# ORBIT RESPONSE MATRIX ANALYSIS FOR FAIR STORAGE RINGS

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*Abstract*

The Orbit Response Matrix (ORM) analysis [1] is a method which allows to find the sources of discrepancies between design and real optics of an accelerator machine. In particular, with this technique one retrieves information about gradient errors, dipole corrector gain errors etc.

Orbit response matrix is computed by measuring orbit deviations caused by single kicks of corrector magnets. With fitting the matrix one obtains the ion optics which best describes the real accelerator. The ORM analysis, presented in the paper, is employed to find error sources in the FAIR storage rings CR [2] and HESR [3] during and after the beam commissioning. The algorithm itself was implemented in Python programming language with a help of linear algebra libraries. The ORM analysis accuracy as well as its limitations are addressed in the paper.

## INTRODUCTION

### *FAIR Storage Rings*

The FAIR circular machines, which are to be built in the first phase, consist of heavy ion synchrotron ring SIS100, Collector Ring (CR) and High Energy Storage Ring (HESR). During the beam commissioning of each of these rings it will be extremely important to track down the errors like:

- normal quadrupole gradients errors;
- skew quadrupole gradients errors;
- polarity and gain errors in beam position monitors (BPMs) and dipole correctors;
- sextupole gradient errors.

These types of errors can be effectively eliminated by an iterative method called Orbit Response Matrix (ORM) analysis.

## ORM ANALYSIS

### *Theory*

The Orbit Response Matrix, or ORM, is a matrix which defines how the beam responds to the dipole correctors. In other words, it associates the beam offsets at the BPMs with the correctors kicks:

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = R \begin{pmatrix} \Delta\theta_x \\ \Delta\theta_y \end{pmatrix} \tag{1}$$

Here $\boldsymbol{R}$ denotes the ORM, $\Delta x$ and $\Delta y$ are the beam offsets in a horizontal and in a vertical plane respectively, $\theta_x$ and $\theta_y$ are the horizontal and the vertical corrector kicks.

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One should distinguish between designed (model) matrix $\boldsymbol{R}^{(mod)}$ and real (measured) matrix $\boldsymbol{R}^{(meas)}$. In the simplest case without coupling the elements of the $\boldsymbol{R}^{(mod)}$ can be obtained from the optical parameters using the following formula:

$$R_{ij}^{(mod)} = \frac{\sqrt{\beta_i \beta_j}}{2\sin\pi Q}\cos(2\pi|\phi_i - \phi_j| - \pi Q) - \frac{D_i D_j}{(\alpha_c - \frac{1}{\gamma^2})C} \tag{2}$$

where $i$, $j$ denote a certain corrector or, respectively, a BPM at whose positions we measure the optical parameters. $\beta$ is beta function, $\phi$ is phase advance, $Q$ is tune, $D$ is dispersion function, $\alpha_c$ is momentum compaction factor, $\gamma$ is Lorentz factor and $C$ is the circumference of the ring.

Now we can describe how the ORM analysis algorithm works. Let us define the ORM $\boldsymbol{R}$ as a function of machine parameters vector $\boldsymbol{V}$, i.e. $\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{V})$. The vector $\boldsymbol{V}$ contains errors which ultimately are supposed to be found by the algorithm. Expanding $\boldsymbol{R}$ into Taylor series and truncating it after the second term we obtain:

$$\boldsymbol{R}(\boldsymbol{V}) \approx \boldsymbol{R}(\boldsymbol{V_0}) + \boldsymbol{R'}(\boldsymbol{V_0})(\boldsymbol{V} - \boldsymbol{V_0}) \tag{3}$$

Here $\boldsymbol{R}(\boldsymbol{V})$ is the measured matrix $\boldsymbol{R}^{(meas)}$ and $\boldsymbol{R}(\boldsymbol{V_0})$ is our guess for the matrix. The first guess is normally the model matrix $\boldsymbol{R}^{(mod)}$. The $\boldsymbol{R'}(\boldsymbol{V_0})$ represents a linear map (Jacobian matrix) $\boldsymbol{J}$ from the machine parameters to the ORM.

After multiplying both parts of Equation 3 from the left side by $\boldsymbol{J}^{-1}$ and slightly rearranging the expression we obtain:

$$\boldsymbol{V} = \boldsymbol{V_0} + \boldsymbol{J}^{-1}(\boldsymbol{R}(\boldsymbol{V}) - \boldsymbol{R}(\boldsymbol{V_0})) \tag{4}$$

We find $\boldsymbol{V}$ by fitting the model to the measurement. This is done by varying the machine parameters under investigation and minimizing the difference $\Delta\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{V}) - \boldsymbol{R}(\boldsymbol{V_0}) = \boldsymbol{R}^{(meas)} \check{} \boldsymbol{R}^{(mod)}$ within an iterative process. The steps of this process are as follows:

1. Define the model machine parameters as a first guess $\boldsymbol{V_0}$ for the real machine parameters;
2. Compute $\boldsymbol{R}^{(mod)}$ with the help of an accelerator code;
3. Compute $\boldsymbol{R}^{(meas)}$ by varying the correctors and measuring the beam offsets. This is done by accessing the real machine;
4. Find the matrix difference $\Delta\boldsymbol{R}$;
5. Vary the machine parameters, e.g. quadrupole gradients, and find the changes in $\Delta\boldsymbol{R}$. This way a Jacobian matrix $\boldsymbol{J}$ is found;




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6. Compute a pseudoinverse of the Jacobian via singular value decomposition (SVD) method [4];
7. Calculate the next guess for the machine parameters vector $\boldsymbol{V}$;
8. Go to step 1.

We execute the above described steps until the algorithm converges. The $\chi^2$-function will be an indicator of the fit goodness. Note that during the third step, in order to measure the orbit response matrix $\boldsymbol{R}^{(meas)}$, we need to access the real machine only once.

### *Implementation*

The MAD-X [5] accelerator code was used for the ion optical calculations and was embedded into the program body. The main algorithm was designed with a help of Python programming language. Graphical user interface was constructed using PyQt (Python bindings for application framework Qt [6]). SVD operation was carried out by means of Numpy [7] linear algebra library.

### *Preconditions*

In this study the ORM analysis accuracy and effectiveness were tested for the storage rings CR and HESR. The quadrupole families in each of the ring were assumed to have gradient errors. The errors had uniform distribution and were bounded to the range of ±5 % from the initial values.

The ion optics, calculated for these distorted quadrupole strengths, was used as an input for the ORM analysis. As an example, Fig. 1 visualizes the discrepancy between a model and a measured response matrices generated by such errors. Each bar in the figure represents one matrix element which is proportional to the orbit offset discrepancy at $j$-th BPM from $i$-th corrector. Thus one can immediately recognize the corrector-BPM pair at which the disagreement between model and measurement is the largest one. Our task was to investigate how precise and how fast the ORM analysis eliminate the discrepancies and thus finds the simulated gradient errors.

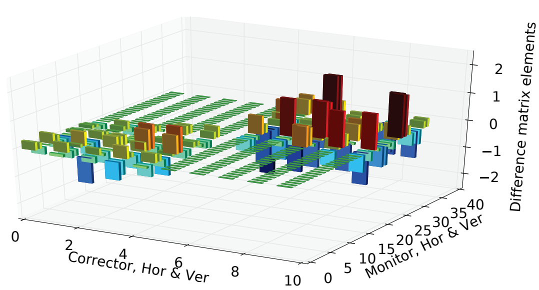


Figure 1: Difference between a model and a measured response matrices $\Delta\boldsymbol{R}$ in the CR for 10 correctors due to the uniform gradient errors in the range of ±5 %. Coupling is off.

## RESULTS

### *CR Collector Ring*

The first ring under investigation was CR Collector Ring which is 221 m long. It possesses 18 BPMs and 50 dipole correctors – 29 horizontal and 21 vertical ones [8]. The BPMs measure the beam position in both planes. The ion optics of the CR with $\gamma_{tr} = 3.85$ for the antiproton mode was examined. The ion optical layout can be seen in Fig. 2.

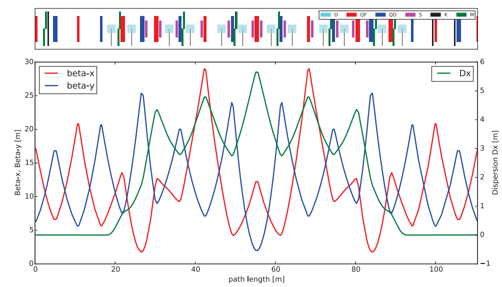

Figure 2: CR ion optical layout for half of the ring. The optical functions for the other half are identical.

The gradients of 11 quadrupole families were given random errors as described previously. The algorithm converged after 3-4 iterations in majority of the cases. For the BPMs with no readout errors the initially generated gradient errors were found almost exactly.

After subtracting the found errors from the distorted quadrupole gradients one obtains virtually ideal agreement with the model optics. This can be observed in Fig. 3 where the beta-beating is plotted before and after the ORM analysis fitting. The beta-beating is defined here as:

$$\frac{\Delta\beta}{\beta} = \frac{\beta^{(meas)} - \beta^{(mod)}}{\beta^{(mod)}} \tag{5}$$

The root-mean-square value of beta-beating in the horizontal plane before the fit is $(\Delta\beta_x/\beta_x)_{rms} = 0.09$. After the ORM analysis the beta-distortion $(\Delta\beta_x/\beta_x)_{rms} \approx 0$. This proves viability and power of the ORM analysis when using for correcting errors in the CR storage ring.

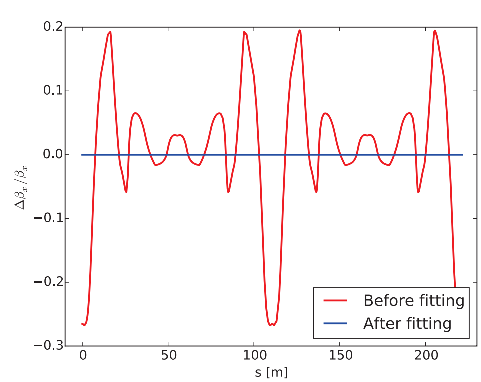


Figure 3: Beta-beating in the CR before and after the ORM analysis.

### *HESR High Energy Storage Ring*

The HESR storage ring is 575 m long. The ion optics for the antiproton mode [9] can be seen in Fig. 4. The maximum values of the horizontal beta functions reach 220 $m$ which is much larger than those of the CR ($\approx 30\,m$).

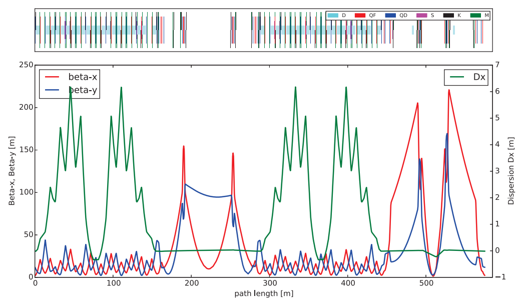


Figure 4: HESR ion optical layout.

The total number of correctors is 52 (30 horizontal, 26 vertical ones) and the number of BPMs is 64. We simulate the quadrupole strengths misset by assigning the ±5 % uniform errors to the 12 quadrupole families of the HESR. Then we could observe what level of accuracy could be achieved by the ORM analysis in the errors search.

The results of beta-beating before and after correction are shown in Fig. 5. One can notice how unstable the ion optics for the HESR is in comparison to the CR. The same relative errors of the quadrupole gradients lead to drastically different behavior of the beta-function. The maximum deviation of $\Delta\beta_x/\beta_x$ of the distorted optics in the HESR is 2.9 whereas for the CR it is less than 0.3. Nevertheless, generally 4 iterations were enough to find the errors and completely restore the initial optics.

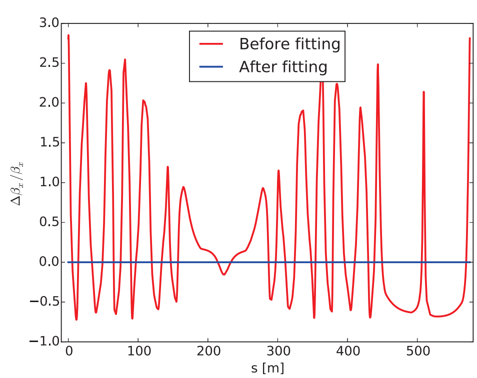


Figure 5: Beta-beating in the HESR before and after the ORM analysis.

### *BPMs with Readout Errors*

The implemented program for the ORM analysis also allows for including the BPM systematic readout errors. For example, in case of the HESR we assumed uniform errors in the range $\pm 0.5\,mm$ for all of the BPMs. 7 quadrupoles families were given the uniform errors bounded in the ±5 % range. The results of the ORM analysis are shown in Fig. 6. The rms beta-beating before the ORM fit was $(\Delta\beta_x/\beta_x)_{rms} = 0.36$. After the correction the beta-beating was reduced to less than 0.01 rms value.

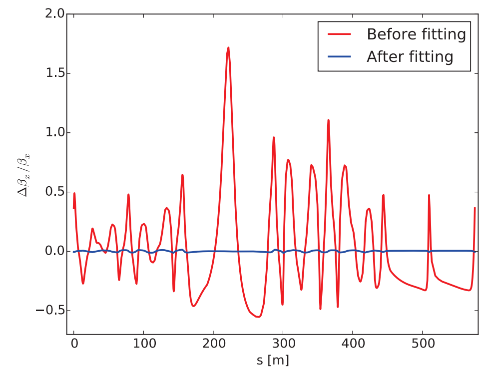


Figure 6: Beta-beating in the HESR before and after the ORM analysis. The BPM systematic readout errors are assumed with uniform distribution within a $\pm 0.5\,mm$ range.

With the BPM readout errors the precision of the search is lowered down basically to the given BPM noise. Simply speaking, we cannot find the gradient error which cannot be resolved by the beam position monitor. This obstacle might however be partially overcome by increasing the strengths of the corrector kicks. This will enhance the beam offsets at the BPMs thus enlarging signal-to-noise ratio. In the CR the limiting factor was corrector strength. In the HESR, on the other hand, a full aperture of most of the magnets, which is 89 mm, did not allow us to use the full strengths of the correctors.

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