

# IDENTIFICATION OF INTRA-BUNCH TRANSVERSE DYNAMICS FOR MODEL-BASED CONTROL PURPOSES AT CERN SUPER PROTON SYNCHROTRON \*

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## Abstract

The high luminosity upgrade plan for the LHC (HiLumi-LHC) increases the bunch intensity and the ultimate intensities require mitigation of possible intra-bunch instabilities in the SPS. Feedback systems can stabilize intra-bunch dynamics. Model based control has promise to stabilize intra-bunch dynamics but it requires a reduced order model which captures the most significant intra-bunch dynamics. We present methods for the estimation of a multi-input multi-output (MIMO) reduced order model of intra-bunch dynamics and demonstrate them using data from nonlinear macro particle simulations (CMAD, HeadTail). These linear models are used to design optimal model-based controllers. We evaluate the effectiveness of the MIMO model-based controllers for future high intensity beam conditions within the nonlinear macro particle simulations. We highlight the use of these techniques to stabilize intra-bunch motion and as an important beam dynamics measurement technique.

## INTRODUCTION

Electron clouds and machine impedances can cause intra-bunch instabilities at the CERN Super Proton Synchrotron (SPS). The high current operation of the SPS for LHC injection requires mitigation of these problems [1], [2]. Design of a new lattice has increased the bunch current limits for strong head-tail instability [3], [4] while carbon coating of the critical chambers allows reduction in the electron-cloud density. Feedback techniques can be used to mitigate both types of instabilities.

Nanosecond-scale bunch stabilization is challenging since it requires sufficient bandwidth to sense transverse motion at multiple locations along the bunch and apply correction signals to the corresponding parts of the bunch. Additionally, modeling the intra-bunch dynamics is more challenging compared to modeling beam dynamics for bunch to bunch interactions.

Due to very fast intrinsic time characteristics of the system, a parallelized control filter architecture has been developed. A similar method has been used for bunch by bunch feedback control systems [5]. Using 3.2 (4) GS/Sec. feedback processing system vertical positions of multiple locations within the nanosecond-scale ( $4\sigma \approx 3.2$  ns) bunch are sampled [6]. The number of samples and the number of control

filters is 16 for the existing implementation. Vertical position (dipole) measurements are sampled across the bunch, the feedback correction signal for slice  $i$  is only computed based on measurements of slice  $i$  over the last  $n$  turns. FIR and IIR filters were implemented with intrinsic limits in the maximum damping that can be achieved and the stabilization of multiple internal modes [7]. Model-based multi-input multi-output (MIMO) controller designs have been evaluated to overcome those limitations, at the expense of a more complex implementation of the filters [8].

Modeling and identification of the intra-bunch dynamics using reduced order linear models are crucial for the model-based controller design. Simulation or measurement data provide time domain trajectories whereas reduced order models provide an analytic form to help us understand system dynamics and the impact of feedback in a more general way. This paper presents methodologies and techniques to represent the dominant dynamics using a discrete-time linear MIMO model, estimate parameters of the reduced model using time domain data from nonlinear macro particle simulation codes and evaluate the design of MIMO controllers. Results of the identification of the dominant intra-bunch dynamics are presented using macro particle simulation codes to generate the original data. Additionally, a model-based controller design is evaluated using the extracted reduced model in MATLAB simulators.

## REDUCED ORDER MODEL AND IDENTIFICATION

Any linear dynamical system can be represented in state space matrix form. A discrete-time system sampled at every revolution period  $k$  with  $p$  inputs and  $q$  outputs is represented by

$$\begin{aligned} X_{k+1} &= AX_k + BU_k \\ Y_k &= CX_k \end{aligned} \quad (1)$$

where  $U \in R^p$  is the control variable (external excitation),  $Y \in R^q$  is the vertical displacement measurement,  $A \in R^{n \times n}$  is the system matrix,  $B \in R^{n \times p}$  is the input matrix, and  $C \in R^{q \times n}$  is the output matrix. For a MIMO system, the model order  $n$  determines the complexity. The transfer function matrix ( $\in R^{q \times p}$ ) from control variable to vertical displacement measurement for a system with  $p$  inputs and  $q$  outputs in  $z$  domain is

$$Y(z) = \left[ D^{-1}(z)N(z) \right] U(z), \quad (2)$$

where  $D(z)$  and  $N(z)$  represent the denominator and numerator of discrete-time transfer function matrix, respectively.

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Based on time-domain data from simulations (or measurements), the input  $U(k)$  and output  $Y(k)$  signals are defined and the estimation of the transfer function coefficient matrices  $N_r$  and  $D_r$  is obtained by solving

$$\begin{bmatrix} N_r & | & -D_r \end{bmatrix} \begin{bmatrix} U(k) \\ Y(k) \end{bmatrix} = 0. \quad (3)$$

Assuming full observability of the system, we can represent our state space model [Eq. (1)] in discrete-time observable canonical form. This defines a direct relationship between the  $N_r$  and  $D_r$  coefficients and the parameters of matrices  $A, B, C$  [9]. Examples of SPS machine measurements based on reduced order MIMO model of intra-bunch dynamics are presented in [10] and [11].

In this study, we use HeadTail to generate time domain data for the identification of the reduced order model of the intra-bunch dynamics. All simulation studies are conducted with  $1.15 \times 10^{11}$  protons at the injection energy of 26 GeV. We study both Q20 and Q26 optic lattices. The betatron tune for Q20 is 0.185 and the synchrotron tune  $f_s$  is 0.017 whereas in Q26 optics the synchrotron tune is 0.0059. To estimate the reduced order model parameters, we excite the intra-bunch dynamics by driving the bunch with specifically tailored swept excitation signals through the modes to study. In this specific case the excitation signal is tailored to excite multiple intra-bunch modes covering the range  $f_\beta + 2f_s$  over 1000 turns.

Figure 1 and 2 show the vertical displacement data across the bunch for 1000 turns for the HeadTail simulation and a reduced order MIMO model. RMS error between the simulation data and the reduced order model response is  $\epsilon < 1.2 \times 10^{-4}$  indicating agreement in the time domain. The corresponding spectrograms are helpful to understand the excited mode numbers based on the spectral content. Given Q20 optics configuration and 0.185 betatron tune, the first upper sideband is at 0.202 and the second upper sideband is at 0.219 fractional tune. As seen in both spectrograms (Fig. 3 and 4), the excitation signal couples energy into modes 0, 1, 2 and the reduced order model captures the dominant dynamics.

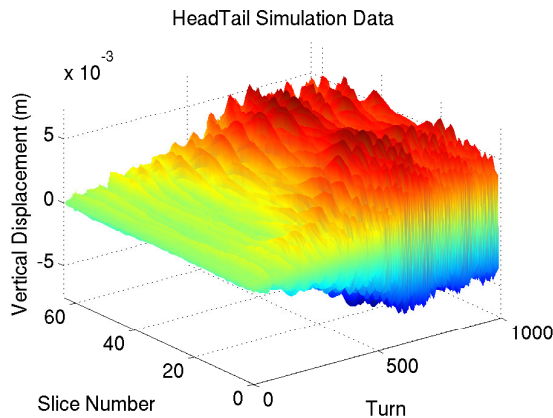


Figure 1: Time domain trajectory of an open loop driven HeadTail simulation.

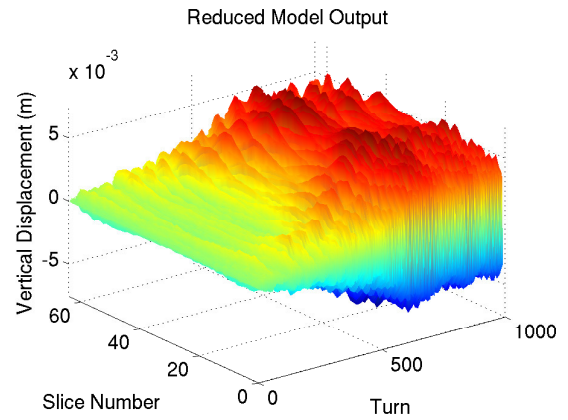


Figure 2: An 8<sup>th</sup> order reduced model with 64 inputs - outputs is able to capture dominant dynamics and replicate the time domain trajectory.

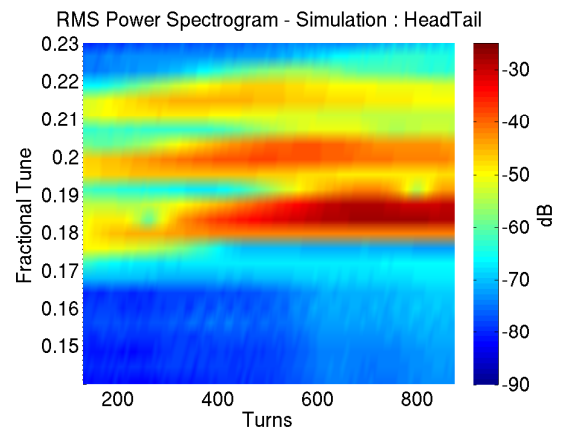


Figure 3: The spectrogram of the vertical displacement - HeadTail simulation. Modes 0, 1 and 2 are excited around turns  $\sim 700$ ,  $\sim 500$  and  $\sim 400$ , respectively.

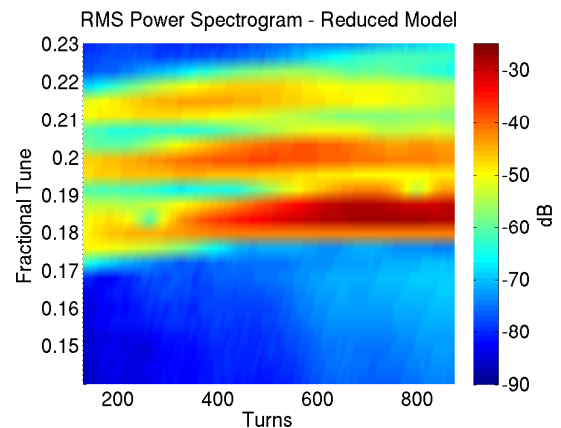


Figure 4: The spectrogram of the vertical displacement (reduced order MIMO model) showing capture of dominant dynamics with an 8<sup>th</sup> order reduced model.

## MODEL-BASED CLOSED LOOP STUDIES

Model-based control design to stabilize the multi-modal intra-bunch dynamics has been presented [8]. Based on the identified reduced order model, an observer is designed to

Table 1: Intra-Bunch Dynamics

Mode	OL (Turns)	CL (Turns)
0	$-0.000 \pm 0.1850i (\infty)$	$-0.0048 \pm 0.1850i (208)$
1	$-0.0011 \pm 0.2015i (909)$	$-0.0058 \pm 0.2012i (169)$
2	$-0.0016 \pm 0.2181i (625)$	$-0.0079 \pm 0.2181i (126)$

gether with state feedback to stabilize intra-bunch motion. Discrete-time linear quadratic optimal control method is used to introduce the desired damping to the system. The main objective is to introduce the minimum necessary damping to each mode for the MIMO system using minimum actuation effort. For this purpose the "pincher" approach is used to design the state feedback and the observer gains [12]. Figure 5 shows the MIMO controller also includes the required 1-turn latency and orbit offset (DC rejection).

Figure 6 shows the open loop response of the reduced order model to an excitation signal tailored to excite modes 0, 1, and 2 (betatron oscillations, 1<sup>st</sup> and 2<sup>nd</sup> upper sidebands). During the first 1000 turns, the response is driven whereas for the remaining 1000 turns, it is the natural open loop response. As it can be seen from the time domain trajectory, oscillations of naturally undamped mode 0 and very lightly damped modes 1 and 2 continue.

Figure 7 shows the closed loop response of the system simulated in MATLAB Simulink while the external driving signal tries to excite modes 0, 1 and 2 for the first 1000 turns. The reduced order model-based MIMO controller significantly increases the damping on each mode and the motion decays to the noise floor within ~200 turns after excitation stops. The eigenvalues of the closed loop dynamics are specified in Table 1.

Figure 5: LQG is designed based on the open loop reduced order MIMO model including the effect of 1 turn delay and DC orbit offset rejection.

## CONCLUSION AND FUTURE WORK

Model-based control design techniques for intra-bunch instabilities require a reduced model of the bunch dynamics. We developed reduced order models and show initial results of the identification of those models. We identified parameters of a reduced order model that captures modes 0, 1 and 2 dynamics from HeadTail macro particle simulation. The natural tunes and damping values seen in simulation data are estimated correctly using a linear model. Dominant dynamics is captured with a reduced order model and simulation data is regenerated successfully in the time domain. The reduced order model-based MIMO controller success-

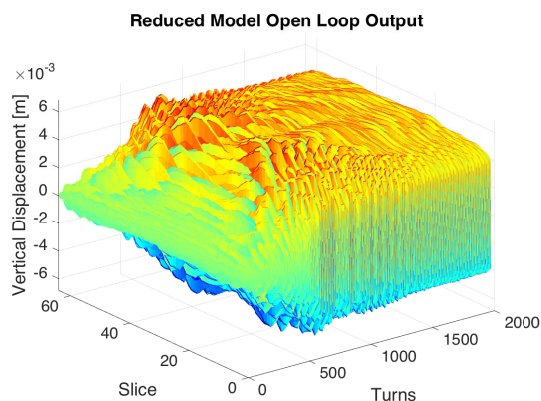


Figure 6: Open loop driven response time domain trajectory - Reduce model response to the excitation signal which is tailored to drive modes 0, 1, and 2.

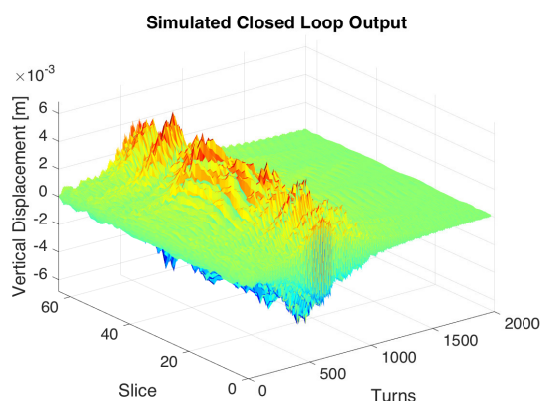


Figure 7: MATLAB simulated closed loop driven response time domain trajectory of reduced order MIMO model.

fully increases the damping in dominant dynamics. These techniques are applicable to design controllers from SPS machine measurements as well. Future work is aimed at estimating more internal modes from physical SPS measurements and comparing these to nonlinear simulation results. The new model-based control architecture appears to have good potential compared to the existing parallelized control filter architecture in terms of stabilization and performance necessary for challenging machine optics operating at high intensities with many unstable modes. We are investigating the implementation of these MIMO controllers in the existing wideband feedback demonstration system installed at the SPS.

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