

# APPLICATION OF STATISTICAL METHODS FOR MEASUREMENTS OF THE COHERENCE PROPERTIES OF THE RADIATION FROM SASE FEL

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## Abstract

Radiation from the SASE FEL operating in the linear regime holds properties of completely chaotic polarized light. Measurements of the SASE FEL gain curve allows to determine saturation length which is strictly connected with coherence time. Statistical analysis of the fluctuations of the radiation energies measured with different spatial apertures allows one to determine the number of the longitudinal and transverse modes. Thus, with these simple measurements it becomes possible to determine the degree of transverse coherence, coherence time, and photon pulse duration. In this report we present theoretical background and experimental results obtained at free electron laser FLASH.

## INTRODUCTION

Amplification process in SASE FEL starts from the shot noise in the electron beam. Initially poor, coherence properties of the radiation are significantly improved in the exponential stage of amplification, and reach their best values at the onset of the saturation regime [1, 2]. Radiation of SASE FEL consists of wavepackets (spikes) having duration of about coherence time. Fields  $\tilde{E}$  are well correlated within one spike, and are statistically independent for different spikes. Coherence properties of the radiation are described with temporal and spatial correlation functions ( $g_1$  and  $\gamma_1$ ), coherence time and degree of transverse coherence ( $\tau_c$  and  $\zeta$ ):

$$\begin{aligned} \gamma_1(\vec{r}_\perp, \vec{r}'_\perp, z, t) &= \frac{\langle \tilde{E}(\vec{r}_\perp, z, t) \tilde{E}^*(\vec{r}'_\perp, z, t) \rangle}{\left[ \langle |\tilde{E}(\vec{r}_\perp, z, t)|^2 \rangle \langle |\tilde{E}(\vec{r}'_\perp, z, t)|^2 \rangle \right]^{1/2}}, \\ g_1(\vec{r}, t - t') &= \frac{\langle \tilde{E}(\vec{r}, t) \tilde{E}^*(\vec{r}, t') \rangle}{\left[ \langle |\tilde{E}(\vec{r}, t)|^2 \rangle \langle |\tilde{E}(\vec{r}, t')|^2 \rangle \right]^{1/2}}, \\ \tau_c &= \int_{-\infty}^{\infty} |g_1(\tau)|^2 d\tau, \\ \zeta &= \frac{\int |\gamma_1(\vec{r}_\perp, \vec{r}'_\perp)|^2 I(\vec{r}_\perp) I(\vec{r}'_\perp) d\vec{r}_\perp d\vec{r}'_\perp}{\left[ \int I(\vec{r}_\perp) d\vec{r}_\perp \right]^2}, \end{aligned} \quad (1)$$

where  $I(\vec{r}_\perp) = \langle |\tilde{E}(\vec{r}_\perp)|^2 \rangle$ . Radiation from SASE FEL operating in the linear regime holds properties of completely chaotic polarized light [3, 4], and the probability distribution of the radiation energy is gamma-distribution:

$$p(E) = \frac{M^M}{\Gamma(M)} \left( \frac{E}{\langle E \rangle} \right)^{M-1} \frac{1}{\langle E \rangle} \exp\left(-M \frac{E}{\langle E \rangle}\right), \quad (2)$$

where  $\Gamma(M)$  is the gamma function,  $M = 1/\sigma_E^2$ , and  $\sigma_E^2 = \langle (E - \langle E \rangle)^2 \rangle / \langle E \rangle^2$ . The parameter  $M$  has physical meaning of the number of modes in the radiation pulse.

## COHERENCE TIME PULSE DURATION

We consider the electron bunch with gaussian longitudinal profile of rms pulse duration  $\sigma_z$ . Figure 1 shows evolution along the undulator of the radiation pulse energy, fluctuations, and rms photon pulse length. Normalized values of these parameters exhibit nearly universal dependencies for  $\rho\omega\sigma_z \gtrsim 1$ . Maximum of fluctuations and minimum of the pulse duration are obtained in the end of the exponential gain regime. Saturation point (corresponding to maximum brilliance of the radiation [1]) is defined by the condition for fluctuations to fall down by a factor of 3 with respect to the maximum value. In the framework of 1D model maximum value of the coherence time and saturation length

$$(\tau_c)_{\max} \approx \frac{1}{\rho\omega} \sqrt{\frac{\pi \ln N_c}{18}}, \quad L_{\text{sat}} \approx \frac{\lambda_w}{4\pi\rho} \left( 3 + \frac{\ln N_c}{\sqrt{3}} \right),$$

are expressed in terms of the FEL parameter  $\rho$  [5] and the number of cooperating electrons  $N_c = I/(e\rho\omega)$  [3, 4, 6]. Here  $\omega = 2\pi c/\lambda$  is frequency of the amplified wave,  $I$  is the beam current,  $-e$  is the charge of electron, and  $\lambda_w$  is undulator period. Practical estimate for parameter  $\rho$  comes from the observation that in the parameter range of SASE FELs operating in the VUV and x-ray wavelength range, the number of field gain lengths to saturation is about 10 [3]. Thus, the parameter  $\rho$  and coherence time  $\tau_c$  relate to the saturation length as:

$$\rho \approx \lambda_w/L_{\text{sat}}, \quad \tau_c \approx \lambda L_{\text{sat}}/(2\sqrt{\pi}c\lambda_w). \quad (3)$$

For the number of modes  $M \gtrsim 2m$  the rms electron pulse length and minimum FWHM radiation pulse length  $\tau_{\text{ph}}^{\min}$  in the end of the linear regime are given by [7]:

$$\tau_{\text{ph}}^{\min} \approx \sigma_z \approx \frac{M\lambda}{5\rho} \approx \frac{M\lambda L_{\text{sat}}}{5c\lambda_w}. \quad (4)$$

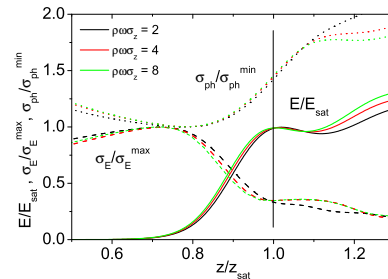


Figure 1: Energy in the radiation pulse  $E$  (solid line), fluctuations of the radiation energy  $\sigma_E$  (dashed line), and rms radiation pulse duration  $\rho\omega\sigma_{\text{ph}}$  (dotted line). Black, red and green lines correspond to the electron rms pulse duration  $\rho\omega\sigma_z$  of 2, 4, and 8. The values are normalized as  $E/E_{\text{sat}}$ ,  $\sigma_E/\sigma_E^{\max}$ , and  $\sigma_{\text{ph}}/\sigma_{\text{ph}}^{\min}$ . Simulations are performed with code FAST [8].

Minimum radiation pulse duration expressed in terms of coherence time (3) is  $\tau_{\text{ph}}^{\text{min}} \approx 0.7 \times M \times \tau_c$ .

Lengthening of the radiation pulse occurs when amplification process enters saturation regime. This happens due to two effects. The first effect is lasing to saturation of the tails of the electron bunch, and the second effect is pulse lengthening due to slippage effects (one radiation wavelength per one undulator period). The effect of lasing tails gives the same relative radiation pulse lengthening as it is illustrated with bottom plot in Fig. 1. At the saturation point pulse lengthening is about factor of 1.4 with respect to the minimum pulse for linear regime given by eq. (3), and it is increased up to a factor of 2 in the deep nonlinear regime. Slippage effect is more pronouncing for relative lengthening of short pulses.

### Experimental Hints

Statistical measurements of the coherence time and of the radiation pulse duration are used at FLASH since start of its operation [7, 9, 10]. There was also trial experiment at LCLS [11]. Experimental technique is as follows. Gain curve of the SASE FEL is measured at the first step (average radiation energy and fluctuations versus undulator length). Saturation length is derived as we described above. This quantity gives us an estimate for the FEL parameter  $\rho$  and coherence time  $\tau_c$  (3). Then FEL process is stopped in the end of the high gain linear regime (FEL power is by a factor of 20 below saturation level, see Fig. 1). Fluctuations are measured with a pinhole aperture selecting central part of the photon pulse. Essential electron beam and machine parameters (charge, orbit, compression signal, rf parameters) for each shot are recorded as well. Final step of the experimental procedure is gating of the experimental results with machine parameters. Final data set contains mainly fundamental fluctuations of the radiation pulse energy related to SASE FEL process. Inverse squared value of fluctuations gives the number of longitudinal modes  $M_L$ . Radiation pulse length is derived from eq. (4).

## DEGREE OF TRANSVERSE COHERENCE

Total number of the modes in the radiation pulse is the product of the number of longitudinal and transverse modes. This is an origin of an idea to use measurements of fluctuations for derivation of the degree of transverse coherence. Measurements of the fluctuations of the total pulse energy and of the radiation energy after a pinhole gives us the total number of modes, and number of longitudinal modes, respectively. Their ratio gives the number for the degree of transverse coherence. Numerical simulations with code FAST [8] confirm this physical considerations. We see from Fig. 2 that in the exponential gain regime, the squared ratio of the fluctuations exactly follows the degree of transverse coherence calculated with rigorous statistical definition (1). We should stress that simple statistical measurements give fundamental quantity without making any additional assumptions. This happens due to the fundamen-

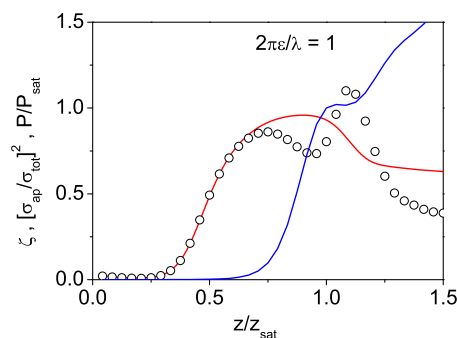


Figure 2: Evolution of the degree of transverse coherence  $\zeta$  (red line) and FEL power  $P$  (blue line). Circles show the ratio of fluctuations of the radiation energy in a pinhole to the fluctuations of the total energy,  $\sigma_{E,ap}^2/\sigma_{E,tot}^2$ . Simulations are performed with code FAST [8].

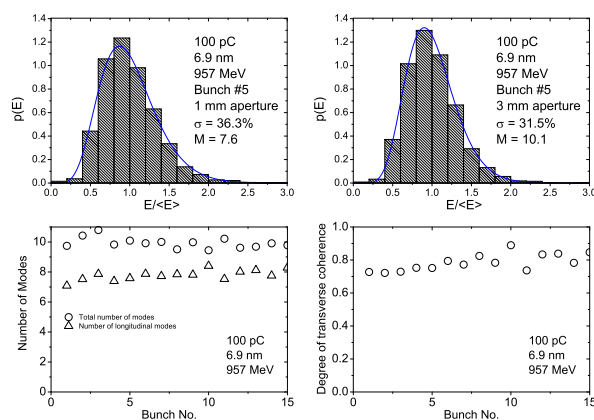


Figure 3: Measurement of mode content and of the degree of transverse coherence at FLASH. Top: probability distributions of the radiation energy, full (right) and in the pinhole (left). Bottom left: number of modes (circles - total, triangles - longitudinal). Bottom right: degree of transverse coherence.

tal nature (gaussian statistics) of the light produced by the SASE FEL in the exponential gain regime. Pinhole techniques allows to trace the evolution of the degree of transverse coherence up to onset of the saturation regime.

### Experiment at FLASH

We performed measurements of the degree of transverse coherence at FLASH1 in the framework of the program aiming development of techniques for characterization of spatial coherence of the SASE FELs. MCP detector has been used for radiation energy measurements [12]. The electronics of MCP-detector has low noise, about 1 mV at the level of signal of 100 mV (1% relative measurement accuracy). Measurements are performed in the same way as it was described in the previous section with one more statistical run with full pulse energy to define total number of modes in the radiation pulse. Then the degree of transverse coherence is given by  $\zeta = M_L/M_{tot}$ . Energy of electrons is 957 MeV, Bunch charge is 100 pC, number of bunches in the train is

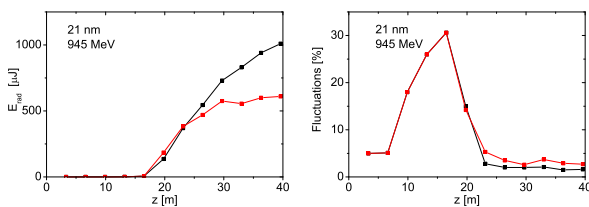


Figure 4: Pulse energy (left plot) and fluctuations of the radiation pulse energy (right plot) versus undulator length measured at FLASH2. Electron energy is 945 MeV, radiation wavelength is 21 nm, bunch charge is 400 pC. Color codes are: red for untapered case and black for optimum undulator tapering.

15, and radiation wavelength is 6.9 nm. Measurements have been performed after 5 undulator modules corresponding to the end of the exponential regime. Top plots in Fig. 3 show probability distribution of the radiation energy for bunch number 5, full and in the pinhole of 1 mm. Then, following (2), we derive number of transverse and longitudinal modes (left lower plot). Finally, ratio of the number of modes gives us the degree of transverse coherence for every individual bunch (right lower plot). Obtained values are in reasonable agreement for the values expected at FLASH in these parameter range [13].

### OPTIMUM UNDULATOR TAPERING

Tapering of the undulator gap leads to an increase in radiation power in the post-saturation regime [14, 15]. Experimental procedure for tuning of the tapering parameters involves statistical measurements of the radiation energy. Optimum conditions of the undulator tapering assume the starting point to be by two field gain lengths before the saturation point corresponding to the maximum brilliance of the SASE FEL radiation [1]. Saturation point on the gain curve is defined by the condition for fluctuations to fall down by a factor of 3 with respect to their maximum value in the end of exponential regime. Then quadratic law of tapering is applied (optimal for moderate increase of the extraction efficiency at the initial stage of tapering [15]). This experimental techniques has been successfully tested at FLASH2 as illustrated in Fig. 4. Saturation occurs at the undulator length of 20 meters, and saturation energy is about 150  $\mu\text{J}$ . Optimized tapering increases the pulse energy by a factor of 6, up to 1000  $\mu\text{J}$ . Untapered undulator delivers only 610  $\mu\text{J}$  at full undulator length of 40 meters. Thus, tapering of the FLASH2 undulator demonstrates great benefit in the increase of the radiation pulse energy.

### SUMMARY

Statistical measurements of the radiation pulse energy is extremely powerful technique for characterization of the main SASE FEL parameters: FEL parameter  $\rho$ , coherence

time, photon pulse duration, and the degree of transverse coherence. It is based on fundamental principles, and measured values have strict physical meaning (1). By now FLASH is the only facility where statistical measurements are routinely used for SASE FEL characterization. Statistical measurements are conceptually simple, but rely on two important technical requirements. The first requirement is availability of fast and precise radiation detector capable to measure radiation energy of every pulse with high relative accuracy in a wide range of radiation intensities. At FLASH we use MCP detector with relative accuracy of measurements better than 1%. The second requirement is small jitter of the machine parameters, much less than the fundamental SASE FEL fluctuations. Good phase stability of the superconducting accelerator FLASH helps a lot. In addition, success of the technique depends on the quality of diagnostics allowing to detect jitters of the electron beam and machine parameters.

### ACKNOWLEDGEMENT

We thank the FLASH team for collaboration and support of the experiment.

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