# BEAM-BREAKUP STUDIES FOR THE 4-PASS CORNELL-BROOKHAVEN ENERGY-RECOVERY LINAC TEST ACCELERATOR

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## *Abstract*

Cornell University and Brookhaven National Laboratory are currently designing the Cornell-BNL ERL Test Accelerator (CBETA) [1–4]. To be built at Cornell's Wilson Lab, CBETA utilizes the existing ERL injector and main linac cryomodule (MLC). As the electron bunches pass through the MLC cavities, higher order modes (HOMs) are excited. The recirculating bunches interact with the HOMs, which can give rise to beam-breakup instability (BBU). Here we present simulation results on how BBU limits the maximum achievable current, and potential ways to improve the threshold current.

## INTRODUCTION

BBU occurs in recirculating accelerators when a recirculated beam interacts with HOMs of the accelerating cavities. The most dominant HOMs are the dipole HOMs which give transverse kick to the beam bunches. The off-orbit bunches return to the same cavity and excite more dipole HOMs which, if in phase with the existing dipole HOMs, can kick the bunches more in the same direction. The effect can build up and eventually result in beam loss. Therefore, BBU is a primary limiting factor of the beam current, and the maximum achievable current is called the threshold current $I_{\text{th}}$. With more recirculation passes, bunches interact with cavities for more times, and $I_{\text{th}}$ can significantly decrease [5]. The target current of CBETA is 100 mA for the 1-pass machine, and 40 mA for the 4-pass machine. Simulations are required to check whether $I_{\text{th}}$ is above this limit.

## Bmad SIMULATION OVERVIEW

Cornell University has developed a simulation software called Bmad to model relativistic beam dynamics in customized accelerator lattices. Subroutines have been written to simulate BBU effect and find $I_{\text{th}}$ for a specific design. A complete lattice provided to the program must include at least one multi-pass cavity with HOM(s) assigned to it. It is possible to assign HOMs of different orders to a single cavity, and also a different set of HOMs to other cavities. Parameters such as bunch frequency and numerical tolerances can also be specified to the program.

For each simulation, the program starts with a test current and records the voltage of all assigned HOMs over time. As the beam pass by the cavities, the momentum exchange between the bunches and wake fields are calculated, as well as the new HOM voltages. If all HOM voltages are stable over time, the test current is considered stable, and a new greater current will be tested. In contrast, if at least one HOM voltage is unstable, the test current is regarded unstable, and a smaller current will be tested. Usually $I_{\text{th}}$ can be pinned down within 30 test currents.

In BBU simulation, only cavities with HOMs assigned are essential, so other lattice structures can be hybridized. Hybridization is a process of merging certain lattice components into an equivalent transfer matrix. A single BBU simulation on a CBETA 1-pass hybridized lattice takes up to 20 minutes, in contrast to hours without hybridization. To efficiently find $I_{\text{th}}$ for various HOM assignments or small changes in lattice, hybridization is necessary.

## Bmad SIMULATION RESULT

Dipole HOMs of a single CBETA cavity have been obtained via simulation. Random errors were introduced to each ellipse parameter of the cavity shape, resulting in a spectrum of dipole HOMs, and their characteristics ( shunt impedance ($R/Q$), quality factor $Q$, and frequency $f$) were recorded. Each random error comes from a uniform distribution, with 4 different error cases: ± 125, 250, 500, and 1000 $\mu m$. For simplicity, we use $\epsilon$ to denote the error case: "$\epsilon = 125\ \mu m$" means the errors introduced come from a ± 125 $\mu m$ uniform distribution. A cavity with smaller $\epsilon$ has better manufacture precision. For each error case, 400 unique cavities were provided, and the top 10 "worst" dipole HOMs (ones with greater HOM figure of merit $\xi = (R/Q)\sqrt{Q}/f$) were recorded for each cavity.

Practically the 6 CBETA cavities are not identical, but manufactured with similar precision. Thus, for simulation each cavity is assigned with a different (randomly chosen) set of 10 dipole HOMs, and all 6 sets have the same $\epsilon$. Hundreds of simulations with different HOM assignments were run, and to statistical distributions of $I_{\text{th}}$ were obtained for each specific design and choice of $\epsilon$. Three distributions will be presented as histograms in this section:

1) CBETA 1-pass with $\epsilon = 125\ \mu m$
2) CBETA 4-pass with $\epsilon = 125\ \mu m$
3) CBETA 4-pass with $\epsilon = 250\ \mu m$

Since modern cavities are built with manufacture precision below 250 $\mu m$, the $\epsilon = 500\ \mu m$ and $\epsilon = 1000\ \mu m$ cases will not be investigated.

### *(1) CBETA 1-pass with $\epsilon = 125\ \mu m$*

The design current of CBETA 1-pass is 1 mA (the lower goal) and 40 mA (the higher goal). Figure 1 shows that all 500 simulations results exceed the lower goal of 1 mA, and 499 of them are above 40 mA. The result is quite promising.

### *(2) CBETA 4-pass with $\epsilon = 125\ \mu m$*

The design current of CBETA 4-pass is the higher goal of 40 mA. Figure 2 shows that out of 500 simulations, 494 of

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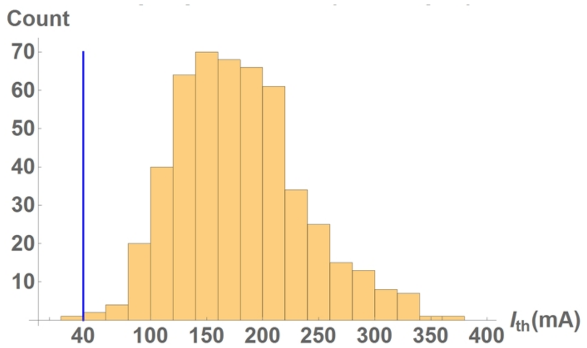

Figure 1: 500 BBU simulation results of $I_{\text{th}}$ for the CBETA 1-pass lattice. Each cavity is assigned with a random set of 10 dipole HOMs ($\epsilon = 125\ \mu m$). The blue line indicates the higher current goal of 40 mA.

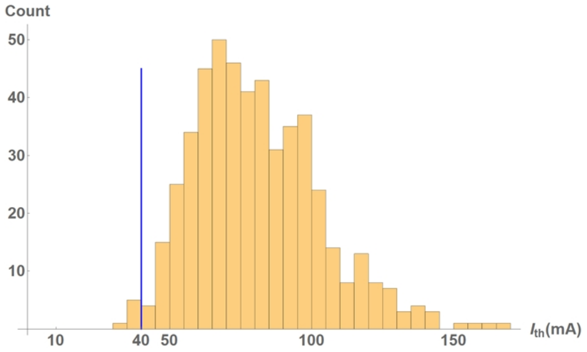

Figure 2: 500 BBU simulation results of $I_{\text{th}}$ for the CBETA 4-pass lattice. Each cavity is assigned with a random set of 10 dipole HOMs ($\epsilon = 125\ \mu m$). The blue line indicates the higher current goal of 40 mA.

them exceed the 40 mA goal. This implies that with certain undesirable combinations of HOMs in the cavities, $I_{\text{th}}$ can be limited.

### *(3) CBETA 4-pass with $\epsilon = 250\ \mu m$*

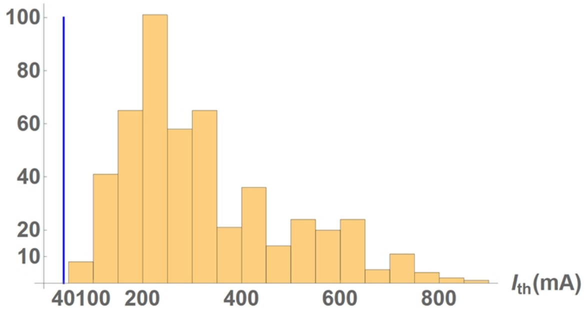

Figure 3: 500 BBU simulation results of $I_{\text{th}}$ for the CBETA 4-pass lattice. Each cavity is assigned with a random set of 10 dipole HOMs ($\epsilon = 250\ \mu m$). The blue line indicates the higher current goal of 40 mA.

It is interesting to see how $I_{\text{th}}$ behaves differently with a different $\epsilon$ for the 4-pass lattice (see Fig. 3). For $\epsilon = 250\ \mu m$, all 500 simulations are above 40 mA, which is better than the $\epsilon = 125\ \mu m$ case. Some might wonder if a greater $\epsilon$ could statistically result in a higher threshold current. Indeed, the more the cavity shapes deviate, the HOM frequency spread becomes greater. A greater spread means the HOMs across cavities act less coherently when kicking the beam, thus statistically increases the $I_{\text{th}}$. However, a greater deviation also tends to undesirably increase the $Q$ (and possibly $R/Q$) of the HOMs, which usually lowers $I_{\text{th}}$. A compensation between the frequency spread and HOM damping means a greater manufacture error in cavity shapes can not reliably improve $I_{\text{th}}$.

There are several ways to improve the accuracy of the simulation results. Perhaps the most important one is to assign HOMs measured directly from the built SRF cavities. Although the dominant HOMs in the measured spectrum can be identified, it is challenging to calculate the R/Q of each mode. Besides improving the simulation accuracy, another important concern is to achieve a greater $I_{\text{th}}$, as discussed in the following section.

## AIM FOR HIGHER $I_{\text{th}}$

To achieve a higher $I_{\text{th}}$, three ways have been proposed, and their effects can be simulated. The first way is to change the bunch frequency $f_b$ (repetition rate) by an integer multiple. Simulations on a CBETA 1-pass and 4-pass lattice show a change of $I_{\text{th}}$ fewer than 5% over several choices of $f_b$, implying that varying $f_b$ is not effective in improving CBETA $I_{\text{th}}$. Rigorous calculation [5] has shown that $I_{\text{th}}$ depends on $f_b$ in a non-linear way for a multi-pass ERL, and it will be interesting to experiment this effect on the realistic CBETA. The other two ways involve varying the phase advances and introducing x-y coupling between the cavities. The simulation results for these two methods are presented in the following sections.

## EFFECT ON $I_{\text{th}}$ BY VARYING PHASE ADVANCE

$I_{\text{th}}$ can potentially be improved by changing the phase advances (in both x and y) between the multi-pass cavities. This method equivalently changes the $T_{12}$ (and $T_{34}$) element of the transfer matrices, and smaller $T_{12}$ values physically correspond to a greater $I_{\text{th}}$ in 1-pass ERLs [5]. To vary the phase advances in Bmad simulations, a zero-length matrix element is introduced right after the first pass of the MLC linac. In reality the phase advances are changed by adjusting the quad strengths around the accelerator structure. In simulation the introduction of the matrix may seem arbitrary, but this gives us insight on how high $I_{\text{th}}$ can reach as phase advances vary.

For each simulation, each cavity is assigned with three "$\epsilon = 125\ \mu m$" dipole HOMs in x, and three identical HOMs in y (polarization angle = $\pi/2$). The $I_{\text{th}}$ is obtained for a choice of $(\phi_x, \phi_y)$, each from 0 to $2\pi$. Several simulations were run for both the 1-pass and 4-pass CBETA lattice, and mainly 4-pass results are presented below.

Figure 4 shows a typical way $I_{\text{th}}$ varies with the two phase advances. Depending on the HOM assignment, the $I_{\text{th}}$ can reach up to 200 mA with an optimal choice of $(\phi_x, \phi_y)$. This implies that changing phase advances does give us advantages in improving $I_{\text{th}}$ for the 1-pass CBETA lattice (the improvement can range from +200 mA to +400 mA depending on the HOMs assigned). Note that $\phi_x$ and $\phi_y$ affect $I_{\text{th}}$ rather independently. That is, at certain $\phi_x$ which results in a low $I_{\text{th}}$ (the "valley"), any choice of $\phi_y$ does not help increase $I_{\text{th}}$, and vise versa. It is also observed that $I_{\text{th}}$ is more sensitive to $\phi_x$, and the effect of $\phi_y$ becomes obvious mostly at the "crest" in $\phi_x$. Physically this is expected since many lattice elements have a unit transfer matrix in the vertical direction, and the effect of varying $T_{12}$ is more significant than $T_{34}$. In other words, HOMs with horizontal polarization are more often excited. As we will see this is no longer true when x-y coupling is introduced.

It is also observed that the location of the "valley" remains almost fixed when HOM assignments are similar. Physically the valley occurs when the combination of phase-advances results in a great $T_{12}$ which excites the most dominant HOM. Therefore, the valley location depends on which cavity is assigned with the most dominant HOM, and is consistent with the simulation results.

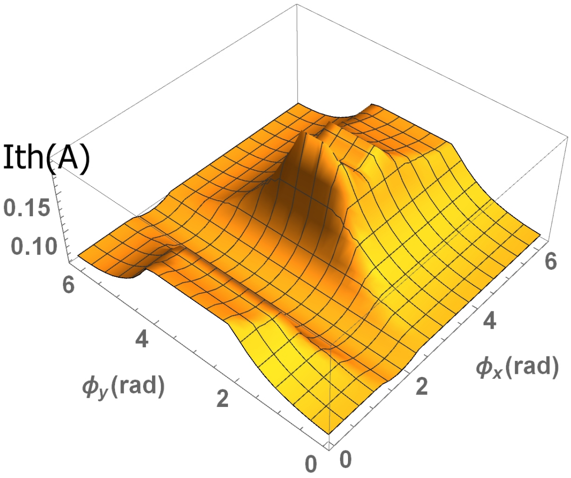

Figure 4: A scan of BBU $I_{\text{th}}$ over the two phase advances for the CBETA 4-pass lattice. Each cavity is assigned with a random set of 3 dipole HOMs in both x and y polarization. ($\epsilon = 125\ \mu m$). For this particular HOM assignment, $I_{\text{th}}$ ranges from 61 mA to 193 mA.

## EFFECT ON $I_{\text{th}}$ WITH X-Y COUPLING

The third way involves x/y coupling in the transverse optics, so that horizontal HOMs excite vertical oscillations and vise versa. This method has been shown very effective for 1-pass ERLs [6]. To simulate the coupling effect in Bmad simulation, a different non-zero length is again introduced right after the first pass of the linac. The matrix couples the transverse optics with two free phases $(\phi_1, \phi_2)$ to be chosen. These two phases are not the conventional phase advances, but can also range from 0 to $2\pi$. The HOM assignment is the same as in the second method and the 4-pass results are presented below.

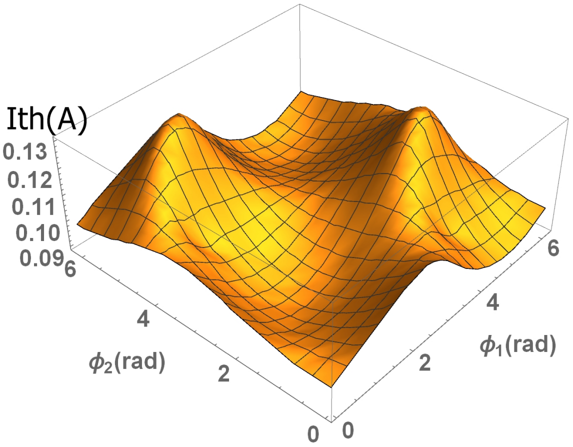

Figure 5: A scan of BBU $I_{\text{th}}$ over the two free phases for the CBETA 4-pass lattice with x-y coupling. Each cavity is assigned with a random set of 3 dipole HOMs in both x and y polarization. ($\epsilon = 125\ \mu m$). For this particular HOM assignment, $I_{\text{th}}$ ranges from 89 mA to 131 mA.

Figure 5 shows a typical way $I_{\text{th}}$ varies with the two free phases for the 4-pass lattice. Depending on the HOM assignment, the $I_{\text{th}}$ can reach up to 131 mA with an optimal choice of $(\phi_1, \phi_2)$. Because the transverse optics are coupled, the two phases no longer affect $I_{\text{th}}$ in an independent manner. That is, there is no specific $\phi_1$ which would always result in a relatively high or low $I_{\text{th}}$. Both phases need to be varied to reach a relatively high $I_{\text{th}}$. Therefore introducing x-y coupling can still improve $I_{\text{th}}$ for the 4-pass lattice (about +60 mA), but not as significantly as varying phase advances.

## SUMMARY

Bmad simulation has shown that with the current design lattice, both the 1-pass and 4-pass machine can always reach the low design current (1 mA), and can surpass the high goal of 40 mA over 98% of time depending on the HOMs assigned.

To potentially increase the $I_{\text{th}}$, we can either adjust the injector bunch frequency, or vary the lattice optics (by introducing additional phase advances or x-y coupling). While the former is shown ineffective by simulation, the later provides room for improvement. For the 1-pass lattice, both optic-varying methods allow great improvement in $I_{\text{th}}$ (about +200 mA to +400 mA). For the 4-pass lattice, the method of varying phase advances allow more improvement (about +150 mA) than x-y coupling (about +60 mA).

In short, varying phase advances is the most promising and cost-effective method to increase $I_{\text{th}}$ of CBETA.

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