COMPARISON OF DIFFERENT METHODS TO CALCULATE INDUCED VOLTAGE IN LONGITUDINAL BEAM DYNAMICS CODES

D. Quartullo*, CERN, Geneva, Switzerland and Università di Roma La Sapienza, Italy
M. Migliorati, Università di Roma La Sapienza and INFN-Roma1, Italy
J. Repond, CERN, Geneva and EPFL, Lausanne, Switzerland

Abstract

Collective effects in longitudinal beam dynamics simulations are essential for many studies since they can perturb the RF potential, giving rise to instabilities. The beam induced voltage can be computed in frequency or time domain using a slicing of the beam profile. This technique is adopted by many codes including CERN BLonD. The slicing acts as a frequency filter and cuts high frequency noise but also physical contributions if the resolution is not sufficient. Moreover, a linear interpolation usually defines the voltage for all the macro-particles, and this can be another source of unphysical effects. The MuSiC code describes interaction between the macro-particles with the wakes generated only by resonator impedances. The complications related to the slices are avoided, but the voltage can contain high frequency noise. In addition, since the computational time scales with the number of resonators and macro-particles, having a large number of them can be cumbersome. In this paper the features of the different approaches are described together with benchmarks between them and analytical formulas, considering both single and multi-turn wakes.

INTRODUCTION

In particle accelerators nowadays, the number of charges in a beam N_P can be of the order of 10^{13} (or higher). Since such numbers are problematic to treat in tracking codes, macro-particle models are used. A macro-particle represents a collection of charges that can be treated as a single entity. The equations of synchrotron motion do not present any particular conceptual or computational difficulties, since the particles can be treated independently. The problems arise when collective effects are added, since the number of macro-particles N_M necessary to simulate the system is not easily defined. Increasing N_M in a given simulation is then necessary to show small dependence of the results on the simulation parameters.

The beam induced voltage acting on a macro-particle is

$$V_{ind}(\phi_i) = \frac{Q_{tot}}{N_M} \sum_{j=1}^{N_M} w_{\parallel}(\phi_i - \phi_j), \tag{1}$$

where ϕ is the longitudinal coordinate, $w_{\parallel}$ the single-particle longitudinal wake and Q_{tot} the total charge of the beam.

Different numerical approaches are available to compute the induced voltage [1]. From Eq. (1), fitting a general wake by wakes from resonant impedances, a propagation matrix can be constructed to compute the induced voltage from particle to particle, this approach is used by MuSiC [2]. On the other hand, since N_M can be large and the resonator impedances coming from the fit of the realistic impedance can be numerous, a slicing of the bunch profile is used in BLonD [3, 4] to compute Eq. (1) either in time domain with a convolution between the wake and the line density, or in frequency domain using Fourier-transform of the bunch spectrum multiplied by the impedance. The length of one slice defines the maximum frequency f_{max} taken into account (Nyquist theorem) and the resolution in frequency domain Δf is related to the length of the induced voltage to be included in simulations. Finally, a linear interpolation is used to define the voltage for particles in-between slices.

This work shows the consistency of the MuSiC and BLonD approaches for two different scenarios characterised respectively by a broad-band (short-range wakefield) and a narrow-band (long-range wakefield) resonators.

SHORT RANGE WAKEFIELD

A broad-band resonator impedance with a resonant frequency higher than the bunch cut-off frequency can be difficult to simulate: physical contributions are lost if f_{max} is set too low and noise can be included if f_{max} is too high.

In the considered example, the resonant frequency is f_r=100 MHz, the quality factor is $Q = 1$ and the shunt impedance is $R_{sh} = 10^7\Omega$. A Gaussian proton bunch with rms bunch length $\sigma_t = 3/f_r$ is used. The RF system has harmonic number h=1, RF frequency $f_{rf} = 1.9$ MHz and peak voltage $V_1 = 1$ MV, the design energy is $E_0 = 13$ GeV, the circumference is $C_{ring} = 157.08$ m and the slip factor is $\eta = 0.0549$. The bunch intensity, $N_P = 10^{12}$, is chosen so that the initial induced voltage V_{ind} has a peak value of 0.8 MV (high intensity effects). The unmatched to the RF bucket initial distribution in phase space is used to see different dynamics (filamentation, losses and later equilibrium in phase space) during the same simulation.

Figure 1 shows the initial normalised bunch spectrum, the resonator impedance and their product using $N_S = 5000$ slices of the bucket length (frequency axis in log scale). It can be seen that 50 slices are enough to consider the main frequency components of the induced voltage, but the impedance peak at high frequency is not included. Increasing N_S up to 1000, the resonator impedance is resolved. Figure 1 shows also that, for fixed N_S, the number of macro-particles should be sufficient to avoid noise at high frequency.

The induced voltage generated by the bunch using $N_M = 4 \times 10^6$ macro-particles is shown in Fig. 2 using the Mu-

* danilo.quartullo@cern.ch

ISBN 978-3-95450-182-3

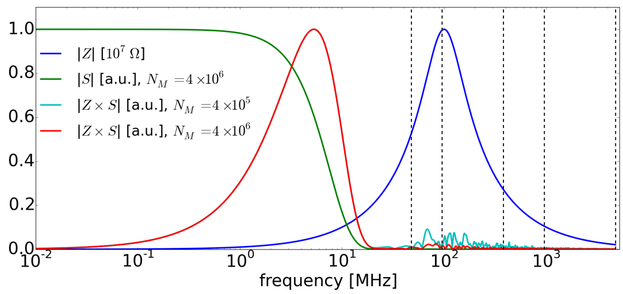

Figure 1: Bunch spectrum $S(f)$ (green), impedance $Z(f)$ (blue) and their product (red) for $N_S = 5000$ and $N_M = 4 \times 10^6$. The vertical dashed lines, from left to right, mark f_{max} corresponding to 50, 100, 400, 1000, 5000 slices. For $N_M = 4 \times 10^5$ high frequency noise is observed (cyan). The main contribution of $|Z \times S|$ is covered using 50 slices.

SiC and BLonD approaches. The MuSiC voltage, compared to the analytical formula, is characterised by high amplitude and high frequency components, since N_M is not high enough (increasing N_M the voltage converges to the analytical one) and the average distance between the macro-particles is relatively small. Using $N_S = 1000$ in frequency domain calculations in BLonD, the voltage is close to the MuSiC one. This is reasonable since, once N_M is chosen, the system is defined, and high values for N_S allow higher frequencies to be sampled. On the other hand, using $N_S = 1000$ in time domain approach in BLonD, a voltage with a significant vertical offset with respect to the MuSiC solution is obtained. Finally, choosing $N_S = 50$ gives an output close in amplitude to the analytical one, but without high frequency components. Since the frequency domain algorithm converges faster than the time domain one in this case (Fig. 2), the first is used. The difference between the two BLonD algorithms is explained by the broad-band nature of the resonator. The wakefield contains many high frequency modulations and needs more accuracy whereas in frequency domain fewer points are needed.

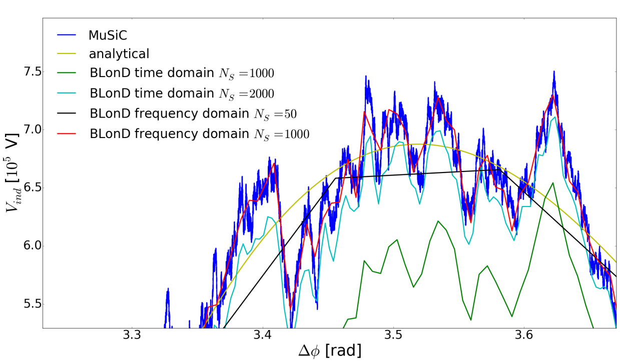

Figure 2: Zoom in induced voltage generated by the initial Gaussian bunch, $N_M = 4 \times 10^6$ and $\Delta\phi = \phi - \phi_s$ ($\phi_s = \pi$ is the synchronous phase). Different methods are shown.

Figure 3 shows that increasing N_S in BLonD, the dipole oscillations converge to the MuSiC one. With just 50 or 100 slices there is no instability and particle loss (low resolution in frequency domain and resonant peak is not resolved). The instability and losses start for $N_S = 400$. Convergence between MuSiC and BLonD results is reached for $N_S = 1000$ (same dipole oscillations) and it is even better with $N_S = 5000$, when also the losses agree.

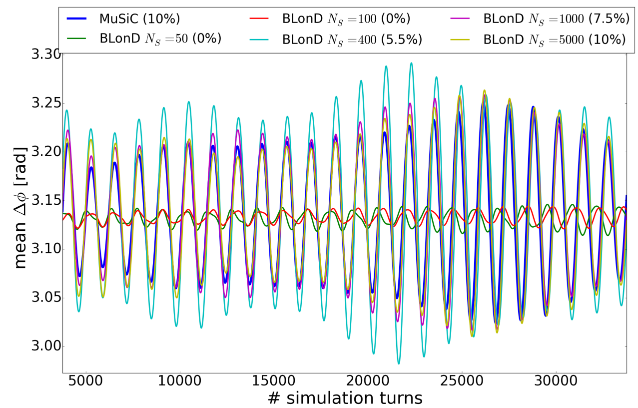

Figure 3: Comparison of MuSiC and BLonD dipole oscillations at equilibrium for $N_M = 4 \times 10^6$ and different N_S. The percentage of particle losses is in parentheses.

Finally in BLonD, for fixed $N_S = 1000$, N_M was increased from 4 to 50 million to verify convergence of simulation results. No significant difference in dipole and quadrupole oscillations was observed, but discrepancy in particle losses (<3%) need further studies. This convergence analysis shows that the instability seen using $N_M = 4 \times 10^6$ and $N_S = 1000$ with MuSiC and BLonD is realistic and not caused by the high frequency noise observed in Fig. 2.

Relative to the computational cost, the broad-band resonator and the wake decaying in one turn allow the largest possible Δf in BLonD to be chosen ($\Delta f = f_0/N_S$, where f_0 is the revolution frequency) or equivalently to consider only the revolution harmonics in the voltage computation. The result is that the BLonD algorithm is much faster than the MuSiC one (factor 27 for $N_M = 4 \times 10^6$ and $N_S = 5000$).

LONG RANGE WAKEFIELD

For a resonant impedance with quality factor $Q \gg 1$ and frequency f_r, the wakefield can couple multiple bunches or even the same bunch on multiple turns (multi-turn wake). If the resonant frequency is close to an integer multiple p of the revolution frequency, then Robinson instability can be observed. The growth rate of the instability is obtained from the imaginary part of the synchrotron frequency shift computed with the linearised Vlasov's equation [2]. Supposing a Gaussian bunch, the analytical growth rate is

$$\frac{1}{\tau_a} = \frac{-\eta e^2 N_P}{2E_0 T_0^2 \omega_s} \sum_{m=\pm 1} m(p\omega_0 + m\omega_s)\mathrm{Re}Z(p\omega_0 + m\omega_s)G_m(x), \tag{2}$$

where T_0 is the revolution period, ω_s the angular synchrotron frequency, e the proton charge and $G_m(x) = 2e^{-x^2} I_m(x^2)/x^2$

is the form factor with $x = (pf_0+mf_s)\sigma_t$ and I_m the modified Bessel function of the first kind.

In the case studied below $p = 2$, the resonator parameters are $f_r = 2f_0 + f_s$, $Q = 5000$ and $R_{sh} = 40\,k\Omega$. In addition $N_P = 4\times10^{12}$, $E_0 = 13$ GeV, $\eta = 0.0217$, $T_0 = 2.1\mu s$, $f_s = 264.1$ Hz. The RF system has $h = 7$, $f_{rf} = 3.3$ MHz and $V_1 = 165$ kV, while $C_{ring} = 628.32$ m. Then the instability growth time is $\tau_a = 59.3$ ms for $\sigma_t \leq 3.3$ ns and the results from MuSiC and BLonD should converge to τ_a for short bunches (no Landau damping).

The initial bunch spectrum decays above 200 MHz whereas the resonant impedance is negligible above 1 MHz. It is then not straightforward to choose f_{max} in BLonD. In addition Δf is another key parameter, since the wake decays over thousands of revolution periods and it is not evident how many turns to take into account. The time domain approach is used in BLonD to simulate this case since the narrow-band resonator requires a very small Δf in frequency domain making simulations computationally heavy. The MuSiC approach avoids all these difficulties since no slices are used and N_M is the only parameter to be studied.

The instability growth time was examined in MuSiC as a function of N_M and compared with $\tau_a = 59.3$ ms from Eq. (2) (Fig. 4). When increasing N_M convergence is observed (63.0 ms) but not to τ_a. This can be explained by the non-linearities of the RF wave (Landau damping). Decreasing sufficiently the bunch length, non-linearities are suppressed and, as shown in Fig. 4, the inverse growth rate converges to τ_a. This proves the validity of the MuSiC algorithm and helps the number of macro-particles to be used for comparison with the BLonD approach to be chosen.

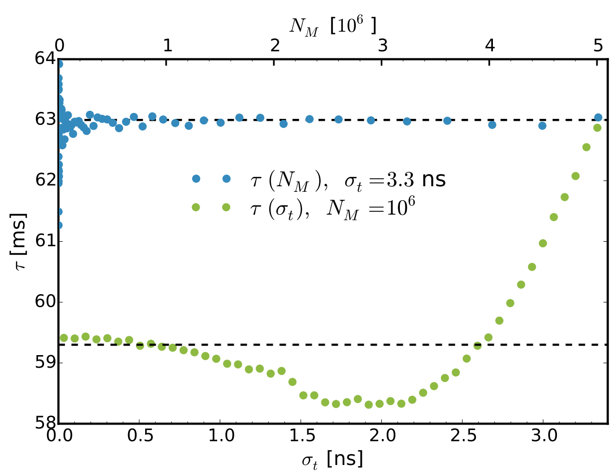

Figure 4: Instability growth time as a function of N_M for $\sigma_t = 3.3$ ns (blue) and as a function of σ_t for $N_M = 10^6$ (green) from MuSiC. Intermediate bunch lengths give effect opposite to Landau damping.

Using $\sigma_t = 3.3$ ns and $N_M = 10^6$ in BLonD, the dependence of the inverse growth rate on Δf and f_{max} was also studied (Fig. 5). Fixing $f_{max} = 200$ MHz to cover the bunch spectrum, the inverse growth rate convergences to 63.0 ms for small Δf as expected from the MuSiC simulations. A scan of f_{max} for two fixed Δf shows consistency of results and therefore agreement between BLonD and MuSiC.

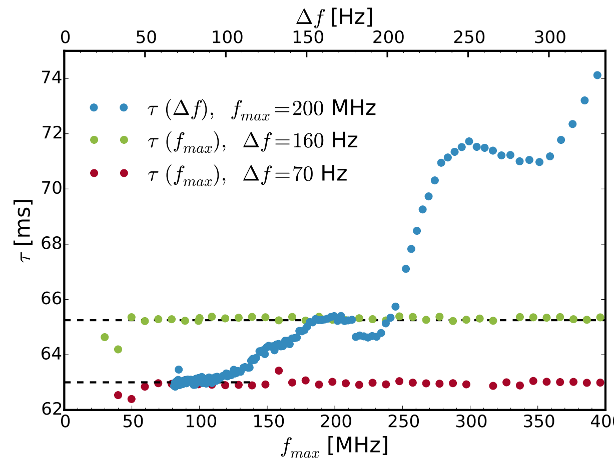

Figure 5: Instability growth time as a function of Δf for $f_{max} = 200$ MHz and versus f_{max} for $\Delta f = 160$ Hz and $\Delta f = 70$ Hz from BLonD ($\sigma_t = 3.3$ ns, $N_M = 10^6$).

However the computational time T in MuSiC (T $\approx N_M = 10^6$) is lower by factor 5, since the number of slices used in BLonD to resolve $f_{max} = 200$ MHz is multiplied by the number of memory turns in which the wake decays (T $\approx f_{max}/\Delta f = 200$ MHz/70 Hz $\approx 3\times10^6$). One way to speed up the calculations in BLonD would be to act only on not-empty buckets. In case of multi-bunch, MuSiC is expected to be less efficient than BLonD since in the first case T scales with N_M while in the second case T scales mostly with N_S and Δf which would not change.

CONCLUSION

Consistency of the MuSiC and BLonD approaches in induced voltage calculation was shown for two different cases (broad and narrow-band resonators). For short-range wakefields and fixed number of macro-particles, the BLonD results converge to the MuSiC one by increasing the number of slices, as expected. Particular care should be taken in BLonD to avoid under-sampling the bunch profile as physical contributions can be lost and wrong results obtained. The absence of slices in MuSiC allows all these parameter space studies to be avoided, however the computational time is much higher than for BLonD, since Δf can be chosen to be large because of the broad-band nature of the resonator.

For long-range wakefield, the analytical value of the Robinson instability growth rate was used as reference for benchmarking and consistency of the two codes has been shown. In this case high computational time was required in BLonD, since too many points have to be taken in frequency or time domain to have reliable results.

ACKNOWLEDGEMENT

We would like to thank E. Shaposhnikova for her continuous support. Our CERN colleagues K. Iliakis and A. Lasheen gave precious contributions to this work.

ISBN 978-3-95450-182-3

REFERENCES

[1] M. Migliorati and D. Quartullo, “Impedance-induced beam instabilities and damping mechanisms in circular machines – Longitudinal - Simulations,” ICFA Beam Dynamics Newsletter No. 69, December 2016.

[2] M. Migliorati and L. Palumbo, “Multibunch and multiparticle simulation code with an alternative approach to wakefield effects,” *Phys. Rev. ST Accel. Beams* **18** (2015) no.3, 031001. `doi:10.1103/PhysRevSTAB.18.031001`

[3] CERN BLonD code, `https://blond.web.cern.ch/`

[4] H. Timko, J. Esteban Müller, A. Lasheen, and D. Quartullo, “Benchmarking the Beam Longitudinal Dynamics Code BLonD,” `doi:10.18429/JACoW-IPAC2016-WEPOY045`