A NOVEL APPROACH IN ONE-DIMENSIONAL PHASE RETRIVAL PROBLEM AND ITS APPLICATION TO THE TIME PROFILE RECONSTRUCTION

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Abstract

Phase retrieval problem occurs in a number of areas in physics and is the subject of continuing investigation [1-15]. One dimensional case, for example, an electron bunch temporal profile reconstruction, is particularly challenging. Frequently applied methods, are reliable if the Blaschke phase [10-12] contribution is negligible. This, however, is neither known a priori nor can it be assumed for an arbitrary profile. In this work we present a novel algorithm with additional constraints which gives reproducible, stable solutions for profiles, both artificial and experimental, otherwise unresolved by existing techniques.

INTRODUCTION

Accurate knowledge of the longitudinal (time) profile of an electron bunch is important in the context of linear colliders and X-ray FELs, but it is a parameter that becomes progressively more difficult to determine for fs-long bunches [1-7]. These, however, are the bunch lengths expected from the next generation of high-gradient particle accelerators which will be based on laser-plasma or wake-field acceleration. Apart from the desirability of determining the profile in a non-destructive manner, and because of the low repetition rate of these accelerators, it is equally desirable to be able to achieve this in a single shot. In this paper we will discuss the technique to recover phase information from power spectrum which one can measure using different spectroscopic technique.

The problem of retrieving the phase of a signal from a measurement of its power spectrum alone is well known and has been under investigation for a few decades. Some information about the missing phase can be retrieved from the so-called 'minimal' phase calculated by the Kramers-Kronig (KK) method under the assumption that the signal is a holomorphic function. There are number of iterative techniques [8, 9, 13-15] which also allowing recovering missing phase and reconstructing the bunch profile. In this work we discuss the method, which combine KK and iterative techniques. The KK feeds iterative method information about initial (minimal) phase and it is also used as a boundary condition limiting possibilities of generating profile with unphysical phase. The conditions which indicates that algorithm is converging to a solution and the second that this solution is likely (i.e. most probable) to be the correct one will be also discussed.

ALGORITHM DESCRIPTION

The new algorithm is based on a combination of the minimal phase θ_m and an iterative procedure (with repeated iterations between the frequency and time domains). The Kramers-Kronig (KK) method is used to calculate the minimal phase. We assume for clarity reason that the power spectrum is known (measured) on the whole frequency domain. It is important to note that the Blaschke contribution (if it exists) is a monotonically increasing function of frequency and that the minimal phase θ_m provides a lower limit for the possible phase values. The new phase-constrained iterative (PCI) algorithm is named to indicate this new phase boundary condition. In addition to the minimal phase, the integral value F (bunch charge) is also used as a limiting condition and to monitor the convergence (as discussed below) to the solution. The profile recovered from the minimal phase also provides an initial estimate of the support function (see below). These points listed above are essential for the new algorithm operation. We note that displacements along the time axis, mirror imaging of the temporal profile or equivalently, the sign of the spectral phase cannot be determined unambiguously. We will not be addressing those issues as it is only for few cases these variations have physical significance. The features of the PCI algorithm are summarised by (2). The algorithm iterative part is similar to the Gerchberg-Saxton (GS) and Hybrid Input-Output (HIO) group of algorithms and can be summarized as follows:

$$f_{n+1}(t) = \begin{cases} f'_n(t) \ \ if \ \ f'_n(t) \in \Upsilon \\ f_n(t) - \beta f'_n(t) \ if \ \ f'_n(t) \notin \Upsilon \\ \theta_n = \theta' \ if \ \theta' > \theta_m \\ \theta_n = \theta_m \ if \ \ \theta' \le \theta_m \end{cases} \quad (1)$$

where Υ is the set of constrains of function *f(t)*. In this letter the abbreviation FT and FT^{-1} are used to denote the forward and the inverse Fourier Transforms respectively. The above expressions define the result of the *n+1* iterative step where f_n and θ_n denote the modulus and phase respectively of the time profile function calculated by FT from the frequency to time domain, while θ_m is the minimal phase.

1. First, the measured amplitude spectrum and θ_m are used for the initial FT^{-1} e.g. from the frequency into the time domain to derive the zeroth order approximation $f'_0(t)$ to the unknown profile *f(t)*. Any values of that function inside the defined window (τ) which do not

satisfy the constraints $f'_n(t) \notin \Upsilon$ (e.g. $f'_n(t) < 0$ for electron bunch profiling) are corrected.

2. The FT of $f_0(t)$ back into the frequency domain gives $g(\omega) = \rho(\omega)e^{i\theta(\omega)}$, $\rho(\omega) = |g(\omega)|$

3. If $\theta < \theta_m$ then θ is set equal to θ_m. The values of the modulus are discarded.

4. The original values of the moduli and the new phases are used for a FT^{-1} back into the time domain leading to $f'_1(t)$ and after satisfying the constraint conditions for the function *f*, a new approximation $f_1(t)$ of the unknown *f(t)* is obtained.

5. Above steps are repeated and at each iteration, the values of the quantities are checked

$$\frac{1}{F}\left(\int_0^\tau |f_{n+1}(t) - f_n(t)|dt\right) \le \varepsilon_1 \quad (2)$$

$$\frac{1}{F}\left(\int_0^\tau f_{n+1}(t)dt - F\right) = \delta_{n+1} \le \varepsilon_2 \quad (3)$$

where *F* is the independently known value of the integral of the target function *f(t)*; (*F*=1 for a charge-normalised distribution), $\varepsilon_{1,2}$ are the predefined acceptable errors appropriate for a specific experiment and $\varepsilon_{1,2} << 1$. The expression (2) is the derivative of (3) (i.e. $\delta_{n+1} - \delta_n < \varepsilon_1$) and the expressions represent the relative (2) and absolute (3) integral errors. The (2) is an indication that the algorithm is converging to a solution and the second that this solution is likely (i.e. most probable) to be the correct one.

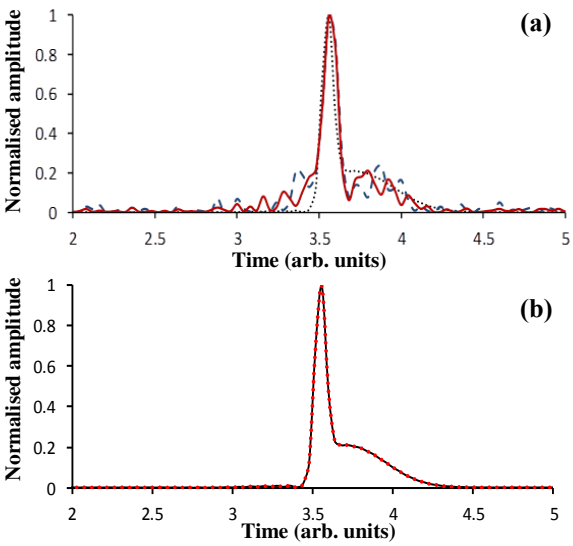

Figure 1: Reconstructions of "synthetic" function (superposition of two Gaussian functions (dotted line) of varying widths (σ_i), amplitudes (A_i): $A_1 = 1$; $A_2 = 0.2$; $\sigma_1 = 0.0085$; $\sigma_2 = 0.425$; $t_1 = 0.043$; $t_2 = 0.543$) based on: (a) non-PCI (dashed line) and PCI (solid line) algorithms; (b) minimal phase retraction (solid line) algorithms.

PROFILE RECONSTRUCTION USING PCI ALGORITHM

To compare new algorithm with other iterative as well as Kramers–Kronig techniques, we have reconstructed a number of 'synthetic' profiles generated by superposition of two Gaussian functions of varying widths (σ_i) and amplitudes (A_i) $f_g = \sum_{i=1}^{i=2} A_i e^{-(t-t_i)^2/2\sigma_i^2}$ (Fig 1-4) and the Lorentz function $f_L = \frac{1}{\pi}\frac{\varepsilon}{(t-t_0)^2+\varepsilon^2}$ (Fig. 5); the Gaussians are centred at t_i and the Lorentzian at t_0 (see Fig. 1-5). To compare different algorithms with PCI we used conditions (2) and (3).

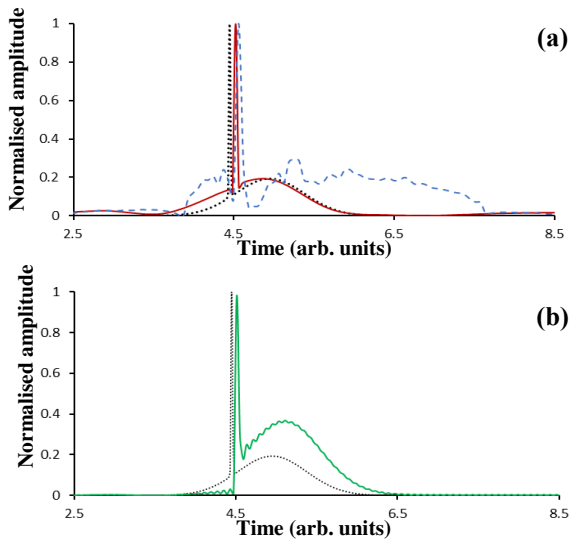

Figure 2: Reconstructions of "synthetic" function (superposition of two Gaussian functions of varying widths (σ_i) and amplitudes (A_i): $A_1 = 1$; $A_2 = 0.6$; $\sigma_1 = 0.13$; $\sigma_2 = 0.21$; $t_1 = 0.64$; $t_2 = 1.64$) based on: (a) non-PCI (dashed line) and PCI (solid line) algorithms; (b) minimal phase retraction (solid line) algorithms.

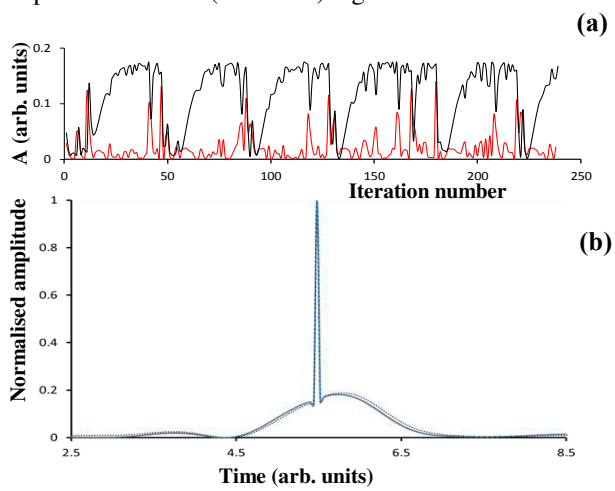

Figure 3: (a) The values of conditions (2) and (3) as a function of iteration number for the PCI algorithm. (b) The time profiles recovered at the end of the process when both conditions are satisfied.

In all cases shown the original function is indicated by the dotted lines. We note that the conventional iterative algorithm generates different solutions for each run and it requires "expert knowledge" to select an appropriate solution. In Fig. 1b the profile reconstructions using minimal is shown and compared with the original function. The minimal phase approach generated a perfect result in this case but it is unknown a priori if the minimal

phase is sufficient. We note that in our studies the PCI algorithm gave at least as good, and usually better results than the non-PCI algorithms. The figures(1a-5a) show the profiles reconstructed by means of PCI and non-PCI iterative algorithm [15], as well as the original function. Figures (1b-5b) show the profile reconstruction based solely on minimal phase together with the original profile. The case of Fig. 2 demonstrates the strong deviation of the profile reconstructed using non-PCI algorithm from the original, while the PCI procedure delivers an acceptable approximation to the given function.

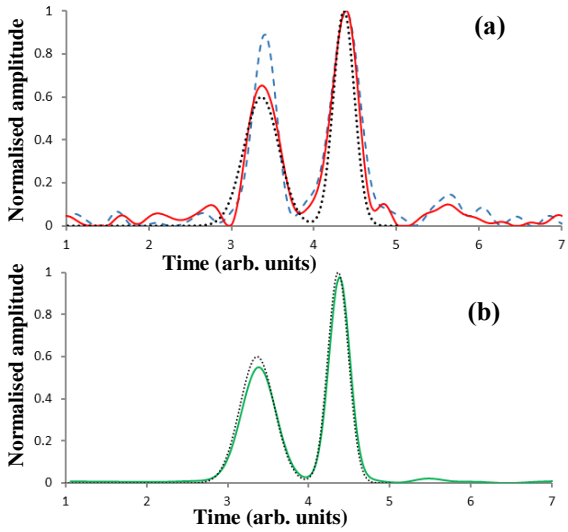

Figure 4: As in Fig.2 but with $A_1 = 1$; $A_2 = 0.6$; $\sigma_1 = 0.13$; $\sigma_2 = 0.21$; $t_1 = 0.64$; $t_2 = 1.64$). The reconstructions are based on: (a) a non-PCI (dashed line) and the PCI (solid line) algorithms; (b) minimal phase reconstruction (solid line). The dotted line is the original function.

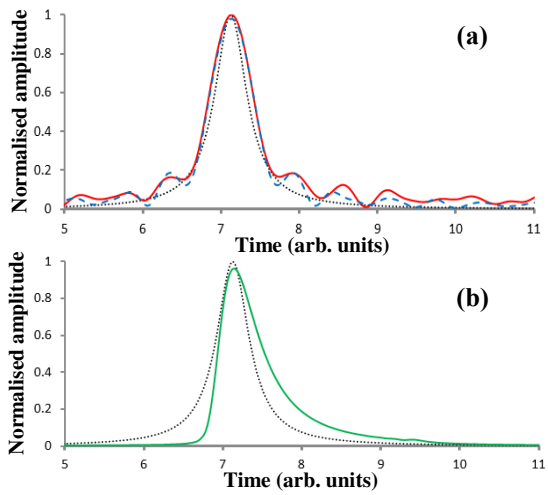

Figure 5: Reconstruction of a Lorentz-type function (α = 0.3; t_0=0) based on: (a) a non-PCI (dashed line) and the PCI (solid line) algorithms; (b) minimal phase reconstruction (solid line). The dotted line is the original function.

CONCLUSION

We have developed a new phase retrieval algorithm based on repeated iterations between the time and frequency domains, with constraints applied in each domain. The two novel features are: (a) the use of the minimal phase as a lower limit for the missing phase and (b) the use of conditions (2) and (3) as criteria for the convergence of the iterations to the most probable and correct solution. In all the cases that we have studied, the new algorithm (PCI) gave at least as good and usually better results than the non-PCI algorithm. In certain cases the minimal phase method would have been adequate for an accurate reconstruction but in both cases the PCI algorithm provided answers that were close to those derived from the Kramers-Kronig method. Since the existence or otherwise, of a significant Blaschke phase contribution cannot be deduced experimentally or known a priori, the PCI algorithm provides an acceptable and reliable reconstruction which does not depend on initial assumptions.

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