COMPARISON OF TRACKING CODES FOR THE DETERMINATION OF DYNAMIC APERTURE IN STORAGE RINGS *

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Abstract

Currently there is a great deal of activity towards making precision measurements utilizing storage rings, for example the Muon g-2 experiment at Fermilab [1], and the Electric Dipole Moment (EDM) program of the JEDI Collaboration [2]. These experiments are expected to perform measurements requiring sub-ppm precision. Of utmost importance in this regard is the capability of tracking codes to treat all nonlinear effects arising from the detailed field distributions present in the system, not the least of which are fringe fields.

In previous work [3, 4], we performed parallel tests of various tracking codes in order to compare and contrast the results. In this study, we continue this line of research and extend the scope to dipoles with edge angles.

DIPOLES WITH EDGE ANGLES

A common replacement for bending magnets of the sector variety are dipoles with edge angles. These differ from traditional sector magnets in that the reference orbit enters and exits at an angle with respect to the face of the magnet (Fig. 1). This difference results in a change in the optics,

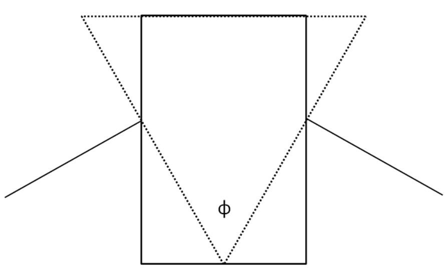

Figure 1: Parallel-faced dipole. A corresponding sector magnet with identical deflection is shown for reference.

especially with respect to fringe fields. For example, in the case of a parallel-faced dipole, the transfer map must be independent of the original horizontal position x_i since the motion is invariant with respect to a change in this direction. This implies that many matrix elements vanish.

On the other hand, under the presence of fringe fields, the motion depends on the vertical position y_i, and several new matrix elements appear. We will show that fringe fields in conjunction with edge angles dramatically limit the dynamic aperture.

We chose to conduct this study using the codes COSY INFINITY v9.1 [5], ZGOUBI v6.0.1 [6], MAD8 v8.51 [7] and GICOSY [8]. The tracking lattice used consisted of four 90 degree dipoles of radius 10 m, each having entrance and exit angles of 22.5 degrees. The dipoles have a 10 cm full aperture and are separated by 5 m drifts.

Implementing this lattice into the aforementioned codes, we find the tunes given in Table 1:

Table 1: y and x Tunes of the Test Lattice as Calculated by Various Codes

Code	Tune y	Tune x
COSY INF. (no fringe fields)	.8655516	.8671905
COSY INF. (with fringe fields)	.8534879	.86714278
ZGOUBI (no fringe fields)	.8657291	.8671926
ZGOUBI (with fringe fields)	.8319596	.8681889
GICOSY (fringe)	.8534629	.8671602
MAD8 (fringe)	.9170898	.9433354

For those codes that support nonlinear transfer map elements, a comparison of map coefficients with and without fringe fields is given in Table 2. The nonlinear coefficients reported by ZGOUBI are not in line with expectations, and we are currently investigating this issue.

When fringe fields are taken into account, these coefficients are dependent upon the aperture of the elements. Starting with the initial aperture of 10 cm, we can reduce the aperture and watch the behavior of the various coefficients (Fig. 2). Moving left to right along the horizontal axis, we reduce the relative aperture by factors of two. The plotted coefficients do not tend to zero, on the contrary, they approach a fixed value asymptotically.

The effect on the dynamics of the fringe fields can be clearly demonstrated by comparing the tracking pictures without fringe fields (Fig. 3) and with fringe fields (Fig. 4). The orbits are at 1 cm increments from the reference orbit, out to 30 cm. The limitation of the dynamic aperture due solely to effect of fringe fields is readily apparent.

ZGOUBI can also do tracking with fringe fields; these are specified using the same Enge coefficients [9] as those used by COSY INFINITY (Fig. 5). Because of the non-symplectic behavior of the tracking, the y-b phase space orbits are seen to rapidly decay, making it impossible to see any of the effects due to the fringe fields. For comparison we can generate a tracking picture with COSY INFINITY running at an artificially low third order, and with symplec-

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Table 2: Higher Order Coefficients for the Test Lattice by the Various Codes, With and Without Fringe Fields

	COSY INF. (w/o ff)	COSY INF. (w/ ff)	GICOSY (w/ ff)	ZGOUBI (w/ ff)
(x|xx)	-.02388155	-.02388975	-.02388974	-1.09
(x|xa)	-.2363646	-.2361127	-.2361128	11.4
(x|aa)	2.428572	2.427770	.4711359	-77.9
(x|xxx)	-.001413245	-.001415586	-.001416513	N/A
(x|xxa)	.02259853	-.02259955	.02259934	N/A
(x|xaa)	-.4062936	.01012976	-.4069460	N/A
(x|yy)	-.003902462	.02357621	.02357620	.784
(x|yb)	-.2027640	.8139067	.8139072	-38.2
(x|bb)	-.2202186	-14.42333	-14.42333	1052

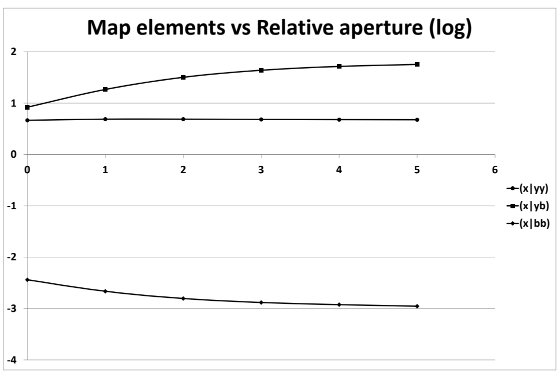

Figure 2: The effect reducing the dipole aperture on the non-linear coefficients. Moving left to right along the horizontal axis decreases the aperture by the indicated power of two. The coefficients do not go to 0 as the aperture is decreased, rather they approach a fixed value asymptotically.

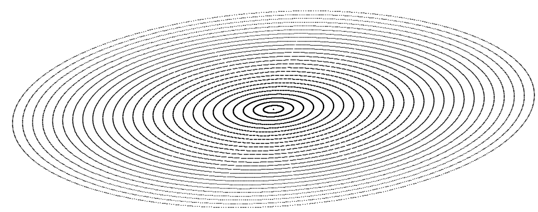

Figure 3: COSY INFINITY y-b tracking in a simple test lattice composed of four 90 degree sector magnets with 22.5 degree edge angles (ignoring fringe fields). The orbits are separated by 1 cm. There is no apparent limit to the dynamic aperture.

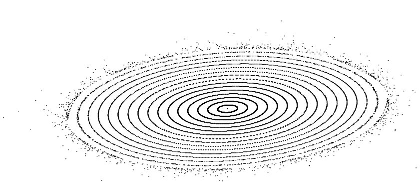

Figure 4: COSY INFINITY y-b tracking in the same test lattice, this time with fringe fields enabled. The severe limitation in the dynamic aperture is due to fringe fields is readily apparent.

tic tracking disabled (Fig. 6). Even with these restrictions, the deterioration of dynamic aperture due to fringe fields is clear, although the details of the dynamics are not faithfully represented.

The results of tracking in MAD8 to determine the effect of fringe fields on dynamic aperture were negative. MAD8 does not use Enge coefficients to model the fringe fields; rather, it uses the fringe integral. Calculating the fringe integral for the fringe fields used in the above tests for COSY INFINITY and ZGOUBI, we find the value to be .369458. The results of MAD8 tracking throught the test lattice using this value of the fringe integral is shown in Fig. 7.

As a final note, we attempted to run similar tests for MAD-X 5.02.00 (64-bit linux) [10], but were unable to change the edge angles in this version of the code. We are currently looking into this issue.

CONCLUSION

The vertical aperture is limited exclusively by the nonlinear fringe field effects, which introduce non-vertical field components. Ignoring these effects (as is done in MAD), or computing them only approximately (as in ZGOUBI, or by forcing COSY INFINITY to run at unusually low order), yields inaccurate results.

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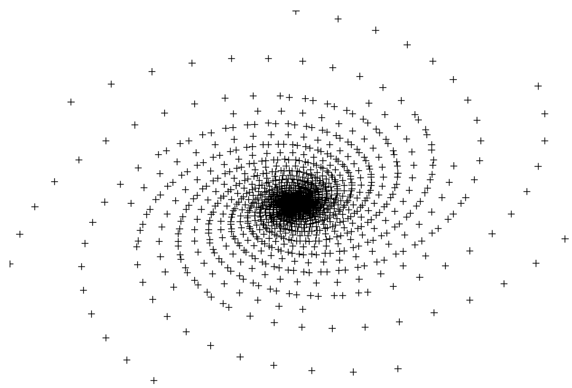

Figure 5: ZGOUBI y-b tracking of the test lattice. The tracking pictures are dominated by the nonsymplectic behavior, and do not show an accurate picture of the dynamic aperture.

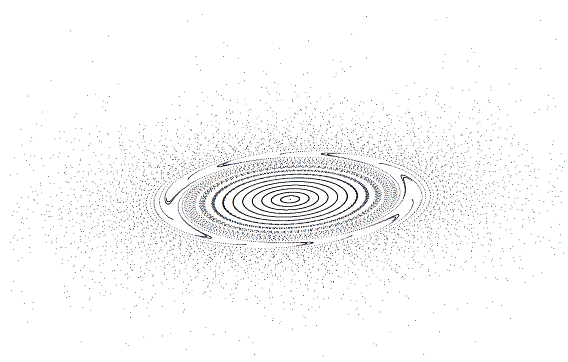

Figure 6: For comparison with ZGOUBI, the test lattice was again tracked by COSY INFINITY, this time with symplectic tracking disabled, and the order artificially held down to third order. Even with these limitations, the restricted dynamic aperture is readily apparent.

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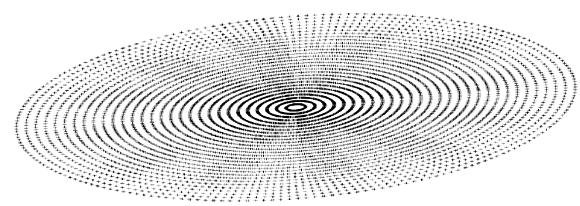

Figure 7: MAD8 has limited support for the specification of fringe fields. Rather than using the Enge coefficients, it uses a single fringe integral. Again tracking the test lattice in the y-b direction in 1 cm steps, we find no evidence of the reduction in dynamic aperture caused by fringe fields.