UNCLOSED LATTICE DISPERSIONS AS A TOOL FOR PARTIAL REMOVAL OF TRANSVERSE TO LONGITUDINAL BEAM CORRELATIONS

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Abstract

We describe how to optimize certain properties of the particle beam with nonzero transverse-longitudinal coupling terms in the beam matrix by the proper choice of the unclosed lattice dispersions.

INTRODUCTION

Careful control of projected emittances is essential for the modern linacs driving short wavelength free electron lasers, and one of the main sources of the growth of projected emittances is creation of transverse-longitudinal correlations in the particle beam due to coherent synchrotron radiation (CSR) effects within magnetic bunch compressors as well as other wake fields along the accelerator. A number of approaches which could help to reduce such emittance growth were developed including different optics tricks, preparation of the initial beam current profile at the bunch compressor entrance, and etc. (see, for example, [1–6] and references therein). Still, the beam considered downstream of the compression system (or at the linac exit) could have nonzero transverse-longitudinal coupling terms in the beam matrix and therefore projected emittances could be further reduced if these correlations will be removed. In general, in order to make complete transverse-longitudinal decoupling, it is necessary to have the possibility to act on the particles depending on their longitudinal positions within the bunch (for example, with transverse deflecting cavity), which means that the system designed for the complete decoupling could be too complicated and somewhat difficult to operate in comparison with the benefit coming from the achievable reduction of the projected emittances. So, in this paper we consider a more simple and more practical question: what one can do having at hand a magnetostatic correction system? We show, in the framework of the linear motion model and with the self fields neglected, how to choose lattice dispersions in order either simply to remove part of the transverse-longitudinal beam correlations or directly to optimize chosen projected emittance. Due to space limitation we consider particle motion only in the horizontal and longitudinal degrees of freedom, and the complete 3D treatment and further discussions can be found in [7].

VARIABLES AND NOTATIONS

We consider the single particle linear dynamics in the horizontal and longitudinal degrees of freedom and ignore the motion in the vertical degree of freedom, which (on the linear level) is assumed to be completely decoupled from the two others. We restrict our consideration to the beam motion through the magnetostatic systems, take the path length along the reference orbit τ to be the independent variable, and use a set of symplectic variables $z = (x, p_x, \sigma, \varepsilon)^\top$ as particle coordinates. In this set x measures the horizontal displacement from the ideal orbit, p_x is the horizontal canonical momentum scaled with the constant kinetic momentum of the reference particle p_0, and the variables σ and ε which describe the longitudinal dynamics are

$$\sigma = c\,\beta_0\,(t_0 - t), \qquad \varepsilon = (\gamma - \gamma_0)\,/\,(\beta_0^2\,\gamma_0), \tag{1}$$

where γ_0, β_0 and $t_0 = t_0(\tau)$ are the Lorentz factor of the reference particle, its velocity in terms of the speed of light c and its arrival time at a certain position τ, respectively.

Let M be an $m \times m$ square matrix. Then $|M|$ denote the determinant of M. Let ω be a nonempty subset of $\{1, 2, \ldots, m\}$ with its elements listed in increasing order. Then $M\{\omega\}$ denote the principal submatrix of M whose entries are in the intersection of those rows and columns of M specified by ω.

Transfer Matrix of a Magnetostatic System

From energy conservation, symplecticity and absence of coupling with the vertical motion it follows that the general horizontal-longitudinal transport matrix of a magnetostatic system has the form

$$R = \begin{pmatrix} r_{11} & r_{12} & 0 & r_{16} \\ r_{21} & r_{22} & 0 & r_{26} \\ r_{51} & r_{52} & 1 & r_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

where the elements r_{16}, r_{26} and the elements r_{51}, r_{52}, r_{56} are the transverse and longitudinal lattice dispersions, respectively. The special form (2) of the matrix R allows to write its symplecticity conditions as follows

$$(R\{1,2\})^\top\, J_2\, R\{1,2\} = J_2, \tag{3a}$$

$$\begin{pmatrix} r_{16} \\ r_{26} \end{pmatrix} = R\{1,2\}\, J_2 \begin{pmatrix} r_{51} \\ r_{52} \end{pmatrix}, \tag{3b}$$

where J_2 is the 2×2 symplectic unit matrix.

Beam Matrix

As usual, we describe the linear properties of a particle beam by a symmetric matrix (beam matrix) of the second-order central beam moments

$$\Sigma = \left\langle (z - \langle z \rangle)\,(z - \langle z \rangle)^\top \right\rangle, \tag{4}$$

where the brackets $\langle \cdot \rangle$ denote an average over a particle distribution. We restrict our considerations to the case when

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the matrix Σ is positive definite and for simplification of notations assume that the beam is proper centered and $\langle z \rangle = 0$. With these assumptions the beam matrix takes on the form

$$\Sigma = \begin{pmatrix} \langle x^2 \rangle & \langle x p_x \rangle & \langle x\sigma \rangle & \langle x\varepsilon \rangle \\ \langle x p_x \rangle & \langle p_x^2 \rangle & \langle p_x\sigma \rangle & \langle p_x\varepsilon \rangle \\ \langle x\sigma \rangle & \langle p_x\sigma \rangle & \langle \sigma^2 \rangle & \langle \sigma\varepsilon \rangle \\ \langle x\varepsilon \rangle & \langle p_x\varepsilon \rangle & \langle \sigma\varepsilon \rangle & \langle \varepsilon^2 \rangle \end{pmatrix}, \tag{5}$$

where $\langle x\varepsilon \rangle$, $\langle p_x\varepsilon \rangle$ and $\langle x\sigma \rangle$, $\langle p_x\sigma \rangle$ are the beam dispersions and the beam tilts, respectively.

The matrix Σ has ten different entries which can be varied independently within the positive definiteness condition. Obviously, not all of them (or their combinations) are equally interesting for any particular accelerator physics application, and in this paper, besides beam dispersions and tilts, we consider only horizontal and longitudinal projected emittances

$$\varepsilon_x = |\Sigma\{1,2\}|^{1/2}, \quad \varepsilon_\sigma = |\Sigma\{3,4\}|^{1/2}. \tag{6}$$

Let R be the matrix which propagates particles from the state $\tau = \tau_1$ to the state $\tau = \tau_2$, i.e let

$$z(\tau_2) = R\, z(\tau_1). \tag{7}$$

Then from (4) and (7) it follows that the matrix Σ evolves between these two states according to the congruence

$$\Sigma(\tau_2) = R\, \Sigma(\tau_1)\, R^\top. \tag{8}$$

To further simplify notations, in the following we will write all propagation rules using the symbol ':=', i.e. in the form

$$\Sigma := R\, \Sigma\, R^\top, \tag{9}$$

where all quantities on the left and right hand sides are assumed to be evaluated for $\tau = \tau_2$ and $\tau = \tau_1$, respectively.

PROPAGATION AND ZEROING OF BEAM DISPERSIONS AND BEAM TILTS

Propagation of horizontal to longitudinal coupling terms in accordance with the transport rule (9) produces the following changes in the beam dispersions

$$\begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x\varepsilon \rangle \end{pmatrix} := R\{1,2\} \left[\begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x\varepsilon \rangle \end{pmatrix} + \langle \varepsilon^2 \rangle J_2 \begin{pmatrix} r_{51} \\ r_{52} \end{pmatrix} \right] \tag{10}$$

and the following changes in the beam tilts

$$\begin{pmatrix} \langle x\sigma \rangle \\ \langle p_x\sigma \rangle \end{pmatrix} := R\{1,2\} \left[\begin{pmatrix} \langle x\sigma \rangle \\ \langle p_x\sigma \rangle \end{pmatrix} + r_{56} \begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x\varepsilon \rangle \end{pmatrix} + (\Sigma\{1,2\} + \lambda\, J_2) \begin{pmatrix} r_{51} \\ r_{52} \end{pmatrix} \right], \tag{11}$$

where the parameter λ is defined by the expression

$$\lambda = \begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x\varepsilon \rangle \end{pmatrix}^\top \begin{pmatrix} r_{51} \\ r_{52} \end{pmatrix} + \langle \sigma\varepsilon \rangle + r_{56} \langle \varepsilon^2 \rangle. \tag{12}$$

Zeroing of Beam Dispersions

The formula (10) tells us that there exists a unique solution for the lattice dispersions which zeros the beam dispersions and this solution can be expressed as follows:

$$r_{51} = \frac{\langle p_x\varepsilon \rangle}{\langle \varepsilon^2 \rangle}, \quad r_{52} = -\frac{\langle x\varepsilon \rangle}{\langle \varepsilon^2 \rangle}. \tag{13}$$

Zeroing of Beam Tilts

According to the propagation rule (11) the beam tilts can be zeroed at the correction system exit by an appropriate choice of the lattice dispersions if and only if the equations

$$(\Sigma\{1,2\} + \lambda J_2) \begin{pmatrix} r_{51} \\ r_{52} \end{pmatrix} = -\begin{pmatrix} \langle x\sigma \rangle \\ \langle p_x\sigma \rangle \end{pmatrix} - r_{56} \begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x\varepsilon \rangle \end{pmatrix} \tag{14}$$

have a real solution with respect to the variables r_{51}, r_{52} and r_{56}. The system (14) is nonlinear, but, as we prove below, it always has at least one real solution for every fixed real value of the r_{56} coefficient, which therefore can be treated as a parameter. To show this, let us assume that λ in (14) is not simply a notation introduced for brevity, but an additional real-valued variable, and let us consider an extended system consisting of equations (14) and (12). In order to apply the method of successive elimination of variables to the system obtained, let us observe that

$$|\Sigma\{1,2\} + \lambda J_2| = \lambda^2 + |\Sigma\{1,2\}| > 0, \tag{15}$$

and therefore the matrix $\Sigma\{1,2\} + \lambda J_2$ is invertible. It means that for every real value of λ equations (14) can be solved with respect to the variables r_{51} and r_{52}, and the solution is unique. Substituting this solution into equation (12) and multiplying both sides of the result by $|\Sigma\{1,2\} + \lambda J_2|$ we obtain the polynomial equation of the third degree with respect to the single variable λ, and because the order of this equation is odd, it always has at least one real root.

So we proved that zeroing of the beam tilts by an appropriate choice of the lattice dispersions is always possible. At least one solution can be found for an arbitrary value of r_{56} coefficient and, for the fixed r_{56}, the number of different solutions can vary from one to three, and the corresponding numerical example can be found in [7].

Conditions for Complete Decoupling

The necessary and sufficient conditions for the complete decoupling can be obtained by the requirement that the solution for the lattice dispersions (13) which removes the beam dispersions also zeros the beam tilts, and substituting (13) into equations (14) and (12) we obtain

$$\begin{pmatrix} \langle x\sigma \rangle \\ \langle p_x\sigma \rangle \end{pmatrix} = \frac{\langle \sigma\varepsilon \rangle I_2 - \Sigma\{1,2\} J_2}{\langle \varepsilon^2 \rangle} \begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x\varepsilon \rangle \end{pmatrix}, \tag{16}$$

where I_2 is the 2×2 identity matrix.

We would note that the question of the physical interpretation of the relations (16) is rather complicated, and further discussions can be found in [7].

PROPAGATION AND OPTIMIZATION OF PROJECTED EMITTANCES

To obtain convenient representation for the emittance transport problem, let us introduce a 2×2 symmetric matrix

$$A = \langle \varepsilon^2 \rangle \, \Sigma\{1,2\} - \begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x \varepsilon \rangle \end{pmatrix} \begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x \varepsilon \rangle \end{pmatrix}^{\top} \quad (17)$$

and associate with this matrix quadratic form

$$\Psi_A(u_1, u_2) = (u_1, u_2) \cdot A \cdot (u_1, u_2)^{\top}. \quad (18)$$

Because the matrix Σ is positive definite, all its leading principal minors are positive and therefore

$$|A\{1\}| = |\Sigma\{1,4\}| \; > \; 0, \quad (19)$$

$$|A\{1,2\}| = |\Sigma\{4\}| \, |\Sigma\{1,2,4\}| \; > \; 0, \quad (20)$$

which means that the matrix A and the quadratic form Ψ_A are positive definite according to the Sylvester criterion.

With the help of the quadratic form Ψ_A the evolution of the projected emittances can be expressed in very compact and physically meaningful form as follows:

$$\varepsilon_x^2 := \varepsilon_x^2 + \Psi_A(r_{51}^x - r_{51}, \, r_{52}^x - r_{52}) - \Psi_A(r_{51}^x, \, r_{52}^x), \quad (21)$$

where

$$r_{51}^x \; = \; \frac{\langle p_x \varepsilon \rangle}{\langle \varepsilon^2 \rangle}, \quad r_{52}^x \; = \; -\frac{\langle x\varepsilon \rangle}{\langle \varepsilon^2 \rangle}. \quad (22)$$

$$\varepsilon_\sigma^2 := \varepsilon_\sigma^2 + \Psi_A(r_{51}^\sigma - r_{51}, r_{52}^\sigma - r_{52}) - \Psi_A(r_{51}^\sigma, r_{52}^\sigma), \quad (23)$$

where

$$\begin{pmatrix} r_{51}^\sigma \\ r_{52}^\sigma \end{pmatrix} = A^{-1} \left[\langle \sigma\varepsilon \rangle \begin{pmatrix} \langle x\varepsilon \rangle \\ \langle p_x \varepsilon \rangle \end{pmatrix} - \langle \varepsilon^2 \rangle \begin{pmatrix} \langle x\sigma \rangle \\ \langle p_x \sigma \rangle \end{pmatrix} \right]. \quad (24)$$

One sees that while the beam tilts and the beam energy chirp $\langle \sigma\varepsilon \rangle$ can influence the evolution of the longitudinal emittance through r_{51}^σ and r_{52}^σ, they do not enter the formula for the evolution of the horizontal emittance at all.

Optimization of Projected Emittances

With elegant formulas (21) and (23), the problem of the minimization of projected emittances by an appropriate choice of the lattice dispersions becomes fairly simple and straightforward, and has the same geometry for the both emittances. So, due to space limitation, we consider in details only optimization of the horizontal emittance.

The formula (21) tells us that the change in the horizontal emittance after the system passage is the same for all lattice dispersions r_{51} and r_{52} belonging to the same level set

$$\Psi_A(r_{51}^x - r_{51}, r_{52}^x - r_{52}) = const \geq 0. \quad (25)$$

Because quadratic form Ψ_A is positive definite, its level sets for $const > 0$ are ellipses all centered at the same point

$$r_{51} = r_{51}^x, \quad r_{52} = r_{52}^x \quad (26)$$

and contracting to this point as $const \to 0$. The level set

$$\Psi_A(r_{51}^x - r_{51}, r_{52}^x - r_{52}) = \Psi_A(r_{51}^x, r_{52}^x) \quad (27)$$

plays a special role. It separates lattice dispersions which lead to the emittance increase from those which provide emittance reduction or preservation. The level surface (27) is an ellipse if at least one of the beam dispersions is nonzero, and it is a point coinciding with the common center (26) of all ellipses (25) otherwise. In any case, there exists unique optimal choice for the lattice dispersions which is given by the equations (26) and which provides the largest possible reduction of the horizontal emittance. Due to (13) this optimal solution simultaneously zeros the beam dispersions and also turns (21) into equation

$$\varepsilon_x^2 := \varepsilon_x^2 \; - \; \frac{1}{\langle \varepsilon^2 \rangle} \cdot I_{cs}^x(\langle x\varepsilon \rangle, \, \langle p_x \varepsilon \rangle), \quad (28)$$

where

$$I_{cs}^x(u_1, \, u_2) = \langle p_x^2 \rangle \, u_1^2 - 2 \, \langle x p_x \rangle \, u_1 u_2 + \langle x^2 \rangle \, u_2^2 \quad (29)$$

is the nonnormalized Courant-Snyder quadratic form.

By analogy, the largest possible reduction of the longitudinal projected emittance is reached for

$$r_{51} = r_{51}^\sigma, \quad r_{52} = r_{52}^\sigma, \quad (30)$$

and (without big surprise) conditions for the simultaneous minimization of both projected emittances are again equations (16), which were derived as conditions for the complete horizontal-longitudinal decoupling.

Let us finish this section with the remark that the choice (30) for the lattice dispersions makes the vectors of beam dispersions and tilts linearly dependent (parallel) at the correction system exit.

SUMMARY

We have shown that if in the beam matrix there are correlations between energy of particles and their horizontal positions and momenta (beam dispersions), then the horizontal projected emittance always can be reduced by letting the beam pass through the magnetostatic correction system with the specially chosen nonzero lattice dispersions. The maximum emittance reduction occurs when the beam dispersions are zeroed, and the values of the lattice dispersions required for that are completely determined by the values of the beam dispersions and the beam rms energy spread, and are independent from any other second-order central beam moments.

We also have shown what can be done for optimization of the longitudinal projected emittance and found conditions on the beam matrix which guarantee that both projected emittances can be minimized simultaneously.

Besides that, we have proven that one can also use lattice dispersions to remove linear correlations between longitudinal positions of particles and their transverse coordinates (beam tilts), but in this situation solution for the lattice dispersions is nonunique and reduction of none of projected emittances is guaranteed.

We would finish with the note that the optimization of the beam properties by the proper choice of lattice dispersions is a potential source of the beam transverse jitter due to the beam energy jitter, and one has always to look for an appropriate compromise.

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