phase of the RF generated by the beam, which will differ from cavity to cavity as they are uncoupled from each other. The actual beam dynamics will reduce the efficiency, as will the addition of the aperture for the beam to pass through the cavity. While this will reduce the efficiency per Ampere, we expect the total efficiency can be compensated for by increasing the beam current.

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