FREQUENCY SCALING STUDY OF CRAB CAVITY FOR FUTURE COLLIDERS WITH CRAB CROSSING*

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Abstract

Crab crossing is an essential concept in the newly proposed colliders or the upgrades. It enables crossing angles to achieve lower β^* without a loss of luminosity. The frequency of the crab cavity shall be chosen with various considerations, including the luminosity degradation, emittance growth due to synchro-beta resonances and RF noises. Here, as the first step, we use the figure of merits to establish the frequency scaling relations with important design parameters, which guide the choice of crab cavity frequency for new designs.

INTRODUCTION

The future electron ion collider (EIC) in US aims on achieving high luminosity [1]. Therefore, all designs adopt crossing angles between the two beams, which allows fast beam separation, simpler managing protection for synchrotron radiation and smaller beta function at interaction point (IP). The crossing angle creates the geometric luminosity loss. The figure of merit to characterize the loss is 'Piwinski Angle':

$$\theta_P = \frac{\sigma_z}{\sigma_x}\theta_c$$

where θ_P is the Piwinski angle, $\sigma_{z/x}$ are the rms longitudinal/transverse bunch size and θ_c is the half crossing angle. Table 1 compares the crossing angles and Piwinski angle of various designs of future EIC and the on-going LHC upgrade project.

All of the designs of future EIC in US adopt the crab crossing scheme to avoid the geometric luminosity loss. Ideally it should provide a transverse kick to the particle which is linearly proportional to its relative longitudinal position within the bunch and form a tilting angle of half of the crossing angle on both colliding beams to compensate crossing angle. As a result, the two beam will collide equivalently as the head-on collision. The crab cavities are placed on both sides of the interaction region area to ensure that the beam rotation does not propagate to the outside of the interaction region.

If the bunch length is comparable with the wavelength of the crab cavity, the sinusoidal form of the crab-cavity voltage leads to the transverse deviation of particle at the head and tail of the bunch from the perfect linear x-s correlation. This nonlinearity not only leads to the luminosity loss, but also induces transverse kicks to the both beams which depend on the longitudinal position during the interaction. This effect presents a challenge for eRHIC bunch, especially without cooling present. On the other hand, an unnecessary low frequency leads to higher voltage of the cavity, larger cavity size and more challenges of manufacturing and post processing. Therefore, finding proper frequency in the design is essential to the average luminosity of the collider and reducing the manufacture risks of the cavities. In this paper we present the considerations of choosing crab cavity frequency for eRHIC. The preliminary choices of eRHIC and the frequency of the crab cavities in other machine or design are list in Table 1.

LUMINOSITY CONSIDERATION

Ideally, the crab cavity exerts a linear kick on the beam that create a transverse tilt that fully compensate the geometric loss due to the crossing angle, and restore the luminosity of the head-on collision. However, the kick from the cavity is sinusoidal and it is expected that there will be luminosity reduction compared with head-on collision. Although the cavity voltage depend on the local beta function and the phase advance from IP, the kick must generate a transverse offset which depend on the longitudinal position.

$$\Delta x_c = \frac{\theta_c}{k}\sin(kz)$$

where k is the wavenumber of the crab cavity. The luminosity with the crab crossing scheme then can be integrated numerically [2], including both the above crab-cavity kicks, hourglass effect and crossing angle.

We use the luminosity degradation parameter H_L as one figure of merit to determine the proper frequency of the crab cavity.

$$H_L = \frac{L_{crab-crossing}}{L_{head-on}}$$

The $L_{head-on}$ is the luminosity of head-on collision, which also includes the hourglass effect. For ideal crabbing scheme, or the $kz \ll 1$, the parameter H_L is 1.

We first consider that only one frequency of the crab cavity is involved and calculate the factor H_L as the function of the frequency, shown in Figure 1. The gridlines show the possible frequency choice, which are the harmonic of the 28MHz, the frequency of the accelerating cavity in RHIC. The figure indicates that very low frequency (<100 MHz) are needed to achieve 10% luminosity loss for the 15cm or 20cm rms bunch length. To avoid manufacturing difficulties related with low frequency cavity, one need to consider higher harmonics to make the crab kick more linear.

To avoid the over-complicated system, we limited ourselves by only adding one harmonic frequency. Therefore,

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Table 1: Comparison of Crossing Angle Parameters

	Crossing angle (mrad)	Piwinski angle (ion beam)	RMS bunch length (cm)	Freq. of crab cavity (MHz)
eRHIC (ring-ring)	15	10.7	20	168 + 504
eRHIC (linac-ring)	10	49.0	15	140 + 420
JLEIC	50	25	1	952
LHC upgrade	0.59	2.95	7	400

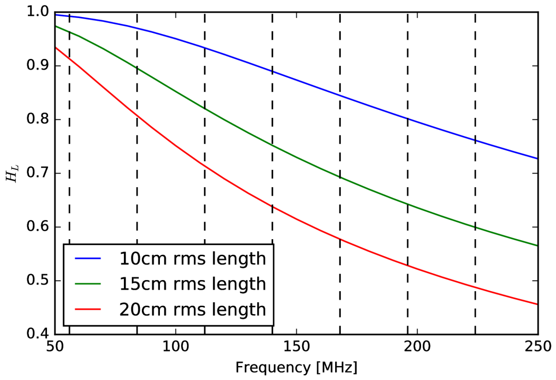

Figure 1: The luminosity degradation parameter as function of frequency for various rms bunch length.

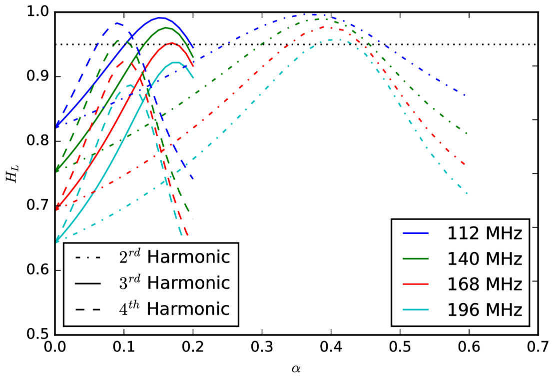

Figure 2: The luminosity degradation parameter as function the harmonic strength α with various harmonic numbers and fundamental frequencies. The grid line marks 5% luminosity loss due to crab crossing scheme.

at IP, the transverse crabbing offset becomes

$$\Delta x_c = \frac{(1+\alpha)\theta_c}{k}\sin(kz) - \frac{\alpha\theta_c}{mk}\sin(mkz)$$

here, m is the harmonic number, and α is the the relative strength of the harmonic cavity. Figure 2 illustrates the effect of the harmonic crab cavities based on different fundamental frequencies (112, 140, 168 and 196 MHz) and the harmonic numbers m. The bunch length used in this example is 15cm, as in the eRHIC linac-ring scheme. The high harmonic cavity need to have negative kicking angle in order to linearized the total crabbing kick. It is straightforward that a lower harmonic number is more efficient to make the kick linear, yet need larger strength. From this figure, we learned that the 2^{nd} harmonic cavity requires the ~40% more voltage in fundamental crab cavity; while the 3^{rd} harmonic cavity only need about 15% more. In the other hand, the choice of fundamental frequency is also important for optimizing the luminosity degradation parameter. If we set the luminosity loss to be less than 5% (grid line), the fundamental frequency need to be or less than 140 MHz when using 3^{rd} harmonic cavity, or 112 MHz when using 4^{th} harmonic cavity. Therefore, we selected the 140 MHz cavity with its 3rd harmonic cavity 420 MHz as the crab cavity frequency for the eRHIC linac-ring design. The figure 2 also indicates that the ratio α should set to 0.16.

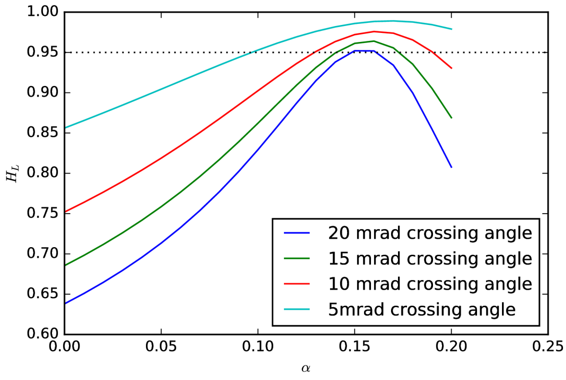

Figure 3: The luminosity degradation parameter as function the harmonic strength α with different crossing angles. The grid line marks 5% luminosity loss due to crab crossing scheme.

We also studied that, if there is small adjustment in the crossing angle or the rms bunch length of the ion beam, how does the luminosity degradation changes with the same frequency choice. Figure 3 shows that the same frequency choice can keep the luminosity loss within 5%, up to 20 mrad. The voltage of both cavities need to scale with the change of the crossing angle from the current 10 mrad.

Figure 4 indicates that the frequency choice will keep the luminosity loss within 5% when the rms bunch length of the ion beam increased to 18 cm (from 15 cm in design). If the bunch length increase to 20 cm, the luminosity loss increase to 7%.

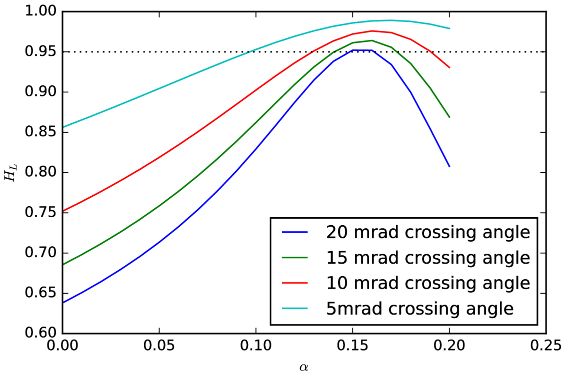

Figure 4: The luminosity degradation parameter as function the harmonic strength α with different rms bunch length of the ion beam. The grid line marks 5% luminosity loss due to crab crossing scheme.

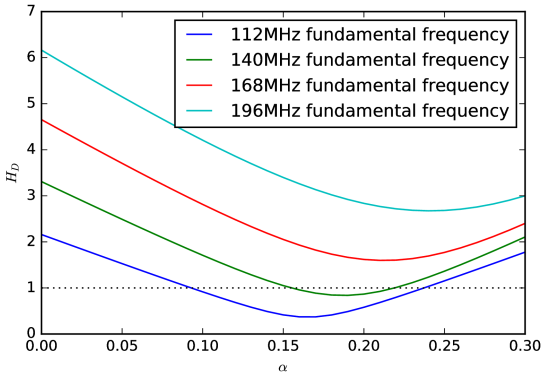

Figure 5: The orbit deviation factor H_D as function of the harmonic ratio α of third harmonic cavities with various fundamental frequencies.

ORBIT DEVIATION CONSIDERATION

In the eRHIC, the ion beam is much longer (50 times) than the electron beam. The collision location of the ion beam at z_1 and electron at z_2 is $s = (z_1 + z_2)/2 \sim z_1/2$. Therefore the orbit deviation of the ion beam determines the dipole kick of the beam-beam force between two beams. We define the orbit deviation factor H_D due to the crab crossing scheme as:

$$H_D = \left[\int_{-\infty}^{\infty} (z\theta_c - \Delta x_c(z))^2 \rho(z)/\sigma_x^2(s = z/2)dz\right]^{1/2}$$

where $\rho(z)$ is the longitudinal distribution of the ion bunch and $\sigma_x(s)$ is the transverse beam size with hourglass effect. The factor is an average of the orbit deviation normalized by the transverse beam size at collision point.

There is no well-defined limit for the orbit deviation factor H_D yet. We planned detailed simulation to study the dynamic aperture (DA) and incoherent beam degradation of the ion beam and the electron beam in both ring-ring and linac-ring scheme of eRHIC. An initial results of DA study can be found in [3].

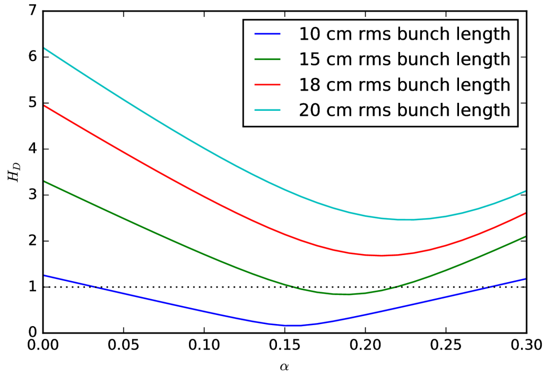

Figure 6: The orbit deviation factor H_D as function of the harmonic ratio α of third harmonic cavities with different bunch length.

In this paper, we temporarily set the limit to 1, as shown in the gridline of figure 5 and 6, which implies that the average orbit deviation should be less than the beam size. Figure 5 supports our frequency choice, which is 140MHz fundamental frequency with its 3rd harmonic cavity. The optimum ratio is about 0.18, very close to the optimum value 0.16 in the luminosity study. Figure 6 shows that the factor H_D is sensitive to the bunch length. Simulation is required to study the adverse effect of the longer bunch.

SUMMARY

We summarized the study of determining the parameter of the crab cavity using two figure of merit, the luminosity degradation factor H_L and the orbit deviation factor H_D. Both parameters indicated the similar frequency choice, 140 MHz and its 3rd harmonic cavity, of the eRHIC linac-ring scheme under our assumption $H_L > 0.95$ and $H_D < 1$. For ring-ring design of eRHIC, an even harmonic of 28 MHz (168 MHz) is preferred due to the future upgradability of doubling the bunch number. Therefore, 168 MHz cavity with its 3rd harmonic cavity are chosen with relaxed constrain of H_L and H_D. The detailed simulation is planned to validate our choices with the DA study and coherent/incoherent effect studies with the self-consistent beam-beam simulations.

REFERENCES

[1] E.C. Aschenauer *et al.*, “eRHIC design study: an electron-ion collider at BNL”, arXiv:1409.1633v2, Dec 2014.

[2] Y-P. Sun *et al.*, “Beam dynamics aspects of crab cavities in the CERN Large Hadron Collider”, in *Phys. Rev. ST Accel. Beams*, vol. 12, p. 101002, 2009.

[3] Y. Luo *et. al.*, “Beam-beam simulation with crab cavities for eRHIC”, presented at IPAC’16, Busan, Korea, May 2016, paper WEPMW026, this conference.