SHORT-WAVELENGTH RADIATION OF A SMALL CHARGED BUNCH IN PRESENCE OF A DIELECTRIC PRISM*

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Abstract

Investigation of radiation of a charged particle bunch in the presence of a large (compared with wavelengths under consideration) dielectric object can be performed using certain approximate methods. We develop here the method based on the known Stratton-Chu formulae which allows calculating the field everywhere outside the object including the Fresnel and Fraunhofer areas, as well as neighborhoods of focal points. The main problem considered here consists in investigation of radiation of a small bunch moving along boundary of a dielectric prism or in channel inside a prism. Approximate analytical solutions of the problem are obtained and typical numerical results are given.

INTRODUCTION

Radiation of charged particles in the presence of dielectric objects are of interests for applications in accelerator and beam physics [1]. One of them is a new method of bunch diagnostics [2]. For realization of this method, it is necessary to calculate Cherenkov radiation outside a dielectric object. As a rule, the complex geometry of the problem does not allow obtaining rigorous expressions for the radiation field. Therefore development of approximate methods for analyses of radiation is necessary.

Some of problems with dielectric objects were considered with use of so-called method of polarization currents [3-5] where these currents are found using some simplifying assumptions. Such approach is valid if the dielectric permittivity is close to 1. In our opinion, this method is in need of additional justification for objects with the permittivity differing essentially from 1.

It is important to develop as well different methods which are similar to ones elaborated in optics and diffraction theory. Earlier we offered a method based on combination of exact solution of problem without “external” boundaries of the object and accounting of these boundaries using the ray optics [6,7].

However this technique have some essential limitations. First, the object size must be much larger than the wavelengths under consideration λ. Exactly speaking, the way of the wave into the target should be much larger than λ, and the same condition is fulfilled for size d of the external border of the object ("aperture"): $d >> \lambda$. Second, the distance L from the aperture to the observation point must not be very large, that is the "wave parameter" must be small: $D \sim \lambda L / \Sigma << 1$, where L is a distance from the border to the observation point, and $\Sigma \sim d^2$ is an aperture square. As well, the observation point cannot be close to focuses and caustics.

Now we consider situation where the first condition is true but the second one can be disturbed. Thus, the cases where $D \sim 1$ or $D >> 1$, as well as neighborhoods of focuses and caustics, are included in our consideration. In the theory of particle radiation, the method developed here was applied for study of a dielectric concentrator of Cherenkov radiation [8] and calculation of radiation from a waveguide with an open end [9]. Now we apply this technique to study radiation from a "truncated" prism.

The method consists in the following. First, the field of the charge in an infinite medium without "external" borders is calculated. Further the incident field is presented as a sum of waves of two polarizations and the field at the external boundary is calculated using the Fresnel transmission coefficients. At final step, we calculate the field outside the target using Stratton-Chu formulae ("aperture integrals"). These formulae allow determining the field in the surrounding space if tangential components of electric and magnetic fields on the aperture are known.

ANALYTICAL RESULTS

We consider an object in the form of "truncated" prism (Fig.1) made from dielectric with permittivity ε and permeability $\mu = 1$. The surrounding medium is a vacuum. We consider two variants: (1) the case of "prism-I" where the charge moves with velocity $\vec{V} = c\vec{\beta}$ along the axis (z) of the vacuum channel having the radius a; (2) the case of "prism-II" where the charge moves along z-axis parallel to one of boundaries at distance a.

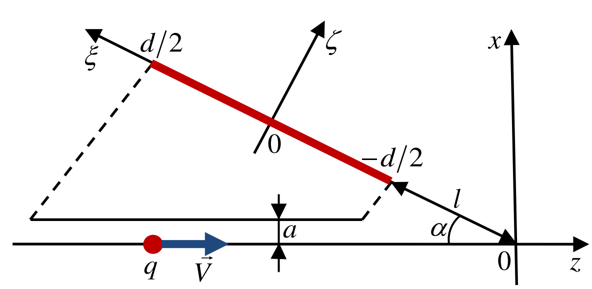

Figure 1: Cross-section of the "truncated" prism and the charge moving along the border. The part of the external surface lighted by Cherenkov radiation is shown by the red heavy line.

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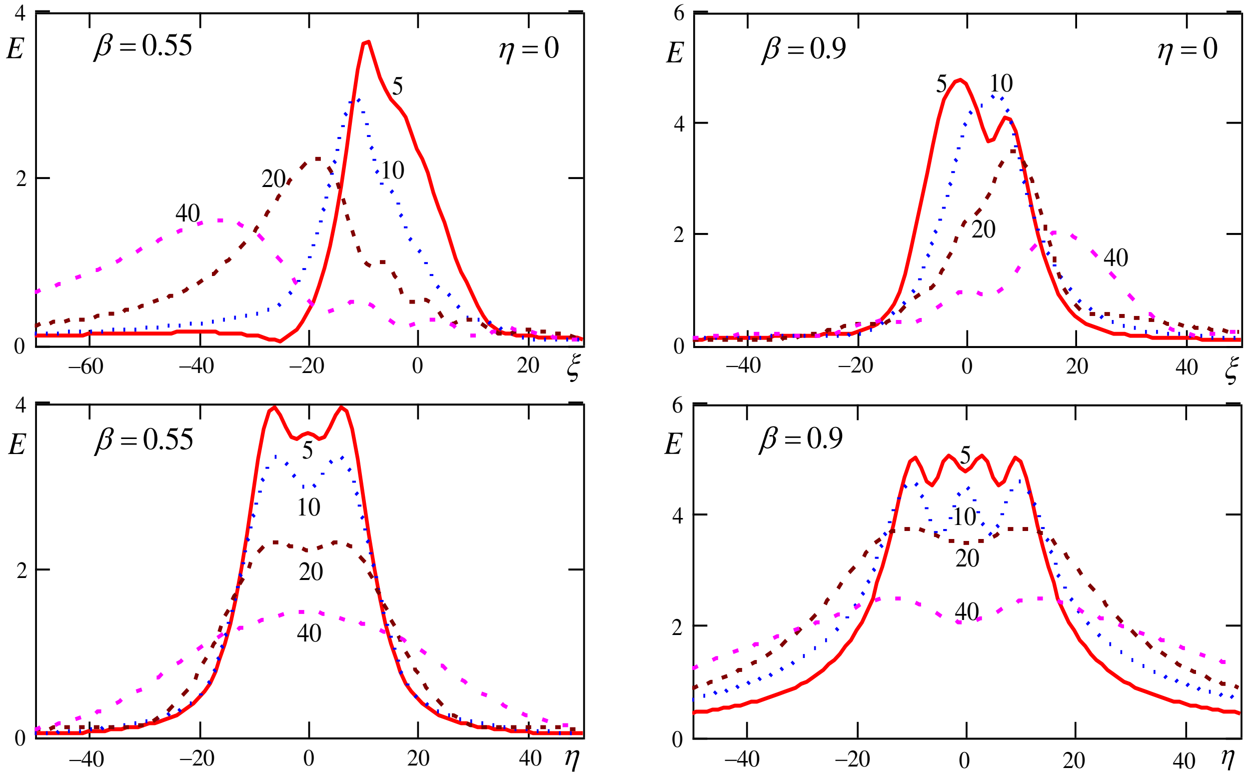

Figure 2: The electric field amplitude (arbitrary units) in case of prism-I. Top: dependency on ξ for $\eta = 0$. Bottom: dependency on $\eta = y$ for the values of ξ which correspond with maximums of curves shown above. Distance ζ is shown near curves. Parameters are: $a = 1$, $\alpha = 45^0$, $b = d = 20$, $l = 11.4$, $\varepsilon = 4$, $\beta = 0.55$ (left) and $\beta = 0.9$ (right). All distances and coordinates are given in unity k^{-1}.

Note that, within the framework of our approximate consideration, the external border of the target can be both the surface of whole object and the part of surface of the object having the bigger size if another part is covered by a metal. Therefore two of target's borders showing by dotted line (Fig.1) can be different. For further consideration, it is important only that the part of external surface ("aperture") lighted by Cherenkov radiation has certain form and sizes. We assume that the aperture is a rectangle with sides d ($|\xi| < d/2$) and b ($|\eta| < b/2$). For convenience we introduce the coordinate system $\xi, \eta = y, \zeta$ associated with the aperture (Fig.1).

First, we should obtain the incident field $\vec{E}^i$, $\vec{H}^i$ assuming that the external boundaries of the object are absent. This is the field of the charge in the channel in unbounded medium (for prism-I) or the field of the charge moving parallel to the boundary of semi-infinite medium (for prism-II). The solutions of these problems are well known [10]. Further we obtain the asymptotic of the incident field into the object for large distance from the charge trajectory.

The obtained wave field is represented as a sum of waves of horizontal (h) and vertical (v) polarization. They are characterized correspondingly by components $\vec{E}^i_h$ and $\vec{H}^i_v$ which are orthogonal to the plane of incidence (containing the wave vector $\vec{k}$ and normal $\vec{n} = \vec{e}_\zeta$). According to Fresnel low, the field components on the external surface of the object are

$$E^t_h = \frac{2\cos\theta_i\, E^i_h}{\cos\theta_i + \varepsilon^{-1/2}\cos\theta_t}, \quad H^t_v = \frac{2\varepsilon^{-1/2}\cos\theta_i\, H^i_v}{\cos\theta_t + \varepsilon^{-1/2}\cos\theta_i}, \quad (1)$$

where θ_i, θ_t are angles of incidence and refraction, correspondingly.

After cumbersome transformations, one can obtain the formulae for electric and magnetic fields on the external boundary in the form

$$\begin{aligned} E^t_\xi &= -UE^t_h + V\cos\theta_t\, H^t_v, \quad E^t_\eta = VE^t_h + U\cos\theta_t H^t_v, \\ H^t_\xi &= -V\cos\theta_t E^t_h - UH^t_v, \quad H^t_\eta = -U\cos\theta_t E^t_h + VH^t_v, \end{aligned} \quad (2)$$

where U and V are certain coefficients depending on the point on the border and parameters of problem.

The last step is applying the Stratton-Chu formulae which have the following view for electric field strength (Gaussian system of units is used):

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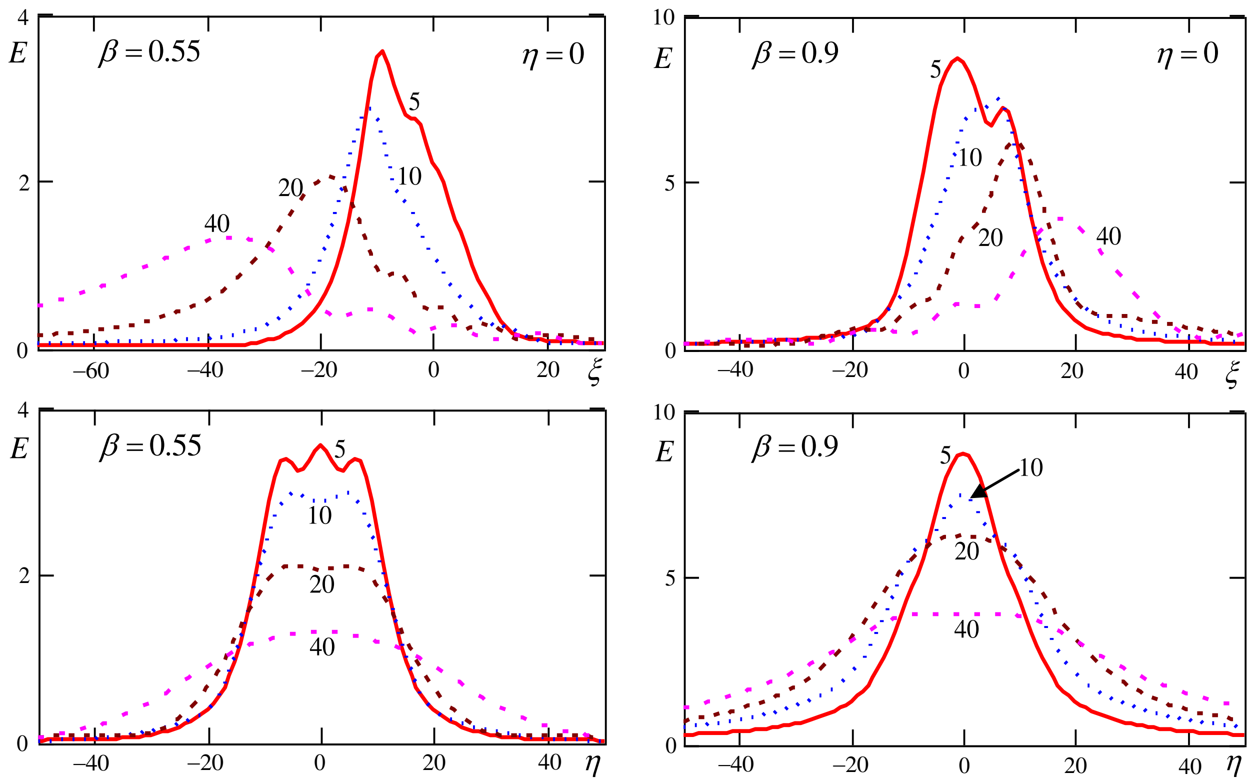

Figure 3: The same as in Fig.1 for the case of prism-II.

$$\vec{E}(\vec{r}) = ik(4\pi)^{-1}\int_{\Sigma}\left[\vec{n}\times\vec{H}(\vec{r}_0)\right]g(\vec{r}-\vec{r}_0)d\Sigma + $$
$$+i(4\pi k)^{-1}\int_{\Sigma}\left(\left[\vec{n}\times\vec{H}(\vec{r}_0)\right]\cdot\nabla_0\right)\nabla_0 g(\vec{r}-\vec{r}_0)d\Sigma - \quad (3)$$
$$-(4\pi)^{-1}\int_{\Sigma}\left[\left[\vec{E}(\vec{r}_0)\times\vec{n}\right]\times\nabla_0 g(\vec{r}-\vec{r}_0)\right]d\Sigma,$$

where Σ is an aperture, $\vec{r}_0$ is a point on the aperture, $\vec{r}$ is an observation point, ∇_0 is a gradient on components of $\vec{r}_0$, $g(\vec{R}) = \exp\left(i\vec{k}\vec{R}\right)/\left|\vec{R}\right|$, $\vec{k}$ is a wave vector in vacuum. The formula (3) allows calculating the field at arbitrary point if the tangential components of electric and magnetic fields are known on the aperture.

NUMERICAL RESULTS

On basis of the method described above we have elaborated the algorithm and Mathcad program. Some typical results are shown in Fig. 2 and 3. Note that the aperture used for computation has the size of several wavelengths only. Therefore the field obtained is not similar to the ray optics field even at distances $\zeta \sim 10\cdot k^{-1}$: one can see that essential diffraction spreading takes place. For the bigger values of ζ this effect is more expressed. Comparison of cases of prism-I and prism-II shows that longitudinal distributions (along ξ) are very similar, but transversal distributions (along η) have some differences. Note that the properties of radiation differ from the ray-optical regularities described in [7] because the ray-optical approximation does not valid for the areas shown in figures.

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