TAPERING OPTIONS AND EMITTANCE FINE TUNING FOR THE FCC-EE COLLIDER

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Abstract

The future electron-positron collider FCC-ee is being optimized for operation at four different collision energies in the range of 90 GeV to 350 GeV. At these high energies, vast synchrotron radiation losses create a large local deviation from the nominal energy, causing orbit offsets and creating the so-called sawtooth effect as well as optics distortions due to quadrupole focusing errors. In order to reduce orbit offsets, dipole fields can be adjusted to the local beam energy, effectively suppressing the sawtooth-effect and decreasing orbit distortions by up to two orders of magnitude. This process is called dipole tapering. This paper will present different dipole magnet tapering scenarios and compare them in terms of effectiveness, feasibility and cost. Furthermore it will be shown, that a similar procedure in quadrupole fields is not necessary, as the residual optics distortions in the form of beta- and dispersion beats after dipole tapering can be corrected solely by rematching the free quadrupoles in the dispersion suppressors and matching sections.

INTRODUCTION

In a lepton storages ring, electrons and positrons constantly lose energy due to synchrotron radiation. This constant loss of energy in combination with the energy gain in RF cavities leads to a periodic deviation of the local from the nominal beam energy. As the bending angle α of a dipole depends on the local particle energy, particles at different energies are forced onto different trajectories, causing the so-called sawtooth-effect of the orbit.

$$\frac{\alpha}{l} = \frac{c}{E} B \tag{1}$$

$$\alpha_{Dipole}(\Delta E) = \alpha_0 (1 - \frac{\Delta E}{E_0}) \tag{2}$$

Expressed in terms of the local dispersion $D(s)$, the relative energy deviation of the particle $\frac{\Delta E}{E}$ leads to an orbit amplitude

$$x_D(s) = D(s) \frac{\Delta E}{E_0}. \tag{3}$$

Schematically, the situation is explained in Fig. 1. Figure 2 shows the effect of a local dipole field correction to compensate the sawtooth-effect.

It should be emphasised, that the actual energy loss within a dipole strongly depends on the particle energy. This is described quantitatively in the critical energy of the radiated photon E_c, as well as the synchrotron power loss within a dipole field P_γ.

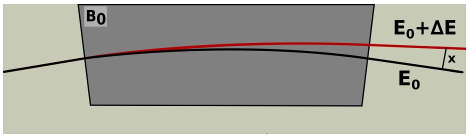

Figure 1: Dipole before tapering. A particle with an energy deviation ΔE is forced away from the ideal orbit.

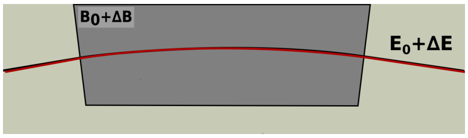

Figure 2: Dipole after tapering. A particle with an energy deviation ΔE now moves on the ideal orbit.

$$P_\gamma = \frac{2}{3} r_e \gamma^4 m_e \frac{l}{\rho^2} \tag{4}$$

$$E_c = \frac{3}{2} \gamma^3 \frac{\hbar c}{\rho} \tag{5}$$

The most critical case in this machine is therefore at the highest energy of 175 GeV and with RF sections installed in only two straight sections in the machine.

The sawtooth-effect is the major contribution to orbit offsets in high-energy lepton machines, causing the orbit in FCC-ee to reach offsets of up to 1.4 mm. If a particle passes a quadruple at such a large offset, it "sees" an additional, perturbing dipole field. In the same way, when the particle passes a sextupole at a large offset, this feed-down effect leads to an additional quadrupole field. While perturbing dipole fields cause orbit and dispersion distortions, perturbing quadrupole fields lead to distortions of the beam optics. Both in turn lead to an increase of the beam emittance.

LATTICE AND MAIN PARAMETERS

The main parameters of the machine at highest energy E=175 GeV are summarized in Table 1 [1]. Figure 3 is a schematic overview of the FCC-ee racetrack lattice. [2]

Table 1: Main Parameters of FCC-ee at 175 GeV

Circumference	100 km
Nominal Beam Energy	175 GeV
Energy Loss/Turn	≈ 7800 MeV
Synchrotron Radiation Power/Turn	50 MV
Critical Energy in the Arc	1.13 MeV
Horizontal Design Emittance	1 nm

TAPERING STRATEGIES

In a lepton storage ring a nonconstant, periodic oscillation of the beam energy leads to both orbit and optics distortions,

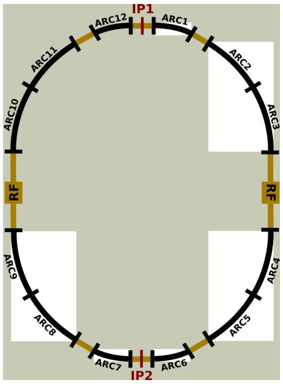

Figure 3: Schematic of the FCC-ee racetrack lattice. Arcs are depicted in black, straight sections in yellow. IPs and RF sections are marked as well.

as all lattice elements (dipoles, quadrupoles, sextupoles,...) are designed for the nominal beam energy. The ideal solution to this problem obviously would be to adjust the strength of each lattice element according to the local beam energy. On the other hand this means, that every magnet has to have its own power supply, which, for a machine the size of FCC-ee, will be tremendously expensive. For that reason, our tapering studies focus on an optimisation of dipole fields and the distortion effects on the orbit. These dipole tapering studies at FCC-ee were conducted at a variety of different lattices, numbers of RF sections and energies.

Table 2: Tapering Studies at FCC-ee

Lattice	Nr. RFs	Energy
12-Fold	12/6/4/2	45.5/120/175
Racetrack	8/6/4/2	45.5/120/175

In this paper, we concentrate on the worst case scenario of the racetrack lattice at 175 GeV and with RF cavities installed in only two straight sections. Here, local energy deviations are the largest and thus the sawtooth-effect of the orbit as well as optics distortions are the most distinctive. In that case, orbit offsets reach an amplitude of about 1.4 mm (see Fig. 4).

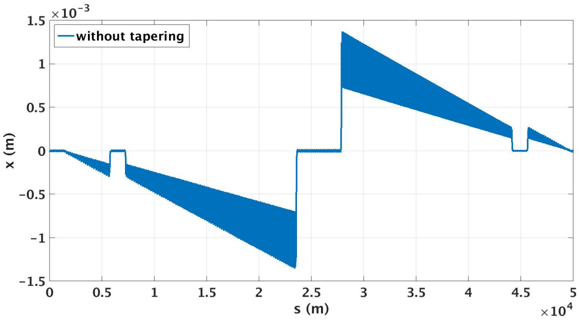

Figure 4: Orbit before tapering for a half ring

If now the strength of each dipole is adjusted individually to the local beam energy the orbit can be reduced to about $20\mu m$, which is an improvement of about a factor 70 (see Fig. 5).

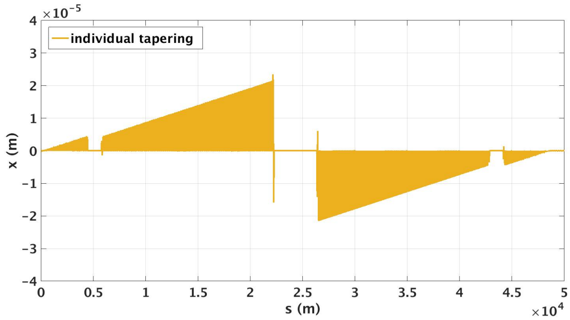

Figure 5: Orbit after individual tapering

Optimisation of the individual dipole strengths can be done in different ways, e. g. by using individually powered correction coils at the end of each magnet. In a real machine however, adjusting the strength of a dipole will most probably be realised with back-leg windings. In both cases however, equipping each dipole with its own correction mechanism will again be a very costly task.

Depending on how large an orbit offset can be considered acceptable, dipoles do not necessarily need to be tapered individually, but families of dipoles can be given the same "averaged tapering strength". This averaged tapering strength is acquired by using Eq. (2) and averaging over the local beam energies of the dipole families. During the course of the studies, two different tapered lattices with families of dipoles were created. Dipole families were created arcwise, for example all dipoles in a short arc (e.g. ARC1 in Fig. 3) were considered one dipole family. At first, all dipoles in a long arc section (e.g. ARC2 and ARC3 combined in Fig. 3) were grouped, resulting in eight dipole families. This was later changed and the long arc sections were divided into two subfamilies (e.g. ARC2 and ARC3 were considered two separate families), resulting in 12 dipole families. In this lattice, the sawtooth-effect orbit can be reduced by about a factor of 5, leading to a residual orbit amplitude of approximately 0.3 mm (see Fig. 6), which is considered tolerable for FCC-ee.

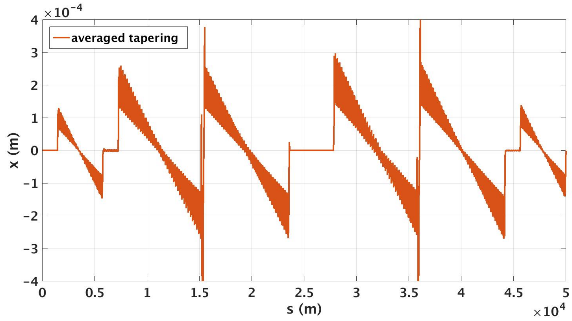

Figure 6: Orbit after averaged tapering

Finally, Fig. 7 shows a comparison of the sawtooth-orbit without tapering with the orbit after both individual and averaged tapering.

INFLUENCE ON OPTICS

With dipole tapering, the orbit can be reduced by almost two orders of magnitude, as shown in the previous paragraph.

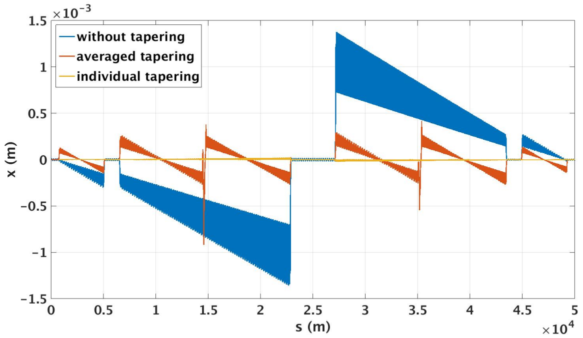

Figure 7: Same scale comparison of the orbit without tapering (blue), with individual tapering (yellow) and with averaged tapering (orange).

However, the periodic oscillation of the beam energy remains unchanged. As the quadrupole strength K_1 shows the same dependence on the local particle energy as the dipole bending angle (see Eq. (6), these periodic energy deviations affect the focusing properties of quadrupoles.

$$K_1 = \frac{c}{E}\frac{dB_z}{dx} \tag{6}$$

A particle with an energy higher than the nominal energy effectively "sees" a weaker quadrupole field and due to this chromatic effect is focused less strongly. These focusing errors can again be viewed as additional quadrupole fields and influence, due to their impact on beta functions and the dispersion function, the emittance of the machine (see. Eq. (7)-(10)).

$$\epsilon_x = C_\gamma \gamma^2 \frac{I5}{I2} \tag{7}$$

where I2 and I5 refer to the well-known synchrotron radiation integrals [3]

$$I2 = \oint \frac{1}{\rho^2}\mathrm{d}x = \sum_i \frac{l_i}{\rho_i^2} \tag{8}$$

$$I5 = \oint \frac{\mathcal{H}_x}{|\rho^3|}\mathrm{d}x = \sum_i \frac{l_i}{|\rho_i^3|} < \mathcal{H}_x >_i \tag{9}$$

and the $\mathcal{H}_x$-function is described as

$$\mathcal{H}_x = \gamma D_x^2 + 2\alpha D_x D_x' + \beta D_x'^2 \tag{10}$$

In FCC-ee at 175 GeV and with 2 RF sections, the distortion of the beta functions (chromatic beta-beat) after dipole tapering would actually be acceptable, as it is below the tolerance limit of 10%. The impact on the dispersion function however is much greater, resulting in a dispersion function reaching values of more than 1 m (see Fig. 8). This is an order of magnitude higher than the ideal dispersion of about 0.12 m (see Fig. (9)).

An obvious solution to this problem would be individual tapering of all quadrupole fields. This, however, turns out to be a very difficult undertaking, as individually powered quadrupoles are very expensive for a machine the size of FCC-ee. However, these studies showed that even in the case of the highest energy deviations, the residual optics and dispersion distortions can be corrected solely by rematching the optics using free quadrupoles in dispersion suppressors and matching sections. With this method, the achieved goal of beta and dispersion beats of below the tolerance limit of 10% could always be achieved.

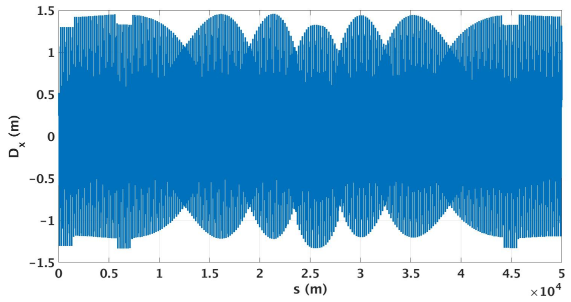

Figure 8: Horizontal Dispersion D_x after dipole tapering

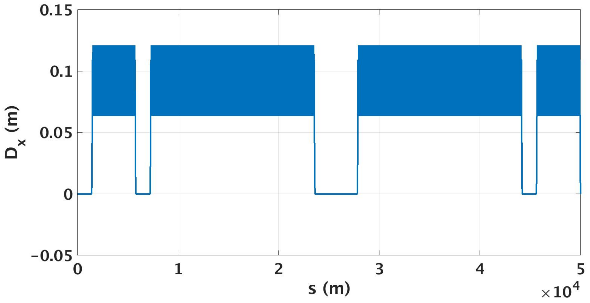

Figure 9: Ideal horizontal Dispersion D_x

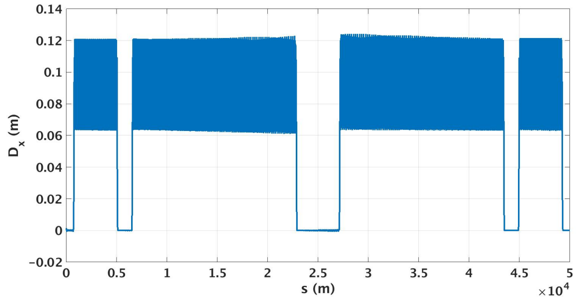

Figure 10: Horizontal Dispersion after matching. The deviations from the ideal dispersion are well below 10%.

CONCLUSION

In FCC-ee, local energy oscillations due to synchrotron radiation loss become so large, that their impact on the beam orbit in form of the sawtooth-effect can no longer be neglected. During the course of these studies it was shown, that through the process of dipole tapering, the sawtooth-effect can be compensated and the orbit can be reduced by almost two orders of magnitude. Residual beam optics distortions can be corrected by rematching of free quadrupoles in dispersion suppressors and matching sections, achieving beta and dispersion beats of below the tolerance limit of 10%.

REFERENCES

[1] F. Zimmermann *et al.*, “Challenges for highest energy colliders”, in *Proc. of IPAC’14*, Dresden, Germany, paper MOXAA01, p. 1.

[2] B. Haerer, B.J. Holzer, F. Zimmermann, and A. Bogomyagkov, “First Considerations On Beam Optics And Lattice Design For The Future Electron-Positron Collider FCC-ee”, in *Proc. of IPAC’15*, Richmond, VA, USA, paper TUPTY059, p. 2162.

[3] R.H. Helm, M.J. Lee, P.L. Morton, and M. Sands, “Evaluation of Synchrotron Radiation Integrals”, in *Proc. of PAC’73*, San Francisco, CA, USA, p. 900.