NUMERICAL SPACE-CHARGE COMPENSATION STUDIES AND COMPARISON OF DIFFERENT MODELS

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Abstract

The design of many Low-Energy Beam Transport sections relies on the presence of space-charge compensation by particles of opposing charge. To improve understanding of the processes involved in the built-up and steady-state, simulations using the Particle-in-Cell code bender were made. We will present the influence of various system parameters on the results. Furthermore, the electron velocity distribution was found to be approximately thermal. The spatial distribution can then be found by solving the Poisson-Boltzmann equation. Such a model for the electron distribution was implemented in a 2D PIC code and applied to typical beam transport situations. We will present results in comparison to the 3D simulations.

SPACE-CHARGE COMPENSATION

When the electron density is not proportional to the beam density, various definitions for a "compensation degree" can be found.

$$\eta_{\text{part}}(z) = \lambda_{e^-}(z) \,/\, \big(\lambda_{\text{beam}}(z) + \lambda_{\text{rgi}}(z)\big),$$

i.e. the ratio of line charge density of positively and negatively charged particles, is a measure of how much charge can be confined by the beam potential. To quantify the effect on the beam,

$$\eta_{\text{beam}}(z) = 1 - \frac{\varphi(R_{\text{beam}}, z) - \varphi(0, z)}{\varphi_{\text{uncomp}}(R_{\text{beam}}, z) - \varphi_{\text{uncomp}}(0, z)},$$

defined as the ratio of the potential drop between the beam's edge and the axis for the compensated as well as the identical beam distribution without secondary particles, can be used.

SIMULATION OF A DRIFT SECTION

The Particle-in-Cell code bender [1] was used to simulate a 50 cm long beam drift through a system terminated by repeller electrodes. The simulations include proton impact ionization and electron impact ionization of an homogeneous background gas, calculated using single-differential cross sections from [2, 3]. Argon at 1×10^{-5} hPa was used as background gas. More on the built-up of the compensation and the model can be found in [1].

Figure 1 shows the density for the different species after 40 µs. Especially around its focus point, the beam has become hollow. This leads to a slightly worse compensation in the focus, $\eta_{\text{beam}} = 77\,\%$, compared to 87 % and 81 % towards the front and the back of the system. A significant portion of the electrons are located beyond the beam radius, effectively not contributing to the compensation. They oscillate radially through the beam volume. Since the total charge density around the axis is close to zero, electrons move at nearly constant velocity and are only reflected by the increase in electric field at the beam's edge.

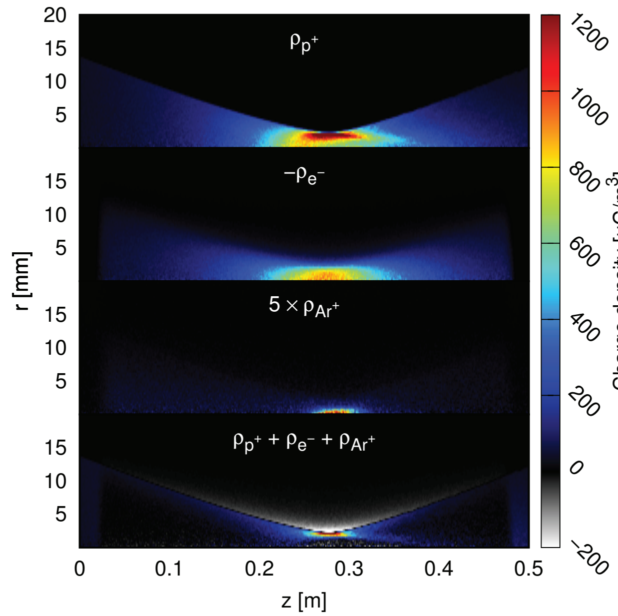

Figure 1: Charge density of the beam particles, compensation electrons and residual gas ions as well as the total charge density from a *bender* simulation of a drift section.

The residual gas ions are continously expelled radially by the remaining electric field. However, within the beams focus, there is a small accumulation of ions. This effect as well as the changes in beam distribution can be understood by investigating the influence of the velocity distribution of the compensation electrons.

Figure 2 shows the distribution of particle energies at arbitrarily chosen locations within the system. For $H < 0$, these show an exponential behaviour. Higher energies are less often populated, since these electrons are able to escape from the system. Over a slice of particles, the velocity distribution can be well approximated by a Gaussian. The "temperatures" differ between the transverse and longitudinal directions and vary between 10 eV and 25 eV along the beam. There is a significant influence from the number of simulation particles on these temperatures – these effects will be discussed in a later contribution.

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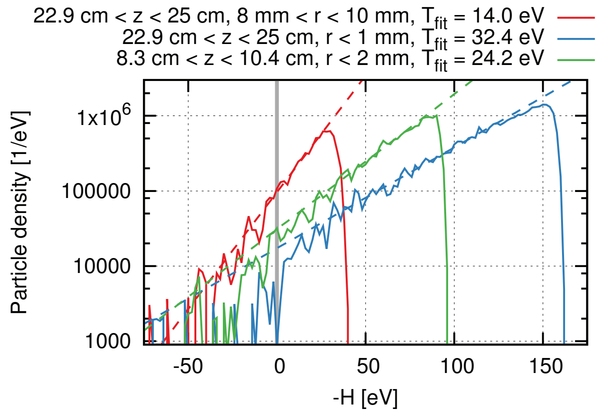

Figure 2: Distribution of particle energies at various locations.

POISSON-BOLTZMANN EQUATION

When the compensation electrons in a slice of the beam are known to be Boltzmann distributed, their density behaves like $n(\mathbf{r}) = n_0 \exp(e\varphi(\mathbf{r})\,/k_\mathrm{b}T)$. The potential can be found from the Poisson-Boltzmann equation,

$$\nabla^2\varphi(r) = -\frac{1}{\epsilon_0}\left(\rho_\mathrm{beam}(r) + \rho_\mathrm{comp}\exp\left(\frac{e\varphi(r)}{k_\mathrm{b}T}\right)\right) \qquad (1)$$

The equation can be non-dimensionalized by introducing $\tilde{\varphi} = e\varphi/k_\mathrm{b}T$ and scaling the coordinates by the Debye length λ_d [4]. Together with

$$\mu = \eta_\mathrm{part}\int \exp(\tilde{\varphi}(r))\,\mathrm{d}V\left(\int f_\mathrm{beam}(r)\,\mathrm{d}V\right)^{-1},$$

it becomes

$$\tilde{\nabla}^2\tilde{\varphi} = f_\mathrm{beam}(r) - \mu\exp(\tilde{\varphi})\,. \qquad (2)$$

Besides η_part, the only degree of freedom is the Debye length. Therefore, for equal beam distributions, when the ratio of the beam density to the electron temperature is equal, the electron distribution will be the same.

Figure 3 shows an example for the solutions of (2) for 90 % charge compensation of a homogeneous beam. When λ_d is much smaller than the beam radius, the density of the electrons follows the beam distribution up to some radius. For the homogeneous beam in Fig. 3, the electric field behaves like that of an hollow beam, i.e. it is zero up to a certain radius and then rises quadratically. For increasingly higher temperatures, a larger amount of electrons has enough energy to be located at larger radii, forming the negative charge density beyond the beam edge also observed in Fig. 1. When the Debye length is much larger than the beam radius (not shown), the electrons form an approximately constant background. Such a situation will not occur in reality due to electron losses on the beam pipe.

The algorithm used to solve (2) was used as a space-charge solver in the two-dimensional beam transport code *tralitrala* to study the evolution of the beam distribution including the

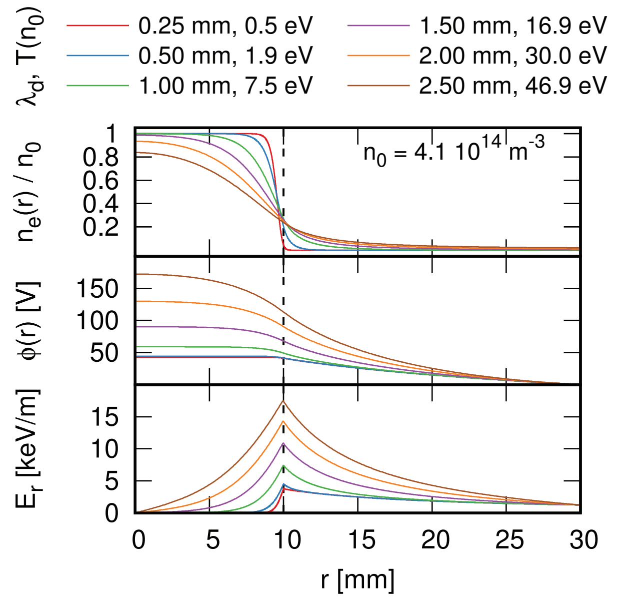

Figure 3: Solution to (2) for a 100 mA proton beam at 120 keV with 1 cm beam radius compensated to 90 % by a thermal electron distribution.

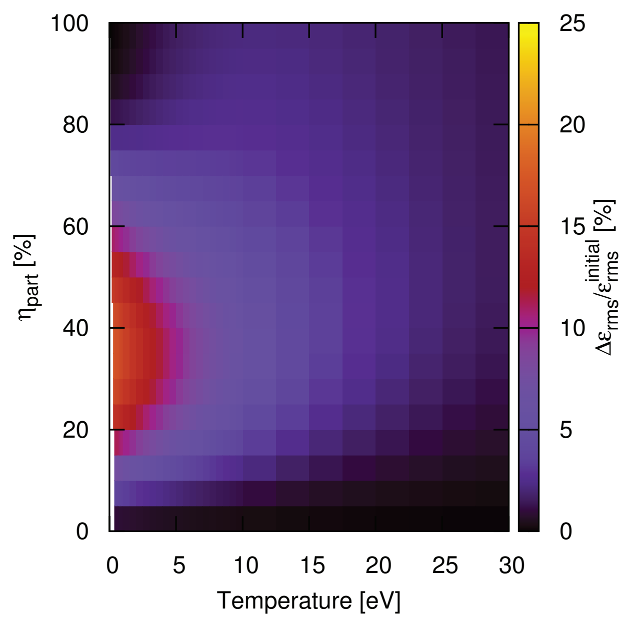

Figure 4: Emittance growth of an initially parallel proton beam (1 cm, 120 keV, 50 mA).

non-linear fields produced by the compensation electrons. Figure 4 shows the increase in rms emittance after a drift of 50 cm. It is zero, when no fields are present, which is the case for $T \approx 0$ eV and full compensation, and when these are fully linear, i.e. $\eta_\mathrm{part} = 0$. The worst case occurs for moderate levels of compensation η around 35 % and low electron temperatures. For these parameters, only particles

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beyond a certain radius see a rapidly increasing electric field. Towards higher temperatures the electron distribution becomes more spread out, mitigating the situation. A small emittance growth also remains when the beam's charge is equal to the electron charge at higher T, since some electrons are always located outside the beam volume.

When the line charge density as well as the temperature distribution $T_{\mathrm{x,y}}(z)$ are taken from the full PIC simulation, many of its features can be reproduced by the model simulation. Figure 5 shows the netto charge density from such a calculation. The increasingly hollow beam towards the focus is the result of the sharp increase of the electric field towards the focus, which decreases the angles x' and y' of the particles at the beam's edge. As is the case in the 3d simulation, the region of elevated density is clearly present after the focus. The effect is more pronounced in the *tralitrala* calculation due to the higher grid resolution in comparison to the *bender* simulation.

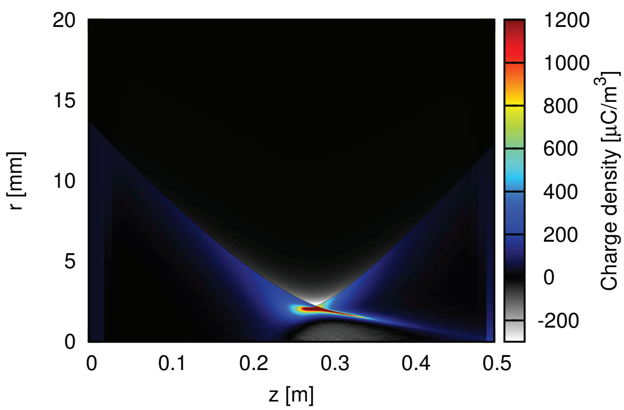

Figure 5: Total charge density when electrons with a thermal distribution are included in a two-dimensional simulation of the same system shown in Fig. 1.

Due to the hollow beam in the focus, the electrons redistribute and their density on the axis becomes larger than that of the beam. This leads to the area of negative charge density, visible between $25\,\mathrm{cm} < z < 35\,\mathrm{cm}$ in Fig. 5. This produces a negative electric field on the axis. When the residual gas ions are included, such as in the *bender* simulation, these become trapped. This is observed in Fig. 1.

VARIATION OF PARAMETERS

Simulations for different gas pressures in the range from 1×10^{-6} hPa to 2×10^{-5} hPa were performed. Over this range, the compensation degree η_{beam} in the simulations was found to vary between 68 % and 86 %. This decrease towards lower gas pressures is a result from both a decrease in the number of captured electrons (i.e. lower η_{part}) as well as an increase of the electron temperature, from 20 eV to 42 eV at the center of the system over the simulated range. A similar dependence between the electron temperature and the compensation degree was also found in other situations.

No influence on the compensation from the beam pipe diameter was found.

The repeller voltage in the previous simulations was set to −1500 V, producing more than twice the beam potential on axis. Reducing this value initially results in a small expansion of the electron column and a small increase in compensation. However, once the potential on axis approaches zero (at about −500 V on the repeller), losses through the apertures increase exponentially until reaching the production rate inside the volume. The losses are increased further by a movement of the focus, as a smaller beam radius within the repeller additionally weakens the repeller. Even for large positive voltages – values up to 1 kV were used – electrons still remain trapped within the potential of the beam focus. This results in a total compensation of about 10 %.

CONCLUSION

Particle-in-Cell simulations of a proton beam compensated by electrons were performed using *bender*. In the calculations, these electrons were found to follow a Boltzmann distribution. From this, it can be shown, that the influence on the beam predicted by the PIC simulation can be reproduced qualitatively by including the solution of the Poisson-Boltzmann equation into a transport code.

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