TRANSVERSE COHERENT INSTABILITIES IN STORAGE RINGS WITH HARMONIC CAVITIES*

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Abstract

Many current and future synchrotron light sources employ harmonic cavities to lengthen the electron bunches in order to reduce the emittance dilution caused by intrabeam scattering. In some cases, the harmonic cavities may be tuned to fulfill the flat potential condition. For this condition, a large increase in the threshold currents of transverse coupled-bunch instabilities has been predicted and recently, the physical content behind this stabilisation has been better understood. With this in mind, an investigation is made into the effectiveness of harmonic cavities for different machines. Frequency domain computations employing Laclare's eigenvalue method have been used to investigate the influence of several machine parameters and the results are presented.

INTRODUCTION

Resonant cavities at a harmonic of the main radio frequency (RF) are employed at many modern synchrotrons. These harmonic cavities alter the RF potential to change the charge distribution of the particle bunches. For synchrotron light sources, it is advantageous to lengthen the bunches since this leads to an increase in the Touschek lifetime and to a reduction in emittance dilution due to intrabeam scattering [1] [2]. The longest bunch that can be achieved without making the distribution double-peaked or asymmetric is obtained with the flat potential condition, where both the first and second derivative of the RF potential are zero at the synchronous phase. With this condition met, the bunch charge distribution in time is not Gaussian, the approximate distribution expected in a single RF system, but has the form $\exp\left(-a\tau^4\right)$ where a is a constant and τ is the time offset from the synchronous particle. The distribution in energy, on the other hand, is unchanged and remains Gaussian. For small amplitudes, the synchrotron tune of a particle is directly proportional to its amplitude of synchrotron oscillation instead of being approximately amplitude independent as it would be in the quasiharmonic RF potential provided by the main RF cavities alone [3].

Coherent betatron motion of particles within each bunch can be coupled by the machine impedance and this can lead to coupled-bunch instabilities. In modern synchrotron light sources, one source of impedance that drives these instabilities particularly strongly is the resistive wall of the vacuum chamber, whose aperture is typically small to accommodate strongly focusing small-bore magnets. It has been shown in simulation that harmonic cavities tuned to the flat potential condition can significantly increase the threshold currents of resistive-wall-driven transverse coupled-bunch instabilities at nonzero chromaticity [4] and the physical mechanisms responsible for this stabilisation have recently been identified and their contributions quantified [5]. The predicted effect across a range of chromaticities for the MAX IV 3 GeV ring is shown in Figure 1, where results from macroparticle simulations using *mbtrack* [6] have been compared to results obtained using Laclare's eigenvalue method [7], a frequency domain approach based on the linearised Vlasov equation which has been solved using the code *rwmbi* [8]. The four features of a bunch in a harmonic-cavity-flattened potential that differ from those of a bunch in a single RF potential and that contribute significantly to the difference in threshold currents are the longer bunch length, the synchrotron tune spread, the non-Gaussian bunch profile and the nonradial bunch distribution in synchrotron phase space, ie. the fact that the bunch distribution is Gaussian in energy but non-Gaussian in time offset. It can be seen that for the case without a harmonic cavity, the agreement between the macroparticle simulations and Laclare's eigenvalue method is very good. In the case with a harmonic cavity, there is a peak in the threshold current at a chromaticity of about 0.6.

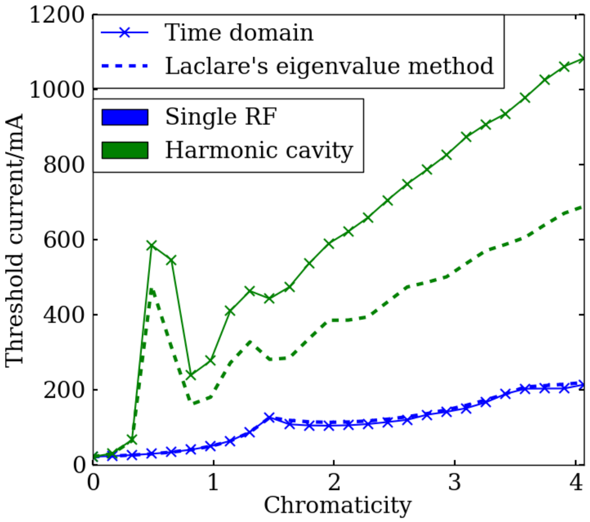

Figure 1: Threshold currents as predicted using macroparticle simulations in comparison with Laclare's eigenvalue method for the MAX IV 3 GeV ring with and without harmonic cavities tuned to the flat potential condition.

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At this point, radial head-tail mode structure in the bunch causes a decrease in the threshold current as the chromaticity is increased. There is no such peak for the case without a harmonic cavity because the threshold current is limited by azimuthal head-tail modes, which are not present in the harmonic cavity case [9]. At chromaticities beyond this peak, the difference between the results of the macroparticle simulations and those from Laclare's eigenvalue method is an almost constant factor of around 1.6. This is because Laclare's eigenvalue method assumes a radial bunch distribution in synchrotron phase space while the macroparticle simulations accurately model the nonradial distribution. A more complete interpretation of these results can be found in [5].

MACHINE PARAMETERS

In order to investigate how the stabilisation of resistive-wall-driven transverse coupled-bunch instabilities by harmonic cavities is affected by the parameters of the machine, several scans were performed using Laclare's eigenvalue method. The enhancement by a factor of ≈ 1.6 attributed to the nonradial bunch distribution in synchrotron phase space is therefore not included. The three parameters scanned were:

- Bunch length
- Radio frequency
- Machine circumference

For each scan, the static parameters were those of MAX IV except for the bunch length which, when not being scanned, was set to 100 ps as this was found to be more illustrative than setting it to the nominal value of 195 ps with harmonic cavity lengthening. Table 1 lists the parameters used. Apart from the three scanned and the betatron tune, changing any other of the listed parameters while keeping all the rest constant results simply in a scaling of the curve of threshold current against chromaticity, either along one axis or both.

Table 1: The Main Parameters of the MAX IV 3 GeV Storage Ring with No Insertion Devices

Parameter	**Value**
Circumference/m	528.0
Design beam current/mA	500
Radio frequency/MHz	99.931
Passive cavity harmonic	3
Average vertical beta/m	6.95
Vertical betatron tune	16.28
Harmonic number	176
Momentum compaction	3.07×10^{-4}
Bunch length before (after) lengthening/ps	40 (195)
Energy spread	7.69×10^{-4}
Energy loss per turn/keV	363.8
Vertical chamber aperture/mm	22.0
Chamber resistivity/Ωm	1.7×10^{-8}

Figure 2 shows the results from the scan of the bunch length. There is an approximate inverse scaling along the chromaticity axis with increasing bunch length. This means that peak seen in the threshold current at the chromaticity at which the radial head-tail mode structure starts to have an impact becomes narrower as the bunch length is increased. On the other hand, its height does not appear to be affected. Therefore, if this peak is to be exploited, selecting a shorter bunch length would increase the range of chromaticities at which the machine could be operated and this would be an advantage since the chromaticity of a machine can only be tuned to within a certain tolerance of the target value. Furthermore, because of uncertainties in the model of the machine impedance, the exact chromaticity at which this peak occurs would have to be measured and this would be easier with a wider peak. Conversely, with a longer bunch, this peak occurs at lower chromaticity which is advantageous in terms of beam lifetime.

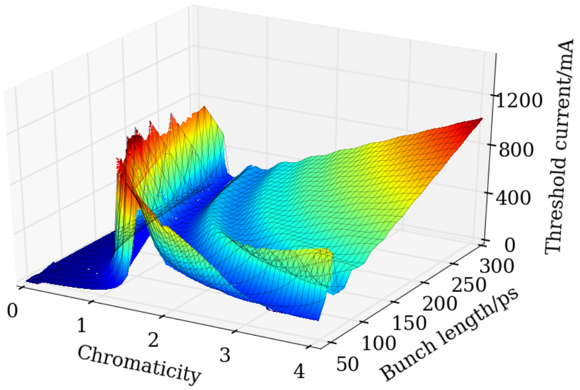

Figure 2: Surface of transverse coupled-bunch instability threshold current against chromaticity and bunch length.

Figure 3 shows the results from the scan of the radio frequency with a bunch length of 100 ps. It can be seen that with a high radio frequency, the peaks seen in the threshold current curve become smaller. In the absence of azimuthal head-tail modes, transverse coupled-bunch modes interact with the machine impedance at discrete angular frequencies ω_p given by

$$\omega_p = (Mp + \mu)\omega_0 + \omega_\beta \tag{1}$$

where M is the number of bunches, $0 \le \mu < M$ is an integer denoting the coupled-bunch mode, p is an integer between $\pm\infty$ and ω_0 and ω_β are the angular revolution and betatron frequencies respectively [7]. The consequence of the increase in the radio frequency is to increase the number of bunches M and therefore, the separation between the frequency lines of a single coupled-bunch mode. Therefore, for each coupled-bunch mode, all lines except the one at lowest frequency are moved to higher frequencies where the resistive-wall impedance is lower so their influence is smaller. At a moderate radio frequency of a few hundred megahertz, there is already one frequency line that is dominant. It belongs to the $\mu = M - 1$ coupled-bunch mode

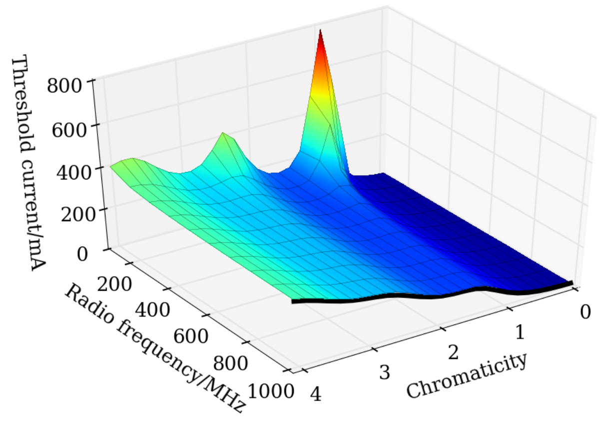

Figure 3: Surface of transverse coupled-bunch instability threshold current against chromaticity and radio frequency for a bunch length of 100 ps. The dark line shows the results when only the lowest frequency lines of the $\mu = -1$ coupled-bunch mode is considered; its position on the radio frequency axis is arbitrary and was chosen for visibility.

since this mode has the line at lowest negative frequency and so tends to have the highest growth rate when driven by the resistive-wall impedance. Taking only this frequency line into consideration ($\mu = M - 1$, $p = -1$), Laclare's eigenvalue method is reduced to a single dimension, the output of which is independent of the RF (and M) and has been added to Fig 3 as a dark line. With a low radio frequency, the peaks in the threshold currents at certain chromaticities are very high and are more dominant in the overall trend in threshold current.

The results of the scan in machine circumference are shown in Fig. 4. In order to maintain the same range of chromatic frequencies throughout the scan, the momentum compaction was reduced as the machine size was increased to compensate for the lower revolution frequency. Similarly, to keep the same impedance, the vacuum chamber wall resistivity was reduced to compensate for the longer vacuum chamber length. This was done in such a way that the MAX IV parameters coincide. These adjustments are justifiable since, as mentioned above, changing the momentum compaction and wall resistivity simply leads to a scaling along the chromaticity and threshold current axes respectively. Furthermore, the radiation damping time was kept fixed suggesting no increase in the radius of curvature with the circumference. Again, it would simply lead to a scaling of the threshold current if a change in the radiation damping were assumed. The results are similar to the scan in radio frequency but the explanation is slightly different. Since, as the machine size is increased, the revolution frequency decreases, the coupled-bunch mode line at lowest negative frequency is brought closer to zero frequency, where the resistive-wall impedance is largest. It is in this way that it becomes more dominant.

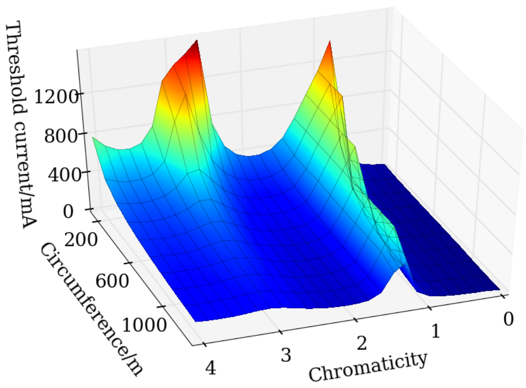

Figure 4: Surface of transverse coupled-bunch instability threshold current against chromaticity and machine circumference where the vacuum chamber resistivity has been scaled so that the impedance does not change and the momentum compaction has been scaled so that the chromatic frequency is the same for a given chromaticity.

CONCLUSION

A harmonic-cavity-flattened RF potential can increase the threshold currents of transverse coupled-bunch instabilities driven by the resistive-wall impedance. Four features, namely the bunch lengthening, the synchrotron tune spread, non-Gaussian bunch distribution in time and nonradial bunch distribution in synchrotron phase space, have been identified as having a significant contribution to this stabilisation. One consequence of the stabilisation is a peak in the curve of threshold current against chromaticity. This peak is located at the chromaticity above which radial head-tail mode structure starts to appear in the particle bunches. The influence of the bunch length, radio frequency and machine circumference has been investigated using Laclare's eigenvalue method. It is found that increasing the bunch length reduces the width of the aforementioned peak in the threshold current as well as reducing the chromaticity at which it occurs. A long bunch would make it harder to exploit the peak as it would be narrower but it would also allow the machine to be run at a lower chromaticity. A small machine circumference and low radio frequency make the peaks seen in the curve of threshold current against chromaticity more dominant. Because the effect of these two parameters is similar, large machines could benefit from some of the beneficial features of smaller machines by choosing a lower RF.

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