

# SIMULATION OF fs BUNCH LENGTH DETERMINATION WITH THE 3-PHASE METHOD AND THz DIELECTRIC-LOADED WAVEGUIDES

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## Abstract

In this paper, we investigate with ASTRA simulations the capability of the 3-phase method to reconstruct the length of a fs electron bunch. We show that a standard 3 GHz travelling wave accelerating structure is not suited for this purpose, because of the too important effect of the space-charge forces and of the too small variations of the induced energy spread with the bunch injection phase. Our simulations demonstrate that the use of dielectric-loaded waveguides driven by THz pulses would allow overcoming these two limitations and possibly achieving an ultimate resolution better than 5% for the determination of a 6.25 fs rms bunch length at 100 MeV energy and 1 pC charge. The next steps of the study to better evaluate, in simulations and experiments, the possible sources of degradation of the 3-phase method resolution are also mentioned.

## INTRODUCTION

The measurement of electron bunch lengths below 100 fs rms, required in many applications (free electron laser, ultrafast imaging, etc.), is very challenging and involves technically complicated methods and devices like the transverse deflecting cavities [1, 2] and the electro-optical sampling [3].

The 3-phase method is a conceptually simple indirect diagnostics which allows determining the bunch length through the measurement of the variation of the energy spread with the injection phase in an accelerating structure (linac) [4, 5]. This is done by inverting, with a least-square algorithm, the following equation [4, 5]:

$$\sigma_{E_f}^2 = 4\pi^2 f^2 e^2 L^2 E_0^2 \sin^2(\varphi) \sigma_{t_i}^2 + 4\pi f e L E_0 \sin(\varphi) \sigma_{E_{t_i}} + \sigma_{E_i}^2$$

where  $f$  and  $E_0$  are the frequency and amplitude of the field in the travelling wave accelerating structure,  $L$  its length,  $\varphi$  the injection phase of the bunch and  $e$  the elementary charge.  $\sigma_{t_i}$ ,  $\sigma_{E_{t_i}}$  and  $\sigma_{E_i}$  are respectively the bunch length, chirp and energy spread at the entrance of the accelerating structure. One can see that to determine them, at least three measurements of  $\sigma_{E_f}$  (energy spread at the exit of the travelling wave accelerating structure) for three different values of  $\varphi$  are required, hence the name of the method. In this paper, we will only detail the measurement of the bunch length  $\sigma_{t_i}$ .

In this paper, we investigate with ASTRA simulations [6] the capability of the 3-phase method to reconstruct the length of a fs electron bunch produced by the ARES linac [7] (see Table 1 for the simulated bunch properties). In the

first part, we introduce the motivation to apply the 3-phase method with THz dielectric-loaded waveguides. In the second part, we investigate the capability of a 3 GHz travelling wave accelerating structure. In the third part, the capabilities of THz dielectric-loaded waveguides are investigated.

Table 1: Input Bunch Properties Used for the Simulations

Bunch property	Value
Charge	1 pC
Mean kinetic energy	93.52 MeV
Rms energy spread	68.03 keV
Rms length	1.875 $\mu\text{m}$ ( $\equiv$ 6.25 fs)
Rms horizontal size	104.3 $\mu\text{m}$
Rms vertical size	104.3 $\mu\text{m}$
Rms horizontal emittance	0.129 $\pi\cdot\text{mm}\cdot\text{mrad}$
Rms vertical emittance	0.130 $\pi\cdot\text{mm}\cdot\text{mrad}$

## MOTIVATION

The resolution of the 3-phase method is limited by two main phenomena. First, for a given combination of frequency and amplitude of the accelerating field, the variations of the bunch energy spread with the injection phase in the structure become too small to be measurable when the bunch is too short. The resolution of the spectrometer used to measure the bunch energy spread is therefore a key parameter for defining the resolution of the 3-phase method. In our study, we consider that it is of  $10^{-4}$ , namely 10 keV at a bunch mean kinetic energy of 100 MeV, and corresponds to the minimal measurable variation of energy spread. Second, the space-charge forces (SCF) also induce a variation of the energy spread in the accelerating structure and in the drift space up to the spectrometer measuring it. This is not taken into account in the analytical model on which the 3-phase method relies and is a disturbance to the measurement. This effect becomes more significant for higher bunch currents. The compactness of the system “accelerating structure + spectrometer” is hence of primary importance for optimizing the resolution of the 3-phase method.

The two aforementioned reasons limit the resolution of the 3-phase method with standard accelerating structures (having few GHz frequencies, few tens of MV/m field amplitudes and lengths  $\geq$  1 m) above 100 fs rms [4, 5]. The use of few cm long dielectric-loaded waveguides driven by high frequencies ( $>$  100 GHz) and amplitudes ( $>$  100 MV/m) accelerating fields [8] would allow overcoming these limitations and achieving fs, and even sub-fs, bunch length measurement.

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## CASE OF A 3 GHz TRAVELLING WAVE ACCELERATING STRUCTURE

Figure 1 shows the bunch rms energy spread, as a function of the injection phase, at the exit of a 4.1 m long 3 GHz accelerating structure as used on the ARES linac [7] and at the exit of a similar structure but being only 0.9 m long. Both cases have been simulated with and without taking into account the space-charge forces.

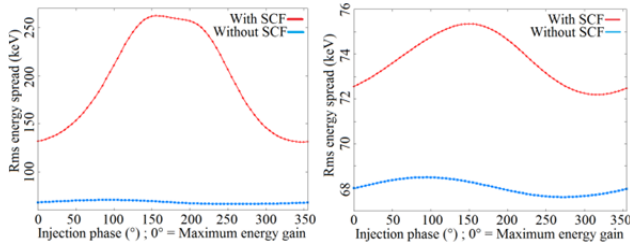


Figure 1: Bunch rms energy spread, as a function of the injection phase, at the exit of 4.1 m long (left) and 0.9 m long (right) 3 GHz accelerating structures with 18 MV/m field amplitude.

The left plot of Fig. 1 shows that, in the case of the 4.1 m long structure used on the ARES linac, the energy spread pattern as a function of the injection phase obtained at its exit is completely dominated by the space-charge forces, in the sense that the amplitude of the curve, the position of the maximum and the absolute value of the energy spread are significantly shifted respective to the case without space-charge forces. This fact totally prevents applying the 3-phase method. Indeed, applying the 3-phase on the red curve of the left plot of Fig. 1 gives an imaginary value for  $\sigma_{ti}$ . The right plot of Fig.1, obtained with a 0.9 m long structure, shows that the effect of space-charge forces can be greatly attenuated, although still significant, by using shorter structures. However, with this shorter structure, the variation of the energy spread with the injection phase is only of 3 keV at maximum. This is well below the resolution of 10 keV we assume and is therefore non measurable experimentally, which also prevents applying the 3-phase method. Figure 1 therefore clearly demonstrates the impossibility to use a 3 GHz accelerating structure (short or long) for performing diagnostics of the fs bunch length shown in Table 1 with the 3-phase method.

## CASE OF THz DIELECTRIC-LOADED WAVEGUIDES

The impossibility to use standard accelerating structures as bunch length diagnostics with the 3-phase method leads to considering using much shorter THz dielectric-loaded waveguides, with higher frequencies and field amplitudes, to overcome the limitations of space-charge forces and too small energy spread variations with the injection phase previously demonstrated.

Figure 2 shows the bunch rms energy spread, as a function of the injection phase, at the exit of a 6.4 cm long dielectric-loaded waveguide simulated by ASTRA [9]. It is visible in the left plot that the effect of space-charge

forces on the energy spread pattern as a function of the injection phase is much weaker than with 3 GHz structures, due to the much shorter length of the THz structure. Moreover the amplitude of the energy spread variations with the injection phase is around 100 keV, due to the higher frequency and field amplitude, which is totally measurable experimentally assuming a 10 keV resolution. The same simulations have been performed for several field amplitudes in the waveguide (see the right plot of Fig. 2).

To apply the 3-phase method on the data, we choose to retain only the successive points for which the energy spread varies by at least the assumed resolution of 10 keV for the spectrometer measuring it (see for example the points in black circles in the left plot of Fig. 2). The results (reconstructed  $\sigma_{ti}$  and number of points which have been retained for analysis) are shown in Table 2. It is visible that the 3-phase method allows reconstructing almost perfectly the 6.25 fs input bunch length (discrepancy  $< 0.5\%$ ) using a 6.4 cm long dielectric-loaded waveguide driven by a 300 GHz field. Table 2 also shows that this is true for field amplitudes down to 50 MV/m, where the variations of the energy spread with the injection phase are still large enough to retain more than 3 points (minimum for the 3-phase method) according to the assumed 10 keV spectrometer resolution.

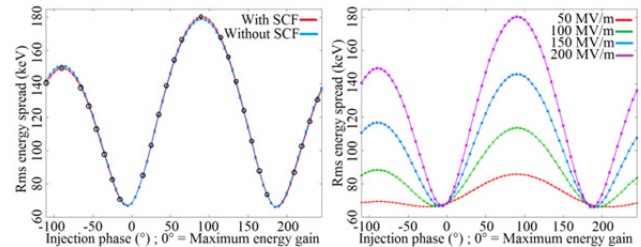


Figure 2: Bunch rms energy spread, as a function of the injection phase, at the exit of a 6.4 cm long dielectric loaded waveguide. Left plot: 300 GHz frequency and 200 MV/m amplitude (with and without space-charge forces). Right plot: 300 GHz frequency for several amplitudes (space-charge forces included).

Table 2: Reconstructed bunch length applying the 3-phase method on the data shown in Fig. 2.

Field amplitude	Number of points used	Reconstructed $\sigma_{ti}$
200 MV/m	26	6.24 fs rms
150 MV/m	21	6.23 fs rms
100 MV/m	10	6.22 fs rms
50 MV/m	4	6.22 fs rms

The data shown in Fig. 2 and reconstructed  $\sigma_{ti}$  shown in Table 2 demonstrate the interesting potential of THz dielectric-loaded waveguide for diagnostics of fs bunch length using the 3-phase method. However, in this case, the spectrometer measuring the energy spread is assumed to be right at the exit of the accelerating structure, which is experimentally difficult. To give a better evaluation of

the potential of the 3-phase method with THz dielectric-loaded waveguides, we perform the same simulation as in Fig. 2 but this time adding drift spaces between the exit of the waveguide and the spectrometer. Then we apply the 3-phase method on the obtained data, following the protocol previously described to decide which points to retain. The results are shown in Fig. 3.

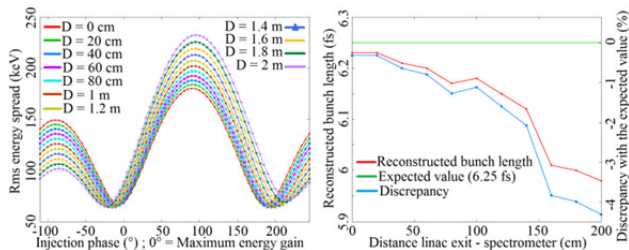


Figure 3: 300 GHz frequency and 200 MV/m amplitude linac. Left plot: Bunch rms energy spread as a function of the injection phase at several distances D from the linac exit. Right plot: Reconstructed bunch length with the 3-phase method as a function of D.

It is visible in Fig. 3 (left plot) that the effect of space-charge forces on the energy spread pattern as a function of the injection phase increases as expected with the distance of the spectrometer from the linac exit. However, compared to standard accelerating structures (see Fig. 2), the effect is significantly weaker, especially in terms of displacement of the injection phases where the curve extrema are located. As a result, as it visible in Fig. 3 (right plot), the reconstructed  $\sigma_{ti}$  with the 3-phase method remains very close to the 6.25 fs of the input bunch in a large range of distances from the linac exit. Indeed, for a spectrometer located at 2 m from the linac exit, the discrepancy is only of 4.3%. This demonstrates the possibility to place the spectrometer at an experimentally realistic distance from the linac exit without implying a big deterioration of the resolution of the 3-phase method due to space-charge forces.

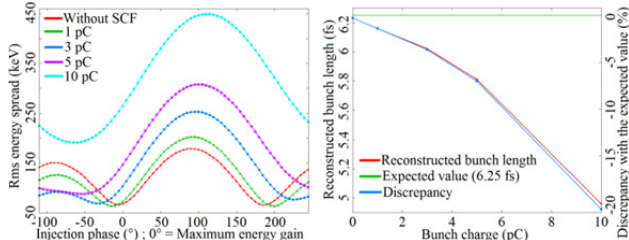


Figure 4: 300 GHz frequency and 200 MV/m amplitude linac, spectrometer located 1 m after the linac exit. Left plot: Bunch rms energy spread as a function of the injection phase for several bunch charges. Right plot: Reconstructed bunch length with the 3-phase method as a function of the bunch charge.

The last study we present in this article is a study as a function of the bunch charge. We chose to fix the distance of the spectrometer from the linac exit to 1 m, and we performed for several bunch charges the same simulations as previously shown. The results of the study are shown in Fig. 4. As our goal is only to study the impact of the

bunch charge on the resolution of the 3-phase method, we just vary it artificially at the exit of the linac. We do not perform again a complete simulation starting from the photocathode and all other bunch properties remain as in Table 1. Especially, we do not demonstrate that it is possible to create on the ARES linac a bunch with more than 1 pC charge and having the properties shown in Table 1.

Figure 4 shows that the resolution of the 3-phase method is rapidly deteriorated by the space-charge forces when the bunch charge increases. In fact, the discrepancy with the expected value becomes higher than 20% above 10 pC bunch charge.

One has to note that the values presented in this section are the ultimate resolution of the 3-phase method in perfect experimental conditions. In reality the resolution will be worse because: the number of measured energy spreads could be lower than the one retained in this study (> 20); Uncertainty on and jitter of the injection phase and field amplitude will be present; transverse focusing due to optics between the linac exit and the spectrometer will increase the effect of space-charge forces; incoherent and coherent synchrotron radiation (CSR) [10] in the spectrometer could affect the bunch energy spread. The impact of all this phenomena on the resolution of the 3-phase method has to be evaluated in simulations and experimentally.

## CONCLUSIONS AND PROSPECTS

Our simulations show that the space-charge forces and the too small variation of the bunch energy spread with the injection phase totally prevent using the 3-phase method with standard RF accelerating structures for measuring fs bunch lengths. We demonstrate that the use of dielectric-loaded waveguides driven by THz pulses would allow overcoming these two limitations, thanks to their much shorter lengths (a few cm against  $\geq 1$  m), higher frequencies and field amplitudes. We especially show that the degradation of the 3-phase method resolution with the increase of the distance between the linac exit and the spectrometer measuring the bunch energy spread, because of space-charge forces, can be kept under control. Indeed, for a distance of 2 m, it is of only 5% for a 100 MeV, 1 pC and 6.25 fs rms bunch.

This ultimate resolution could be degraded due to several effects which have to be carefully studied in future simulations and experiments (expected for end 2018 or beginning 2019 on ARES): uncertainty on and jitter of field amplitude and injection phase in the linac; perturbation of the energy spread measurement due to CSR in the spectrometer; increase of space-charge forces due to transverse focusing of the bunch between the linac exit and the spectrometer.

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