COHERENT X-RAY RADIATION FROM ELECTRON BEAM PROCESSED BY CHANNELING AND EMITTANCE EXCHANGE

I. Lobach*, A. Benediktovitch, Belarusian State University, Minsk, Belarus

Abstract

Presented contribution theoretically studies a novel scheme of compact intense x-ray radiation source. In the scheme, longitudinally modulated electron beam emits x-rays by Inverse Compton Scattering (ICS). The setup's feature is the way how longitudinal density modulation in angstrom scale is created. There are three stages of processing of initial beam of relativistic electrons: 1. First, the electrons cross a crystal plate in channeling regime. It is shown that upon leaving the crystal, the electron beam acquires discernible transverse modulation in angstrom scale. It is taken into account that not all electrons are captured in channeling mode and that some of those that do may leave it as they travel through the crystal slab. 2. Further, the beam is transported to Emittance Exchange (EEX) line, in which the direction of modulation is tilted and the beam becomes longitudinally modulated. The scale of modulation remains the same. 3. Finally, intense quasi-coherent x-ray radiation is emitted by ICS. Numerical estimations show that coherent contribution to intensity is considerable for feasible parameters of used beam, components of EEX line and laser producing photons for ICS.

INTRODUCTION

The operation of synchrotron x-ray light sources and free electron lasers opened new era in the investigation of matter on angstrom length-scale with fs time resolution. However, need for GeV electron accelerators and hundreds meter long undulator modules results in high construction costs, which makes the existing facilities extremely overbooked. In this contribution we theoretically demonstrate one possible compact and relatively cheap alternative of intense coherent x-ray source. The paper is built upon consideration of an example of such scheme with specific and feasible values of all parameters. It is divided into three sections (Channeling, EEX and ICS) in accord with principal components of the scheme.

CHANNELING

In this section it is shown that relativistic electron beam acquires transverse density modulation upon crossing a crystal slab in channeling mode. Parameters of initial electron beam are based on possibilities of ASTA accelerator in Fermilab, see Table 1 and [1]. The beam is assumed to be Gaussian and distribution in $y-y'$ plane is not discussed at this point. In general, state of a particle in a bunch is described by a vector $X=(x,x',y,y',z,\delta)$, where $\delta=\Delta\gamma/\gamma$, the notation is adopted from [2], γ is the Lorentz factor of a reference particle.

* lobachihor@mail.ru

Table 1: Used Parameters of Electron Bunch

Notation	Clarification	Value
E_e	Electron's energy	106 MeV
Q	Charge of one bunch	20 pC
σ_x	Standard deviation of x	10 µm
$\sigma_{x'}$	Standard deviation of x'	0.143 mrad
σ_z	Standard deviation of z	5 µm
σ_δ	Standard deviation of δ	1.02 %

We consider planar channeling in 110 plane of Si crystal slab of thickness 220 nm. Thus, both interplanar spacing and period of obtained transverse modulation are equal to $d=1.91$ Å. Estimations performed using equations from [3] show that classical description of motion of channeled particles at used energy is possible. We used continuous model of interplanar potential, namely, Molier potential, see [3]. Trajectories of electrons (macroparticles) were calculated numerically by Runge–Kutta method. Results of calculations regarding modulation of density along x are shown in Fig. 1. It is clear that considerable modulation is present.

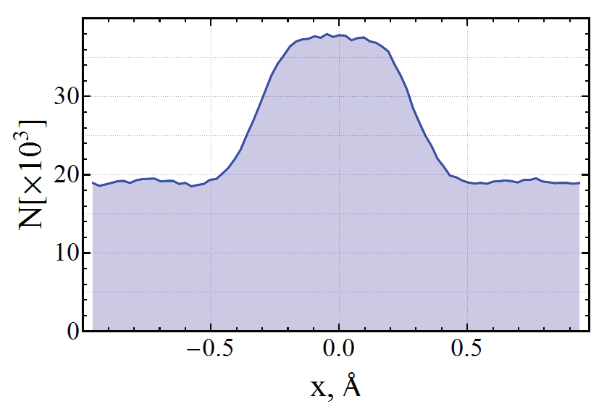

Figure 1: Distribution of macroparticles along x-axis upon leaving a Si crystal slab of thickness 220 nm. Each particle's coordinate was mapped onto the segment $[-d/2, d/2)$ by dropping component of x multiple of d. In the plot x-axis is divided into 80 bins, N is the number of macroparticles in one bin.

We keep only the first harmonic of modulation for further investigation, i.e. the following form of density function of distribution of electrons over x is assumed:

$$p(x)=\left(1+a\cos\frac{2\pi x}{d}\right)\frac{1}{\sqrt{2\pi}\sigma_x}e^{-\frac{x^2}{2\sigma_x^2}}, \qquad (1)$$

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where a is determined through amplitude of corresponding Fourier component of the curve in Fig. 1, it will be called normalized amplitude of modulation. In considered example $a = 0.394$.

Apart from density modulation along x, distribution of electrons over x' is also changed upon leaving a crystal slab. Even though the distribution is no longer Gaussian, it can be described as such with sufficient accuracy. Corresponding value of standard deviation is $\sigma_{x'} = 0.296$ mrad.

Dependence of normalized amplitude of modulation a on crystal's thickness was also obtained, see Fig. 2. Note that the value of crystal thickness 220 nm was chosen for specific numerical example in the beginning of this paper, because highest modulation is achieved at this value. In general, the behavior in Fig. 2 is the result of the following factors. First, any potential well resembling a parabolic one focuses a bunch. That is why there are peaks of focusing. However, anharmonicity of oscillation (due to non-parabolic potential) hinders exact focusing and gradually disrupts the effect, until final value of modulation is achieved. This final value is ascribed to the fact that during any oscillations a particle spends more time when it is far from equilibrium position.

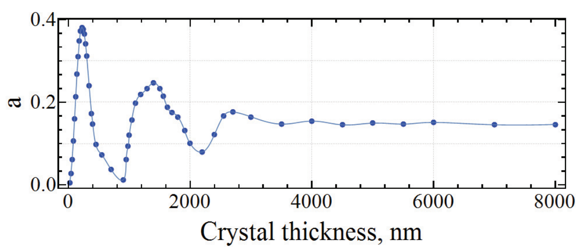

Figure 2: Normalized amplitude of modulation a as a function of crystal's thickness.

Fortunately, the highest modulation is achieved at thickness value 220 nm. It is much smaller than dechanneling length ($\approx$ 2200 nm, see [3]) for the conditions under consideration. Hence, dechanneling effect can be neglected.

EMITTANCE EXCHANGE

On this stage, transverse modulation is transformed into longitudinal one. It is shown in [2] that it is possible to obtain the following form of transfer matrix for EEX line:

$$\mathcal{M}_{\text{eex}} = \begin{pmatrix} 0 & 0 & -\frac{L}{\eta} & 0 \\ 0 & 0 & -\frac{1}{\eta} & -\frac{\xi}{\eta} \\ -\frac{\xi}{\eta} & 0 & 0 & 0 \\ -\frac{1}{\eta} & -\frac{L}{\eta} & 0 & 0 \end{pmatrix}, \tag{2}$$

where blocks corresponding to evolution of y and y' are omitted, this is why the matrix's dimensions are 4×4, see [2]. Required for this particular research feature of matrix of EEX scheme in Eq. (2) is that its element in position (5, 2) equals zero (in general EEX transfer matrix it is not). Owing to this property, exact transfer of transverse modulation to longitudinal direction is possible, because $z_f = -\xi/\eta x_i$, where i means "initial" and f means "final". The way to nullify the above-mentioned element is described in [2]. In a nutshell, one needs to introduce two bitriplets in a conventional EEX line.

The parameters used in Eq. (2) are defined in [2] as well. They depend on several other parameters and are quite flexible, therefore it is enough to only mention used in this contribution values: $L = 1.50$ m, $\eta = 48.7$ cm, $\xi = 15.8$ cm.

The parameters of beam upon passing through EEX line are listed in Table 2.

Table 2: Bunch Parameters upon Leaving EEX

Notation	Description	Value
σ_x	Standard deviation of x	15.4 µm
$\sigma_{x'}$	Standard deviation of x'	3.31 mrad
$\sigma_{xx'}$	$\langle x \cdot x' \rangle$	0.158 µm mrad
σ_z	Standard deviation of z	3.25 µm
a	Normalized amplitude of modulation along z	0.394
λ_M	Period of modulation along z	0.622 Å
σ_δ	Standard deviation of δ	0.0912 %
$\sigma_{z\delta}$	$\langle z \cdot \delta \rangle$	0.00667 µm %

At this point, we assume that distribution of electrons in $y - y'$ plane is the same as in $x - x'$ plane. This significantly simplifies calculations related to radiation by ICS, since angular distribution becomes axially symmetric. There may be many ways to achieve the above-mentioned condition, e.g. one can constrain growth of $\sigma_{y'}$ in the crystal by using axial channeling instead of planar. In this case, $\sigma_{y'}$ should not considerably exceed Lindhard angle, [3]. Also, relevant beam optics should be introduced in EEX line, which would prevent expansion of the beam in y direction as it passes through it.

INVERSE COMPTON SCATTERING

It follows from [4, 5] that spectral-angular density of emitted photons from a bunch of electrons can be calculated by

$$\frac{d^2N}{d\omega d\Omega} = \frac{\alpha \omega a_0^2}{16\omega_L^2 \gamma^2} \left\langle \left| \sum_{\text{all electrons}} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} S\left(\frac{\omega - \bar{\omega}}{\bar{\omega}}\right) \right|^2 \right\rangle, \tag{3}$$

where $\alpha = 1/137$, ω_L is angular frequency of the laser wave (CO_2 laser with $\lambda_L = 10.6$ µm is considered), a_0 is a strength parameter of the laser, see [4]. In considered example $a_0 = 0.1$, $\boldsymbol{k}$ is a wavevector of emitted x-ray wave, $\boldsymbol{r} = (x_0, y_0, z_0)$ represents initial position of electron, $\bar{\omega} = 2\omega_L/(1 - \boldsymbol{n} \cdot \boldsymbol{\beta})$,

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where $\boldsymbol{n} = \boldsymbol{k}c/\omega$. Here and below averaging is performed over states of all particles. The function $S(x)$ depends on model of interaction of electrons and laser field (see [4]), in this contribution we use

$$S(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}, \qquad \sigma = \frac{1}{\sqrt{2\pi} N_0}, \tag{4}$$

where N_0 describes characteristic length of interaction between electrons and laser field (more precisely, number of half-wavelengths). Equation (3) can be transformed in such a way that there is a summand corresponding to incoherent contribution (the first one) and a summand corresponding to coherent contribution (the second one):

$$\frac{d^2N}{d\omega d\Omega} = \frac{\alpha\omega a_0^2}{16\omega_L^2\gamma^2} N_e \langle S^2 \rangle + \\ + \frac{\alpha\omega a_0^2}{16\omega_L^2\gamma^2} N_e (N_e - 1) \left| \langle e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} S \rangle^2 \right|, \tag{5}$$

where N_e is the number of electrons in one bunch.

Averaging in Eq. (5) was performed analytically. Final expressions are not shown in this article. They will be reported in further publications. Coherent contribution to spectral-angular density of emitted photons is presented in Fig. 3. θ is the angle between axis of the beam and direction of radiated x-ray. Only dependence on θ is shown because of axial symmetry (beam parameters were chosen in this way on purpose).

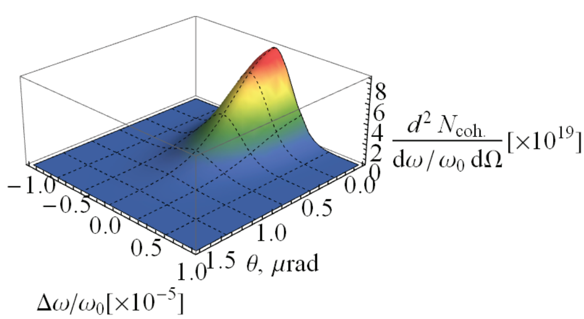

Figure 3: Plot of coherent contribution to spectral-angular density of emitted photons.

It is notable, that the peak is very narrow, compared to incoherent radiation. Therefore, apart from comparison of conventional (integral per 0.1%BW, per π mrad^2) characteristics of synchrotron light sources in Table 3, we also provide values of peak brightness, defined by

$$B_{\mathrm{peak}} = \left. \frac{d^2N}{\Delta t\, d\omega/\omega_0 d\Omega} \right|_{\theta=0}, \tag{6}$$

where $\Delta t = 2\sigma_z/c$, and peak brilliance, defined by $b_{\mathrm{peak}} = B_{\mathrm{peak}}/(4\sigma_x\sigma_y)$.

Table 3: Parameters of Obtained Photon Source

Parameter	Incoherent contribution	Coherent contribution
N, $\frac{\text{photons}}{\text{bunch}}$	770	596
F, $\frac{\text{photons}}{\text{s } 0.1\%\text{BW}}$	$3.55 \cdot 10^{16}$	$2.75 \cdot 10^{16}$
B, $\frac{\text{photons}}{\text{s mrad}^2\ 0.1\%\text{BW}}$	$2.26 \cdot 10^{16}$	$1.75 \cdot 10^{16}$
B_{peak}, $\frac{\text{photons}}{\text{s mrad}^2\ 0.1\%\text{BW}}$	$8.72 \cdot 10^{26}$	$3.96 \cdot 10^{33}$
b, $\frac{\text{photons}}{\text{s mrad}^2\ \text{mm}^2\ 0.1\%\text{BW}}$	$2.39 \cdot 10^{19}$	$1.85 \cdot 10^{19}$
b_{peak}, $\frac{\text{photons}}{\text{s mrad}^2\ \text{mm}^2\ 0.1\%\text{BW}}$	$9.22 \cdot 10^{29}$	$4.19 \cdot 10^{36}$
θ_0	15 mrad	1 μrad
$\delta\omega/\omega_0$	$>10^{-3}$	10^{-5}

In Table 3, θ_0 is a characteristic angular size of peak of spectral-angular density, $\delta\omega/\omega_0$ is a relative peak width in frequency. Characteristic wavelength of the x-ray radiation is λ_M (see Table 2), $\omega_0 = 2\pi c/\lambda_M$.

CONCLUSIONS

It is shown that due to channeling electron beam can acquire considerable transverse modulation of density on angstrom scale. Upon transformation of this modulation into longitudinal one by EEX line, it is possible to obtain increase (in comparison to incoherent radiation) in intensity of radiation from this beam by ICS. It is seen in Table 3 that coherent contribution to integral characteristics is of the order of incoherent one and may be detected in experiment. Moreover, increase in peak brightness and brilliance is enormous, around 7 orders of magnitude in this particular example. Hence, radiation from presented scheme may be useful in many applications requiring highly monochromatic and focused photon beams.

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