DETUNING COMPENSATION IN SC CAVITIES USING KALMAN FILTERS

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Abstract

At Helmholtz Zentrum Berlin two projects are being developed where continuous wave driven SRF cavities are being implemented: bERLinPro and BESSY-VSR. The almost negligible beam loading in these systems, allow operating at high loaded quality factors and thus narrow bandwidths. Thereby, microphonics' influence on the frequency response will dominate the performance of the cavities and the amount of needed forward power. The LTI model describing the cavity's field state is now a time varying system as detuning changes the state of the cavity. Moreover, the received feedback data like transmitted or reflected power will suffer from sensor errors and noises either inherent or caused by the environment. The mentioned uncertainties could be reduced by applying a mathematical process called Kalman filtering. As the object law is well known but not ideal in this case, the role of filter is to judge about which data is more trustworthy under "real" system conditions: theoretical value or the obtained measured. We have designed a Kalman Filter (KF) implementation dedicated for use in mTCA control system as a part of microphonics suppression subsystem. Here the method, design and first data test results are going to be presented.

KALMAN FILTER ADAPTATION TO MICROPHONICS COMPENSATION

Kalman filters are e.g. successfully used for navigation systems aiming at better object position definition in noisy environments. The KF method [1, 2, 3] represents a mathematical method which can predict a system's state with defined probability when the measured system's values contain unpredicted or random error, uncertainty or variation. It accepts a more or less complete physics model describing the equations of motion or just the state of a system by a set of equations as an input. For SC cavities first order systems to describe the field envelope are routinely used in LLRF control. Second order models are applied to emulate the cavity detuning response to external forces like Lorentz forces by the contained field or piezo tuner action. We will use the latter implemented with the Kalman filter observer-estimator-predictor algorithm to develop a new detuning compensation approach.

The KF method is based on iterative calculation of three sequential equations. These equations are: Kalman gain (KG), estimate (EST) and error in the estimate (E_{EST}).

$$\mathrm{KG} = \frac{E_{EST}}{E_{EST}+E_{MEA}} \quad (1)$$

$$\mathrm{EST_t} = \mathrm{EST_{t-1}} + \mathrm{KG}[\mathrm{MEA} - \mathrm{EST_{t-1}}] \quad (2)$$

$$E_{EST_t} = \frac{(E_{MEA})(E_{EST_{t-1}})}{(E_{MEA})+(E_{EST_{t-1}})} = [1-\mathrm{KG}](E_{EST_{t-1}}) \quad (3)$$

KG [Eq. (1)] is defined by two errors: error in the measurement (E_{MEA}) and error in the estimate. It determines how much one can trust a new measurement data sample to use to update the new estimate. The value of KG is between zero and one and it depends on the error in the measurement. The equation for current estimate [Eq. (2)] means that if measurement is considered to be accurate then KG approaches to one and therefore the new estimate will be equal to sum of previous estimate and mostly new measurement (MEA). But if the error in the measurement is comparatively high then KG approaches to zero and the new estimate takes a smaller fraction of the measurement to update the estimate. The new error in the estimate [Eq. (3)] every time decreases to the correct value with a velocity defined by KG.

The transition from cavity behavioural model to KF mathematical description can be introduced by multi-dimension KF model. State matrix and process covariance matrixes, i.e. matrix representing error in the estimate, are used on every stage of KF. The new state is predicted by known theoretical equations describing space-state model of the cavity, while the covariance matrix is calculated by taking into account process noise covariance matrix Q_k. KG is calculated on the next stage and takes into consideration the sensor noise R. The new state then takes a fraction of measured input Y versus the fraction of predicted state by KG. Finally the process covariance matrix is updated and could be used further for prediction loop. An overview of this sequential calculation can be seen in Fig. 1

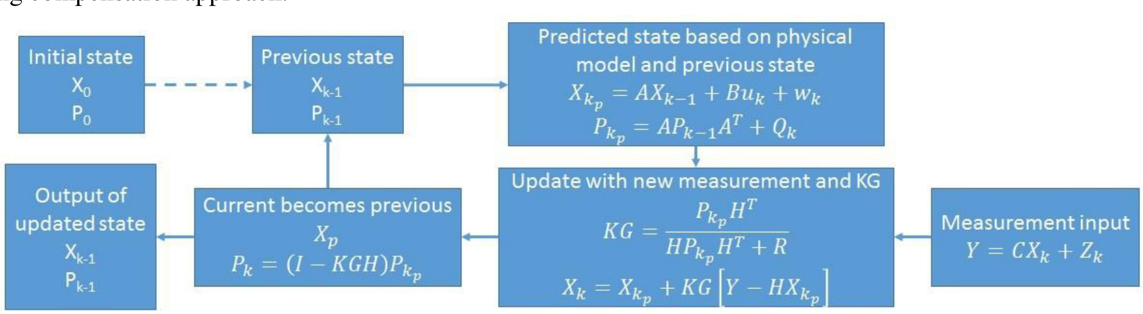

Figure 1: The multi-dimension model of Kalman filter based on matrix arithmetic.

MATHEMATICAL DESCRIPTION OF THE 2ND ORDER MECHANICAL MODEL OF THE CAVITY STRING

The cavity behaviour can be described as a second order system [4]. Moreover it can outline with minor changes either a cavity under Lorentz force presence or piezo-tuner response by changing the coupling coefficient $k_{LF,k}$ by stiffness coefficient.

$$\Delta\ddot{\omega}_{cav,k}(t) + 2\varepsilon_k\omega_{m,k}\Delta\dot{\omega}_{cav,k}(t) + \omega_{m,k}^2\Delta\omega_{cav,k}(t) = \pm k_{LF,k}2\pi\omega_{m,k}^2 E_{acc}^2(t) \quad (4)$$

The damping constant, ε_k, set of eigenmode frequencies, $\omega_{m,k}$ and coupling constant, $k_{LF,k}$, have to be determined experimentally by examining the cavity frequency response in a PLL-lock-in amplifier set up.

Total detuning experienced by the cavity is equal to sum of each individual detuning.

$$\Delta\omega_{piezo}(t) = \sum_k \Delta\omega_{m,k}(t)$$

The presented state-space cavity model, [Eq. (4)], needs to be adopted to transfer it in the matrix form suitable for implementation in VHDL on an FPGA. The final discretited equation looks like:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{n+1} = \begin{pmatrix} 1 & \Delta t \\ -\omega_m^2\Delta t & 1 - \frac{\omega_m}{Q}\Delta t \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_n + \begin{pmatrix} 0 \\ \pm k2\pi\cdot\omega_m^2 \end{pmatrix} E_{acc}^2(t)$$

In fact this equation describes state equation $X_k = AX_{k-1} + Bu_k + w_k$, where A and B are adaptation matrixes, X_k is the system state, u_k is control variable matrix and w_k is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance Q_k.

The second is the measurement equation $Y = CX_k + Z_k$, where the term of adaptation matrix C and state X_k equals $[1 \quad 0][x]$.

It is worth to mention that variables process noise covariance matrix Q_k and measurement covariance matrix R are unavoidable. The first matrix takes into account errors in the parameters because any transfer function of a real cavity is just some approximation not free from model inaccuracy, because the probability to have only a reduced order model compared to reality is high. The second R matrix takes into consideration intrinsic sensor noise or noisy environment affecting the measurement.

In order to estimate the detuning of high-Q low bandwidth cavities to be used in bERLinPro and BESSY-VSR, the KF intellectual property core (IP core) was developed for mTCA system. A preliminary design was made using System generator™ which allows to debug performance and FPGA-related architecture issues. First issue was related to excessive computations and output values range. The traditional approach supported by design tools is to use fixed point arithmetic, which shows disadvantages because number of multiplications constituting Kalman Filter will give predictively rather huge output value. It was decided to develop floating point VHDL library including 3 main arithmetical operations: addition/subtraction (2 clock cycles), multiplication (1 clock) and division (1 clock). The most complicated division operation is based on Patent [5]. First IP core synthesis reveals huge DSP resource consumption of FPGA because System generator directly translates all equations to logic without any optimisation. Therefore it was decided to implement and debug core on Struck SIS8300L digitizer containing relatively rich of resources Virtex-6 XC6VLX130T FPGA. The following performance was achieved:

- Maximum frequency: 17.155 MHz.
- 31 clock cycles for one value calculation but in a pipeline mode.
- 550 KHz actual filter rate for one eigenmode processing.

Table 1: Resource Comparison of two KF IP Cores Synthesised for SIS8300L and DAMC-FMC-20 mTCA Boards

Logic Utilization	Not optimised KF core XC6VLX130T			Optimised KF core XC6VLX130T			Optimised KF core XS6SLX150		
	Used	Available	Utilization	Used	Available	Utilization	Used	Available	Utilization
Slice Registers	6909	160000	4%	2626	160000	1%	2590	184304	1%
Slice LUTs	31373	80000	39%	10235	80000	12%	10373	92152	11%
used as logic	30663	80000	38%	10231	80000	12%	10371	92152	11%
used as Memory	668	27840	2%	0	27840	0%	0	21680	0%
Occupied Slices	11251	20000	56%	4600	20000	23%	4113	23038	17%
RAMB36E1/FIFO36E1s	0	264	0%	3	264	1%			
RAMB16BWERs							3	268	1%
DSP48E1s	98	480	20%	16	480	3%			
DSP48A1s							32	180	17%

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Next improvement leads to system redesign iteration with a purpose to fit design in DAMC-FMC20 carrier board. 3 following main computational blocks were distinguished from preliminary design: 2x2 matrix multiplication, 2x2 matrix addition and combinatorial division. The processing sequence of such ALU was supplied by a finite state machine and memory to keep intermediate results. Minimization efforts gave significant result (see Table 1). And the final performance was achieved:

- Maximum frequency: 159.376 MHz for XC6VLX130T and 87.089 MHz for XC6SLX150.
- 49 clock cycles for one value calculation.
- 3.25 MHz actual filter rate for one frequency.

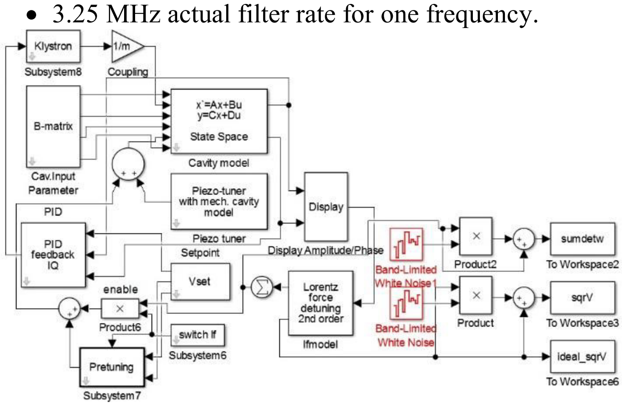

Figure 2: Cavity state-space model equipped with phase and amplitude noisy sources.

KALMAN FILTER PERFORMANCE PROVEMENT AS CAVITY OBSERVER

Several KF IP core tests were done using state-space model (see Fig. 2). In every test the output phase detuning and cavity field were provided by state-space model together with some level of white noise. Three different situations were simulated:

- Only noisy phase input with different noise power.
- Noisy phase and field amplitude input with different noise power.
- Reduced number of eigenmodes instead of full cavity description by 20 eigenmodes [6] used before.

Results were estimated by standard deviation from ideal modelled behaviour. The additional evidence that KF operates correctly was the fact that KG stabilises at some constant level. Figures 3 and 4 show example of system described by 15 eigenmodes instead of 20 and additional noise on phase and amplitude input. Ideal model response (red curve) is distorted by noise (black) and after IP core work the filter response appears (green). The filter response depends on the use of high or low frequencies in the reduced model.

CONCLUSION

The developed and debugged KF IP core is able to track quite noisy experimental equipment. Further test of it as an observer is planned for bERLinPro project together with a Gun cavity in June 2017. FG-ISRF department is going to extend portfolio of IP cores and use KF not only as observer but also as a part of control systems. The Model predictive control with a help of two sequential filters and feedforward detuning control are planned for development. First experimental results from a copper Pill-box cavity demonstrated the necessity to develop IP core which can analyse the transfer function automatically.

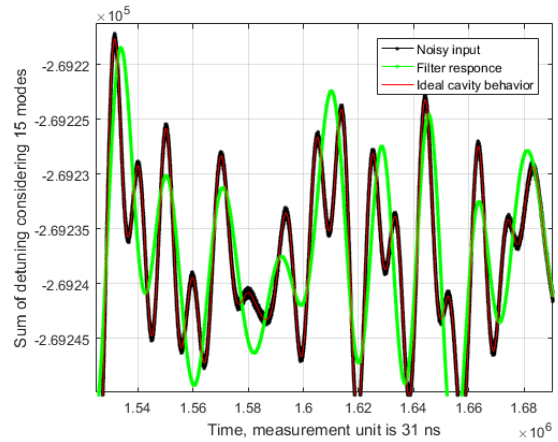

Figure 3: Ideal, noisy phase input and filter response.

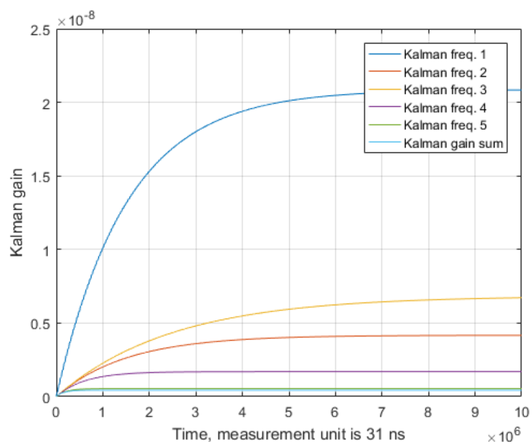

Figure 4: KG stabilises after some time depending on initial noise matrixes choice.

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