

A TRANSVERSE DEFLECTION STRUCTURE WITH DIELECTRIC-LINED WAVEGUIDES IN THE SUB-THz REGIME*

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Abstract

Longitudinal bunch measurements are typically done with rf-powered transverse deflection structures with operating frequencies 1–12 GHz. We explore the use of mm-scale, THz-driven, dielectric-lined cylindrical waveguides as transverse deflectors by driving the fundamental deflecting mode of the structure, the HEM_{11} . We give a brief overview of the physics, history, and provide an example with a 5 MeV beam using ASTRA and CST-MWS.

INTRODUCTION

A full temporal characterization of electron bunches is a prerequisite for the performance optimization of bunch compression schemes and the application of ultrashort electron bunches. Transverse deflecting structures (TDS) impart a transverse streak onto a bunch, i.e. a transverse momentum which is - within limits - linearly correlated to the longitudinal position and which thus allows imaging of the temporal profile on a downstream screen. If the screen is located in a dispersive section even the complete longitudinal phase-space can be imaged. Transverse streaking is the method of choice for ultrashort beams applications for its conceptual simplicity, high resolution, and robustness.

The streaking action S of a time varying field scales with the product of the integrated effective transverse voltage V with the wavenumber k of the deflecting field as $S = ekV/cp_z$ with the elementary charge e , the velocity of light c and the longitudinal momentum of the electrons p_z . The effective voltage is given as integral over the structure length $V = \int E_y(t, z) + c\beta_z B_x(t, z) dz$ where $E_y(t, z)$ and $B_x(t, z)$ are the transverse electric and magnetic field components and $c\beta_z$ is the longitudinal velocity component of the electrons passing through the structure.

Higher frequency structures have by definition higher wavenumbers k but they also promise high field gradients which makes them interesting candidates for transverse deflecting structures. However, the image formed on the screen downstream of a TDS is a convolution of the streaked beam with a distribution that would be formed without the streaking action. For optimized conditions, i.e. a phase advance of 90° between TDS and screen, the resolution is given by the ratio of the uncorrelated beam divergence in the structure ε/σ_y to the streaking action S , i.e. the imparted divergence

per unit length. Here ε stands for the geometrical emittance and σ_y for the transverse rms beam size - in direction of the streak - in the TDS structure. The resolution of a TDS setup reads hence:

$$R = \frac{\varepsilon}{\sigma_y S} = \frac{\varepsilon c p_z}{\sigma_y e k V} = \frac{\varepsilon_n m_0 c^2}{\sigma_y e k V}, \quad (1)$$

with the normalized emittance ε_n and the rest mass of electrons m_0 .

For $\beta_z \approx 1$ the resolution is independent of the beam energy, while for $\beta_z < 1$ the velocity dependence of the effective voltage becomes relevant. A high resolution power R^{-1} requires not only a large streaking action but also a large beam size inside the structure.

The transverse dimensions, i.e. the aperture radius a , of a certain type of transverse deflecting structure scales as the wavelength. If we would scale the beam size as the aperture radius, so that the ratio of beam size to aperture stays constant, the resolution (1) would become independent of the wavenumber k and thus we would lose an advantage higher frequencies promise.

In practice the beam size in a TDS is not a completely free parameter, but it is determined by the conditions in neighboring sections and the limits of intermediate matching sections. For TDS' operating at typical rf frequencies, e.g. 3 GHz, the achievable beam size is often significantly lower than the TDS would allow. On the contrary it appears to be mandatory to design TDS' working in the THz range in a way that they can operate with sufficiently large beam sizes to take full advantage of the higher frequencies.

For the design of a THz TDS two aspects are thus important. On the one hand a large ratio of aperture radius to wavelength in combination with a high transverse shunt impedance, on the other hand a homogeneous deflecting field, so that the beam can occupy a large fraction of the free aperture without distortion of the beam quality.

Dielectric-lined waveguides (DLW) have already been discussed in the 1970s as candidates for beam separators [1], at that time for conventional rf frequencies. Circular waveguides [2] support hybrid HEM_{11} modes [3] which yield very linear deflecting fields within the aperture of the structure and thus are ideal candidates also for high resolution bunch length measurements in the THz range. We note that recently other alternative deflection concepts have risen [4–10] but have more limited resolutions due to field non-linearities and small interaction lengths. In the follow-

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ing we discuss a parameter set for a DLW which supports a deflecting mode at 300 GHz (e.g. 1 mm).

HEM₁₁ DEFLECTOR

In this section we briefly review the main points from [1,2] and scale their work to mm-scale structures. Consider a DLW with inner radius a , outer radius b , and dielectric permittivity ϵ_r (a, b, ϵ_r). The fundamental mode of the structure is the HEM_{11} . For a $\theta = \pi/2$ polarized mode, we have the following fields

$$\begin{aligned} E_z &= k_1^2 A_{11} \frac{E_0}{i\omega\epsilon_0} I_1(k_1 r) \sin(\psi) \\ E_r &= \left[\frac{k_1 A_{11} Z_0}{2v_p} (I_0(k_1 r) + I_2(k_1 r)) - \frac{B_{11}}{r} I_1(k_1 r) \right] \cos(\psi) \\ H_\phi &= \left[\frac{A_{11} k_1}{2} (I_0(k_1 r) + I_2(k_1 r)) - \frac{B_{11}}{v_p Z_0 r} I_1(k_1 r) \right] \cos(\psi). \end{aligned} \quad (2)$$

Here A_{11} and B_{11} are the field amplitude coefficients, v_p is the normalized phase velocity, Z_0 is the impedance of free space, I_n is the modified Bessel function of the first kind, $k_1 = \omega \sqrt{\frac{1}{v_p^2} - \frac{1}{c^2}}$, $k_2 = \omega \sqrt{\frac{\epsilon_r}{c^2} - \frac{1}{v_p^2}}$, $k_z = \frac{\omega}{v_p}$, and $\psi = \omega t - k_z z$.

From [2], and after some algebra, the transverse force acting on a synchronous ($v_p = \beta$) particle can be expressed as

$$F_y = q A_{11} Z_0 k_1 \frac{1}{\gamma \beta} \left\{ \frac{I_1(k_1 r)}{k_1 r} + I_2(k_1 r) \right\} \cos(\psi). \quad (3)$$

In the ultra-relativistic limit ($k_1 \rightarrow 0$), things simplify considerably to

$$F_y \propto eP \cos(\psi), \quad (4)$$

where P is the input power.

A remarkable feature of DLWs is the linearity of the fields for modes with $k_1 = 0$ ($v_p = 1$); this is essential for the transverse independence of Eq. (4). We note that very similarly, the TM mode in a DLW has no transverse dependence for $k_1 = 0$.

As an example we consider a DLW with length L with dimensions (a, b, ϵ_r, L)=(0.3 mm, 0.398 mm, 4.41, 3 cm); the dielectric thickness was determined by solving the dispersion relation for a 5 MeV electron bunch ($v_p = .995c$); see [2], and Fig. 1 for dispersion diagram.

TUNING

Ensuring the synchronous condition above requires a precise knowledge about the structure and THz source; accommodating any potential manufacturing imperfections is wise, and a short discussion on tuning is in order. From visual inspection (Fig. 1), it is also viable to change v_p via the

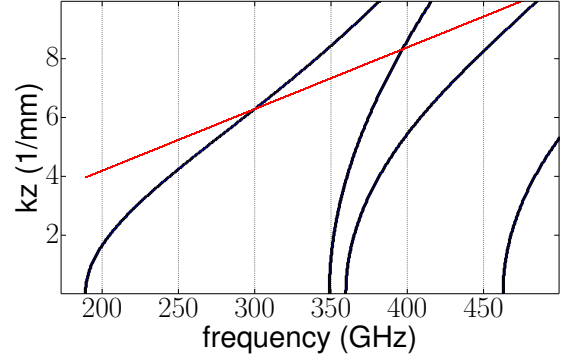


Figure 1: Dispersion curve for the HEM_{1n} modes in a Quartz DLW with dimensions (a, b, ϵ_r)=(0.3 mm, 0.398 mm, 4.41); the structure is matched to a phase velocity $v_p = .995c$ i.e. for a 5 MeV electron bunch. The speed of light is shown for reference as the red trace.

frequency. We can explore this analytically too, consider the regime far from cutoff where $v_g = c/n = c/\sqrt{\epsilon_r}$

$$k_z = \frac{\partial k}{\partial \omega} \omega + k_z^0, \quad (5)$$

and let us consider the example from Fig. 1, where $k_z^0 = \frac{\omega_0}{c} = \frac{2\pi 300 \text{ GHz}}{c}$, then,

$$k_z = \frac{\omega}{v_g} + \frac{\omega_0}{c} \left(1 - \frac{c}{v_g}\right), \quad (6)$$

introducing a dimensionless scaling factor $\omega = \alpha \omega_0$, simplifying and reorganizing leads to

$$k_z = \frac{\omega_0}{c} (n(\alpha - 1) + 1). \quad (7)$$

The phase velocity is given by

$$v_p = \frac{\omega}{k_z} = \frac{\alpha c}{n(\alpha - 1) + 1}, \quad (8)$$

finally, introducing the normalized phase velocity for simplicity $\beta_p = \frac{v_p}{c}$, we can solve for α in terms of β_p ; after some algebra, the simple result is given by

$$\alpha = \frac{n - 1}{n - 1/\beta_p}. \quad (9)$$

As an example, a phase velocity change of $0.01c$ (e.g. from $v_p = c$ to $v_p = .99c$) can be achieved by a frequency change of $\alpha = 1.009$; i.e. a frequency shift from $f = 300$ GHz to $f = 302.7$ GHz. A tunable laser-based THz source meeting these requirements has been demonstrated in [11].

SIMULATION

We used a finite-difference time-domain solver, CST-MWS [12] to verify the fields; simulation results are shown in Fig. 2 for the structure described above, here Fig. 2(a) shows the $E_z(y, z)$ in log-scale, Fig. 2(b) shows $E_z(x, y)$ also

in log-scale, and finally Fig. 2(c) shows $E_z(y)$ at a position $z=1$ cm into the structure. The linearity of E_z is especially relevant for a linear streaking force on an electron bunch and also implies $k_1 \sim 0$.

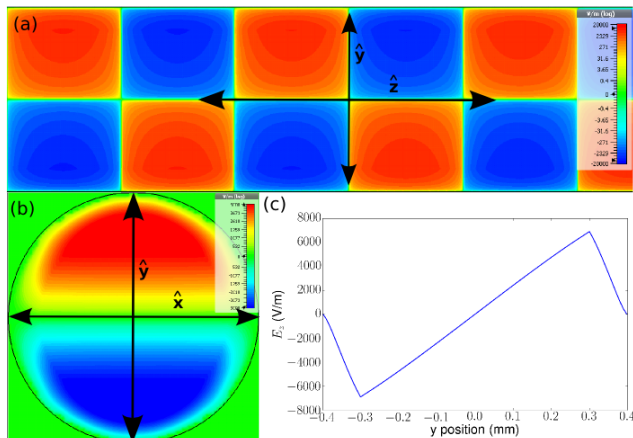


Figure 2: CST-MWS simulation results for a structure with dimensions $(a, b, \epsilon_r, L)=(0.3 \text{ mm}, 0.398 \text{ mm}, 4.41, 3 \text{ cm})$. (a) shows the contour plot of $E_z(y, z)$ in log-scale, (b) shows $E_z(x, y)$ – a cross-section, also in log-scale and (c) shows $E_z(y)$ at a position $z=1$ cm into the structure. The thickness of the structure was calculated to yield a synchronous condition between v_p and β of the electron bunch at 5 MeV. $k_1 \sim 0$ which makes the fields very linear for possible applications like e.g. streaking.

We used CST-MWS to output a 3D field map for beam dynamics simulations in ASTRA [13]. In our CST-MWS simulation, we used a 1% bandwidth (e.g. 100 cycle) 300 GHz pulse with 1.35 kW input power; only 1.08 kW (80%) propagated through the structure. From Fig. 1, the group velocity of the HEM_{11} mode is $v_g \sim .52c$.

We note that the number of cycles N for a wavelength λ needed for a structure of length L is given by $N = \frac{L}{\lambda} (\frac{c}{v_g} - 1)$, therefore only an $N \sim 30$ cycle pulse is required; however it should be noted that with shorter pulse lengths, the appropriate structure should be designed with considerations on mode coupling and dispersive effects, in our example the HEM_{12} cutoff is ~ 350 GHz which alleviates any issues. Here, the total pulse energy required for the deflection is less than a μJ .

SIMULATION IN ASTRA

In order to check the beam dynamics a $10 \mu\text{m}$ long (rms) 5 MeV electron beam with zero initial emittance and zero energy spread was tracked through the THz structure. The integrated voltage of the structure is only 9.2 kV, which is mainly provided by the electric field component. As the analysis of the fields presented above already suggested, the structure provides a linear streak over the aperture and thus behaves nearly as a theoretical structure. The induced energy spread follows the Panofsky-Wenzel theorem and the induced bunch lengthening through the 3 cm long structure

is negligible. The streaking action leads - in the direction of the streak - to an emittance growth which is given by $\epsilon_{n,y}^{\text{lin}} = \frac{eV k}{m_0 c^2} \sigma_z \sigma_y$. Field non-linearities would lead to an additional emittance growth which scales with a high power of the transverse beam size [14]. However, for the present structure no additional emittance growth could be detected within the accuracy of the simulations. The largest transverse rms beam size in the simulation was $125 \mu\text{m}$, corresponding to a radius of $250 \mu\text{m}$ (flat top distribution), thus leaving only $50 \mu\text{m}$ to the inner aperture of the waveguide. Even in this extreme case the simulated result matched the theoretical emittance to within 0.5%.

A comparison of the structure parameters with a conventional S-band structure is instructive. In [14] an S-band structure for a 5 MeV beam is discussed which provides an integrated voltage of 170 kV at 10 cm wavelength. The free aperture radius of the structure is 1 cm, i.e. 1/10th of the wavelength. This large aperture is, however, not really usable for beam operation, thus a higher content of non-linear field components has been accepted in the design in favor of a higher transverse shunt impedance. The field is thus only linear up to a radius of about 0.4 mm. The free aperture of the DLW structure is 1/3rd of the wavelength and the field is highly linear. Since the achievable resolution is inversely proportional to the product of beam size, wavenumber and integrated voltage, cf. Eq. (1), which yields for the S-band structure 4.3 kV and for the THz structure 7.2 kV, the THz structure yields even at the low voltage assumed here a nearly two times better resolution as the S-band structure.

Finally a touch on the transverse shunt impedance, given by $Z_{\perp} = \frac{|V_{\perp}|^2}{P}$ – where V_{\perp} is the transverse integrated voltage, and P is the input power – is a figure of merit for transverse deflection structures. From the S-band structure described above, $Z_{\perp} = 36.7 \frac{\text{M}\Omega}{\text{m}}$, for a similar frequency range based on a slab-symmetric DLW [15], a very similar result is obtained $Z_{\perp} \sim 10\text{--}100 \frac{\text{M}\Omega}{\text{m}}$ for different parameters. In our regime, the ~ 100 times reduction in wavelength leads to a shunt impedance of $Z_{\perp} \sim 2 \frac{\text{M}\Omega}{\text{m}}$.

CONCLUSION

We have resurrected a beautiful and simple idea from 1970 by Kustom, Chang, and Dawson. While their work looked at high-energy particle deflectors based on rf sources for high-energy physics applications, we have scaled the physics down to mm-scale waveguides which is suitable for demands to measure shorter bunch lengths. We have worked through their math, verified it in CST-MWS, and looked at the dynamics with a field-map in ASTRA. The extremely linear fields are promising for relatively short bunch measurements, and the large transverse shunt impedance, $Z_{\perp} \sim 2 \frac{\text{M}\Omega}{\text{m}}$, with a relatively small required input power of 1.35 kW make it an attractive technique for accelerators with fs-scale bunch lengths. Our future work will look at optimizations, higher energy and higher charge beams.

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