CALCULATION AND ANALYSIS OF THE MAGNETIC FIELD OF A TRANSVERSE GRADIENT UNDULATOR*

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Abstract

Transverse gradient undulator (TGU) is attracting more and more attentions, especially for the rapid progress of laser plasma accelerator techniques. The transverse gradient of TGU is usually given by an empirical formula simply derived from the empirical formula of a uniform-parameter undulator. In this paper, we numerically investigate the transverse magnetic field of TGUs using the RADIA code. Through many simulations for TGUs with different magnet structures, we have given the dependences of transverse gradient parameter on the cant angle, the undulator period and the average gap. Based on these results, when the cant angle is small and the variable g/λ_u is in the range of 0.4-0.6, the simulation results agree with the empirical formula well. But, with the growing of the cant angle, or with the growing of the deviation of g/λ_u from the range of 0.4-0.6, the difference between the simulation results and the empirical formula becomes larger.

INTRODUCTION

Undulator is a core component of free-electron lasers (FELs), providing the lateral cyclic static magnetic field for the interaction between the electrons and optical field. In a uniform-parameter undulator, the field strength acting on the beam is identical for all electrons. As a result, the radiation bandwidth is affected much by the energy spread and emittance of the electron beam [1]. Transverse gradient undulator was proposed to reduce the sensitivity to the energy variations for FEL oscillators in the beginning [2, 3] and now is widely used in FEL operations.

In recent years, laser plasma accelerators (LPAs) have made tremendous progress in generating high energy, high peak current, low emittance beams in a very compact scale [4, 5]. The electron beam produced by such an accelerator was successfully sent through conventional undulator to generate spontaneous radiation [6, 7]. However, LPA beams have rather large energy spread on the order of a few percent level, and such energy spread hinders the short-wavelength FELs application. One of the possible ways to solve this problem is the use of transverse gradient undulator [8]. Another important application of TGU is making the transverse-longitudinal phase space coupling, proposed in the phase-merging enhanced harmonic generation (PEHG) scheme [9, 10]. In the PEHG, a transversely dispersed electron beam travels through the TGU modulator, around the zero-crossing of the seed laser, the electrons with the same energy will be merged into a phase with the same longitudinal phase, and then bunch at very high harmonics after dispersion section.

A TGU is usually realized by canting the magnetic poles of a uniform-parameter undulator, as illustrated in Fig. 1. At present, the transverse gradient of a TGU is given by an empirical formula simply derived from the empirical formula of a uniform-parameter undulator, as used in Ref. [8]. In this paper, we investigate the TGU magnetic field by numerical simulations using RADIA code [11]. The comparison of numerical simulation and the empirical formula for a uniform-parameter undulator is given. Next, we simulate the TGU magnetic field and check the results with the existed empirical formula. Finally, we sumarize in the last section.

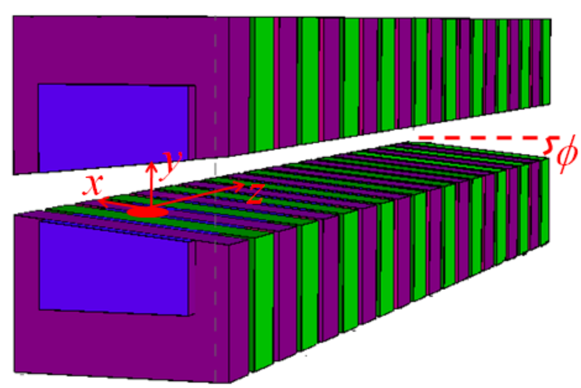

Figure 1: Transverse gradient undulator by canting the magnetic poles. Each pole is canted by an angle ϕ with respect to the *xz* plane.

THE MAGNETIC FIELD OF A UNIFORM-PARAMETER UNDULATOR

For a uniform-parameter hybrid undulator, the peak magnetic intensity B_0 on the axis can be given by the empirical formula [12, 13]:

$$B_0 = a e^{-b\frac{g}{\lambda_u}+c(\frac{g}{\lambda_u})^2}, \quad 0.07 \le g/\lambda_u \le 0.7 \qquad (1)$$

where, g and λ_u are the gap and period of the undulator, respectively, and $a = 0.55B_r + 2.835$, $b = -1.95B_r + 7.225$, $c = -1.3B_r + 2.97$, where B_r is the remanence of the permanent magnet blocks. This formula has been proved that it was effective in practical measurement, however, it was found to be a little larger (<5%) than the practical

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* supported by National Natural Science Foundation of China (No. 11205156 and No. 11375199).

magnetic intensity from measurement in our many years' experience.

Another way to calculate the magnetic field of an undulator is the simulation by using the RADIA code, which has been proved valid for undulator design. We take the hybrid undulator composed of NdFeB magnet blocks and NdFeV magnet poles as an example. Using the parameters of UND-1 listed in Tab.1, the magnetic field with undulator gap of 18 mm is simulated and shown in Fig.2. In this undulator, the value of g/λ_u is about 0.41, which locates in the applicable scope of the empirical formula. From Fig.2, the peak field strength on the axis is about B_0=0.5942 T, while from Eq. (1) it is 0.5998 T. The simulation result is about 1% smaller than that from Eq. (1).

Table 1: Main Parameters of the Undulator

Parameters	UND-1	UND-2
period /mm	44	80
remanence of magnetic blocks /T	1.2	1.2
height of magnetic blocks /mm	45	81
width of magnetic blocks /mm	17	31
height of magnetic poles /mm	25	45
width of magnetic poles /mm	5	9

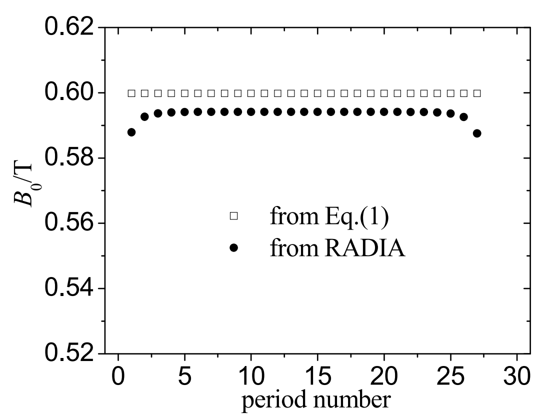

Figure 2: The peak magnetic field B_0 on the z axis of a uniform-parameter hybrid undulator with the gap of 18 mm.

THE MAGNETIC FIELD OF A TGU

As illustrated in Fig. 1, by canting the magnetic poles, one can obtain a linear x dependence of the vertical undulator field so that

$$\frac{\Delta K}{K_0} = \alpha x \quad (2)$$

where K_0 is the rms undulator parameter on the axis and $K_0 = 0.934\lambda_u[\mathrm{cm}]B_0[\mathrm{T}]$. Based on Eq.(1), for a full angle $2\phi \approx \Delta y/\Delta x$, the gradient parameter is

$$\alpha = 2\phi\frac{1}{K_0}\frac{\partial K}{\partial y} = 2\phi(\frac{b}{\lambda_u} - 2c\frac{g}{\lambda_u^2}) \quad (3)$$

Here g is the average gap of the cant poles. This formula has been used in Ref. [8] for calculating the gradient parameter.

Using parameters listed in Table. 1, we firstly simulate the magnetic field of TGU with the cant angles of ϕ= 0.05, 0.1, 0.15 rad, at the fixed average gap of 20 mm. From Fig.3, one can easily find that the peak magnetic field strength has a linear dependence on transverse position x, and the gradient increases with the increasing of the cant angle.

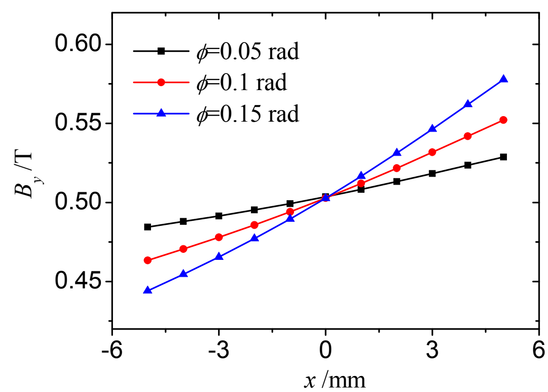

Figure 3: Variation of peak magnetic field strength B_0 with x at the average gap of 20 mm.

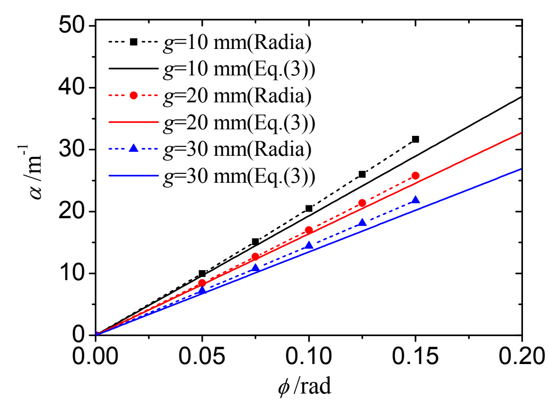

Figure 4: Variations of the gradient parameter with the full cant angle at different average gaps of 10 mm, 20 mm and 30 mm.

Based on the results in Fig. 3, the variations of gradient parameters with the different cant angles can be calculated. They are depicted in Fig. 4, and compared with the result given by Eq. (3). It is obviously that the gradient parameter is in direct proportion to the cant angle, which agrees with Eq. (3). One also can find that the gradient parameter from Radia is larger than that from Eq. (3), and the difference becomes growing as the cant angle increases, even the RADIA result being larger by about 10% while ϕ= 0.15 rad. This may be partially induced by that as mentioned above, the peak magnetic field from RADIA is usually a little smaller than that calculated by Eq. (1). However, we think that the difference mainly

ISBN 978-3-95450-147-2

comes from the misknowledge of the coefficient $(b/\lambda_u - 2c\,g/\lambda_u^2)$ in the proportional relationship. That is to say, Eq. (1) appears needing some modification.

Next, we investigate the dependence of the transverse gradient on the average gap. In FEL applications, we usually need to adjust the undulator gap to tune the resonant wavelength. Normally, the magnet blocks and poles are installed on the mechanical bracket and there is usually only one drive motor or two motors locating at both ends. The undulator blocks and poles at one side can be considered as a whole, and in FEL experiments, the gap only can be changed a same variation for each period simultaneously. Under this condition, it is worth investigating the variation of the transverse gradient with the average gap.

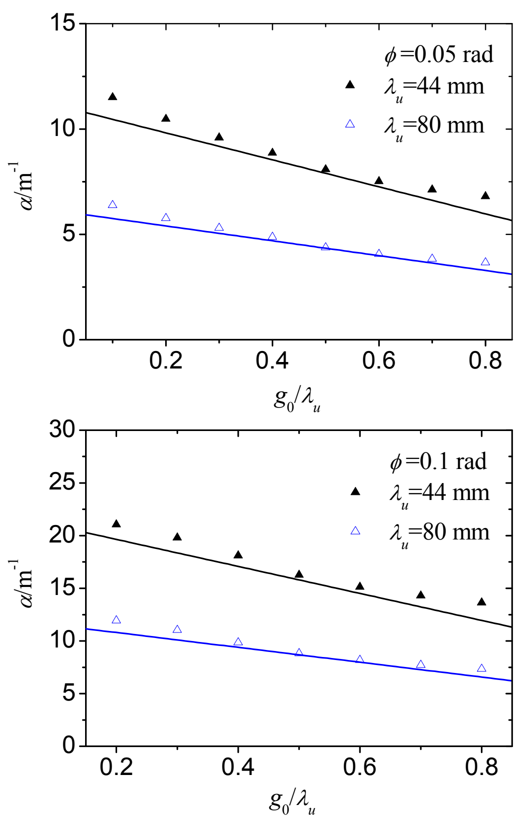

Figure 5: Variation of the transverse gradient with the rate of the average gap and the period, at different undulator periods. Top: ϕ = 0.05 rad; Bottom: ϕ = 0.1 rad.

Figure 5 gives the variation of the transverse gradient with the average undulator gap from RADIA simulation and Eq. (3) at the fixed cant angles of ϕ = 0.05, 0.1 rad, for different periods of 44 mm and 80 mm. From the illustrations, the two kinds of results agree well when the variable g/λ_u locates in the range of 0.4-0.6, and in the two sides of this range, the simulation results are larger than that from Eq. (3) and the difference gradually grows with the increasing of the deviation. Obviously, the transverse gradient does not linearly vary with the average undulator gap. It seems that a term including $(g/\lambda_u)^2$ is omitted in Eq. (3), corresponding to a term including $(g/\lambda_u)^3$ omitted in Eq. (1). This also gives an explanation for the difference of the relation α-ϕ between the simulation results and Eq. (3). We attempted to add a term including $(g/\lambda_u)^3$ into Eq. (1), but it was found that the coefficients b and c also needed to be modified by a small amplitude. However, Eq. (1) is a well-known empirical formula used widely, and the modification must be based on much experience.

SUMMARY

In summary, we have numerically investigated the magnetic field of transverse gradient undulator and compared the results with those from Eq.(3), through much work of RADIA simulations for TGUs with different undulator structures. The dependences of transverse gradient parameter on the cant angle, the undulator period and the average gap have been given. Based on these results, when the cant angle is small and the variable g/λ_u is in the range of 0.4-0.6, the simulation results agree with Eq. (3) very well. But with the growing of the cant angle, or with the growing of the deviation of g/λ_u from the range of 0.4-0.6, the difference between the simulation results and Eq. (3) becomes larger. We speculate that these differences come from that a term including $(g/\lambda_u)^3$ is omitted in Eq. (1) and then a term including $(g/\lambda_u)^2$ is omitted in Eq. (3).

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