USING OF THE MENT METHOD FOR RECONSTRUCTION OF 2D PARTICLE DISTRIBUTIONS IN IFMIF ACCLERATORS

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Abstract

Beam particles are characterized by their coordinates in real spaces or phase spaces that are at least two-dimensional. It is often necessary to reconstruct such a 2D-distribution from the knowledge of only its projections onto some axes. Our objective is to determine the minimum number of parameters to be measured on line or to input into simulations, which can correctly describe the beam distribution. In this article, the use of the MENT (Maximum Entropy) reconstruction method is reported for the IFMIF accelerators where high intensity beam distributions significantly depart from Gaussian.

INTRODUCTION

Particle distributions in a 2D-real or -phase space are totally characterized by the whole set of each of the particle coordinates in this space. Besides this exhaustive knowledge, a summarized parametrization is currently used, consisting in RMS values of the coordinates. In phase spaces, the latter are referred to as emittance and Twiss parameters of the RMS concentration ellipse. In case of Gaussian distributions, due to its regular shape, RMS values are sufficient to get a good idea of its characteristics. For IFMIF-like high intensity accelerators however, particle distributions significantly depart from Gaussian ones. In [1] for example, it is found that two different distributions, one Gaussian and one called "nominal input" for the LIPAc HEBT, having exactly the same emittance and Twiss parameters, will become significantly different after transport through a 3.5 m line equipped with three quadrupoles.

It is therefore necessary to go beyond the usual RMS parameters. The idea is to characterize high intensity distributions by their projected distributions on a few axes. In this paper, we first recall the MENT method used to reconstruct the distribution from its projections, then apply it to typical IFMIF distributions. The number of needed projections is finally discussed. The objective is to determine the minimum number of projections, and which ones, that should be measured online or input into simulations in order to correctly represent the beam.

THE MENT METHOD

The question is to reconstruct a particle distribution from the sole knowledge of its projections. This typical problem with missing data admits in principle an infinity of solutions. In order to obtain a unique solution, an additional assumption must be made, by stating that, where no data is available, the distribution is as regular as possible. In terms of mathematics, this means that the distribution must be described with the least number of parameters and in terms of information science, the least data, which means that the entropy is maximal. This method is referred to as MENT, for maximization of entropy.

G. Minerbo [2] first introduced the MENT approach to reconstruct beam distributions, including thorough analytical calculations and the Gauss-Seidel algorithm for numerical calculations. This method was then successfully used for exploiting results coming from several beam tomography experiments [3, 4 and 5].

The MENT algorithm is known for its very fast convergence, generally in less than 5 iterations. The searched distribution having the imposed projection profiles is quickly found. But which are the projections that allow to reconstruct the actual distribution is another question that remains to be studied. An important part of this question is the choice of projection axis orientations. If a particle distribution is relatively isotropic, then projection axes evenly positioned in angles within 360° is appropriate, but in case of a stretched distribution, it is not. In [6], it is judiciously suggested to use equal angular intervals for projection axes, but in the normalized space instead of the standard one, because an ellipse in the latter is transformed into a circle in the former.

In the course of our studies, we have successfully tested a procedure that would be similar to the above suggestion while always staying in the standard space. It consists in:
1) Reconstructing the distribution with 4 projection axes evenly positioned within 360°, i.e. 0°, 45°, 90° and 135°.
2) Calculating the concentration ellipse of the obtained distribution; determining its major axis angle θ and aperture $\delta\theta$ of which the tangent is given by the ratio of its major to minor axis.
3) Finally, reconstructing the distribution with projection axes regularly positioned, not within 360° but within $\theta \pm \delta\theta$, combined with axes perpendicular to those ones.

APPLICATION TO TYPICAL IFMIF DISTRIBUTIONS

By means of a home-made code, the MENT method was successfully applied to particle distributions along the IFMIF-LIPAc [7]. Results for two typical distributions are presented and discussed here, at the MEBT exit and SRF-Linac exit, two key locations where the beam should be measured and qualified. Examples are shown in z, z' for the first case and in x, x' for the second one. For each example, the actual distribution is shown (Figure 1 and 2),

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to which reconstruction results for 2, 4 and 6 projections should be compared, when angles of projection axes are taken within 360° or within $\theta \pm \delta\theta$ (Figure 3 and 4). In parallel, RMS quantities (emittance and Twiss parameters) of reconstructed distributions are also compared to those of the actual ones.

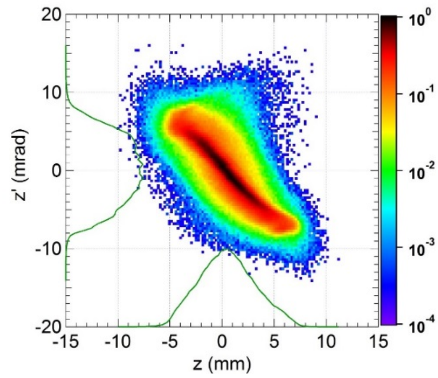

Figure 1: Actual distribution at MEBT exit in z, z', to be reconstructed. Examples of projections onto horizontal and vertical axes are shown.

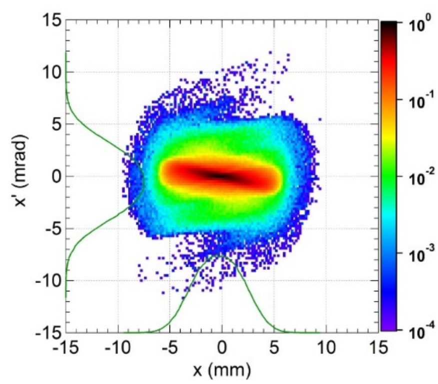

Figure 2: Actual distribution at SRF-Linac exit in x, x', to be reconstructed. Examples of projections onto horizontal and vertical axes are shown.

For the distribution at MEBT exit, as the θ direction is very different from horizontal/vertical axes, the general shape of reconstruction results in case of projection angles regularly distributed in 360°, is very bad for 2 projections, but becomes immediately satisfying from 4 projections. RMS quantities are different from the actual ones, of 100% for 2 projections, 10 % for 4 projections and are further halved from 6 projections. When now considering reconstructions within -57° ± 15° (for one half of projections, the other half being perpendicular), right for 2 projections, the global shape is not very different from the actual one, and from 6 projections, most of the details are well reproduced. Differences of RMS quantities start from 4-3% at 2-4 projections, and come down to less than 1% for more projections.

For the distribution at SRF-Linac exit on the contrary, the θ direction is very close to the horizontal axis, but the general shape, especially in the external parts, is very far from cylindrically symmetric. That is why reconstructions in 360° easily give a satisfying core but much hardly the external parts, even for a big number of projections. RMS quantity differences are only 10% at most (except for α because it is close to zero). Reconstructions with projection angles within -6° ± 23° give satisfying global shapes from 2 projections and reproduce well the very external parts from 6 projections. RMS differences are only 2.5% at most.

If more projections are available, then the reconstructed distribution is all the more consistent with the actual one. Results with 10 projections for example (not shown here) reveal almost all the shades of the density distribution very similarly to the actual one, from the core to the most external halo. For an image of 100x100 bins, computing such a reconstruction takes typically 12 minutes after 3-4 iterations.

CONCLUSIONS

The MENT reconstruction method is very appropriate for recovering particle distributions. For very high intensity beams like IFMIF-LIPAc, which are very different from Gaussian beams, satisfying results are obtained after 3-4 iterations. This method also helps in terms of physical insight. Whenever the different projections are consistent between them, whatever their number, the method will find out for sure a distribution presenting relatively precisely those projections. When some projections cannot be recovered precisely, that means there are errors, inaccuracies or inconsistencies in the projections.

We also pointed out the importance of positioning the projection axes with angles following the distribution main axis (θ) and aperture (δθ). When this is done, the actual distribution can be satisfyingly recovered from 2 projections for the core and from 6 projections for the external parts. Those are the minimum number of profiles to be measured or data set to be input into simulations, so as to correctly represent the beam.

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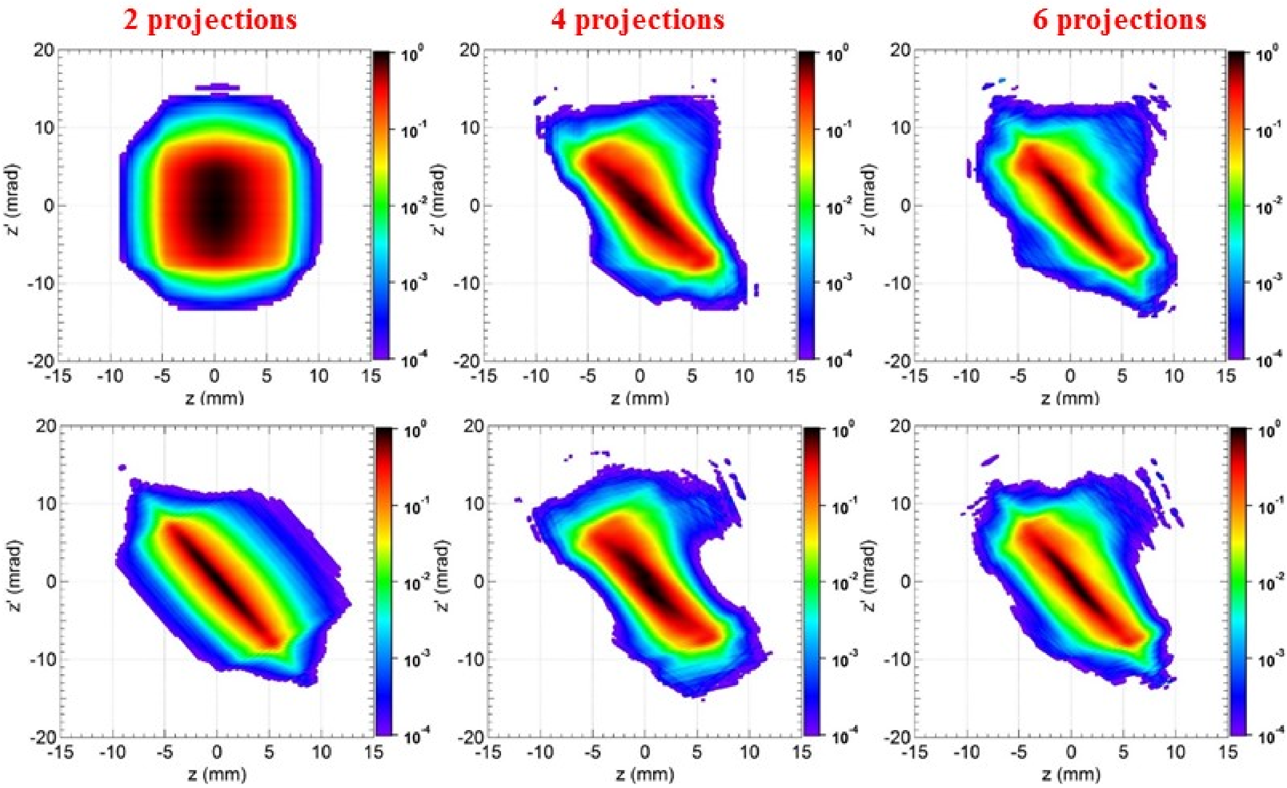

Figure 3: Reconstructed distributions for the MEBT exit with 2, 4, 6 projections. Top three graphs: projection axes are positioned with angles within 360°. Bottom three graphs: projection axes are positioned with angles within -57° ± 15°.

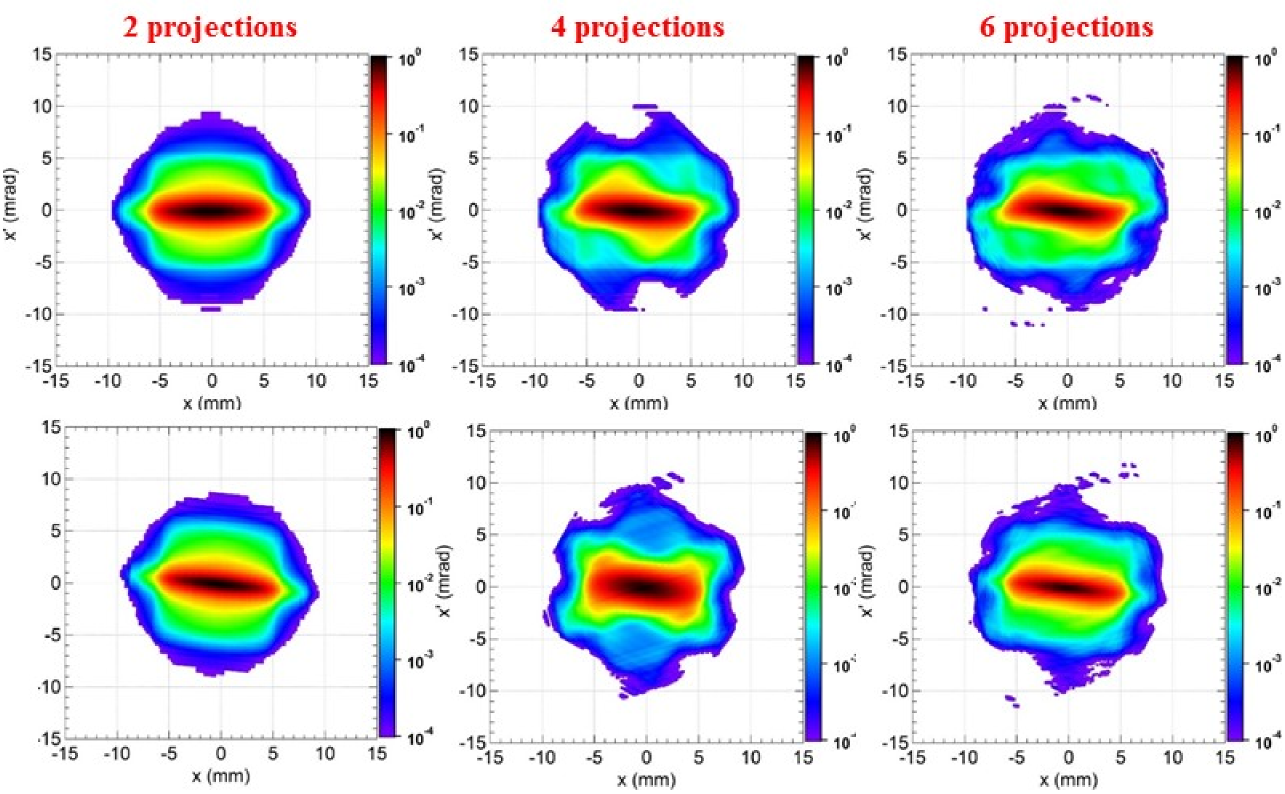

Figure 4: Reconstructed distributions for the SRF-Linac exit with 2, 4, 6 projections. Top three graphs: projection axes are positioned with angles within 360°. Bottom graphs: projection axes are positioned with angles within -6° ± 2.