K-MODULATION DEVELOPMENTS VIA SIMULTANEOUS BEAM BASED ALIGNMENT IN THE LHC

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Abstract

A parasitic effect of k-modulation is that if the modulated quadrupole has an offset the modulation results in a dipole like kick forcing the beam on a new orbit. This paper presents a new method using the orthonormality of singular value decomposition that uses this new orbit to estimate the offset. This could be used to measure misalignments or crossing angles but could also help improve k-modulation β measurements by predicting the parasitic tune change caused by the new orbit not passing through the centre of the sextupoles.

INTRODUCTION

K-modulation is one of the most precise tools for measuring the β^*-function in the Large Hadron Collider (LHC) [1]. By changing the strength of the innermost quadrupole and measuring the subsequent change in tune, ΔQ, one can compute the average β-function in the quadrupole using

$$\beta_{av} = \pm 4\pi \frac{\Delta Q}{\Delta k L} \tag{1}$$

where Δk and L are the change in k and length of the quadrupole.

From β_{av} in both quadrupoles one can compute the location and value of the minimum β function and hence reconstruct the value of β^* by solving the quadratic equation

$$\beta_{av} = \beta_w + \frac{(L^* - w)^2}{\beta_w} \tag{2}$$

where L^* is the length of the final drift going from the quad to the interaction point (IP) and w and β_w are the offset or waist and the value of the minimum β-function, β^*.

A parasitic effect of the modulation is that if the quadrupole is misaligned the change in k causes a kick proportional to the offset, x_q, and Δk [2]. This kick forces the beam on a new closed orbit, given by

$$\begin{aligned}\Delta x &= \frac{\beta \cot(\pi Q)\Delta k L}{2 + \beta \Delta k L \cot(\pi Q)} x_q \\ \Delta x' &= \frac{[1 - \alpha \cot(\pi Q)]\Delta k L}{2 + \beta \Delta k L \cot(\pi Q)} x_q\end{aligned} \tag{3}$$

in the quadrupole [3]. Where α and β are the twiss functions in the centre of the quadrupole, L is the length of the quadrupole and Q is the machine tune. The transfer matrix M of the machine can be used to work out the new orbit at any point in the machine [4] as expressed by Eq. 4.

$$\Delta x(s) = M_{11}(s, s_0)\Delta x + M_{12}(s, s_0)\Delta x' \tag{4}$$

On the one hand this new orbit causes the beam to not go through the centres of the sextupoles resulting in an additional change in tune. On the other hand, measurements of this new orbits can give information about the offset of the quadrupole and help align it or estimate the crossing angle. This information can also be used to predict the new orbit through the sextupoles and hence predict and correct for the parasitic change in tune.

EFFECT OF PARASITIC TUNE CHANGE ON VDM OPTICS

An extreme example of where the parasitic tune change has a drastic effect on the accuracy of k-modulation β-scans is when using van der Meer (vdM) optics. This optics is used during calibration scans and it is essential to know β^* accurately [5]. The significance of the vdM optics is that it is symmetric and has a β^* close to L^*, which turns out to be the maximum possible β^*.

Mathematical Properties of β^* Reconstruction

For simplicity we will assume the case with $w = 0$ and $\beta_w = \beta^*$ to modify Eq. 1 to

$$\beta_{av} = \beta^* + \frac{L^{*2}}{\beta^*} \tag{5}$$

By taking the derivative of this, one can work out that the relative errors of β_{av} and β^* relate as

$$\frac{\sigma_{\beta av}}{\beta_{av}} = \left|\frac{\partial \beta_{av}}{\partial \beta^*}\right| \frac{\sigma_{\beta^*}}{\beta_{av}} = \frac{\left|\beta^* - \frac{L^{*2}}{\beta^*}\right|}{\beta^* + \frac{L^{*2}}{\beta^*}} \frac{\sigma_{\beta^*}}{\beta^*} \tag{6}$$

In the nominal optics the beam is squeezed at the IP resulting in $\beta^* << L^*$. In this case $\frac{L^{*2}}{\beta^*}$ becomes the dominating factor in the numerator and denominator of Eq. 6 resulting in $\frac{\sigma_{\beta av}}{\beta_{av}} \approx \frac{\sigma_{\beta^*}}{\beta^*}$. This means that an error in β_{av} of a few percent caused, for example, by an error in ΔQ would result in an error in β^* of a few percent.

This property drastically changes when the vdM optics are applied. Since for these optics $\beta^* \approx L^*$ the full expression of eq. 6 becomes relevant. In the limiting case of $\beta^* \to L^*$, $\left|\beta^* - \frac{L^{*2}}{\beta^*}\right| \to 0$, meaning that a small error in the β_{av} causes an infinite error in β^*.

A further problem arises when examining the solutions to Eq. 5 given by

$$\beta^* = \frac{\beta_{av} \pm \sqrt{\beta_{av}^{*} - 4L^{*2}}}{2} \tag{7}$$

of which only the '-' root is physical.

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As $\beta^* \rightarrow L^*$, $\beta_{av} \rightarrow 2L^*$ so that $\sqrt{\beta_{av}^2 - 4L^{*2}} \rightarrow 0$, consequently the physical and unphysical solutions move close together. For most numerical solvers there is no way to differentiate between these two solutions. Finally, one has to recognise that in a case where an error in the measurement of β_{av} gives $\beta_{av} < 2L^*$ means that $\sqrt{\beta_{av}^2 - 4L^{*2}}$ will be complex. This would cause solving methods to run into errors and give rise to unphysical solutions.

Testing Detailed β^* Reconstruction

In order to see how much these problems hold up when the reconstruction is done using all its details a short algorithm was written that scans through ΔQ starting at 5 % less and going up to 5% more than the change predicted by the MADX [6] model and then outputs the β^* estimate for each case. The results are shown on a 3D plot in Fig 1 and the result from the correct ΔQ is highlighted by a red circle.

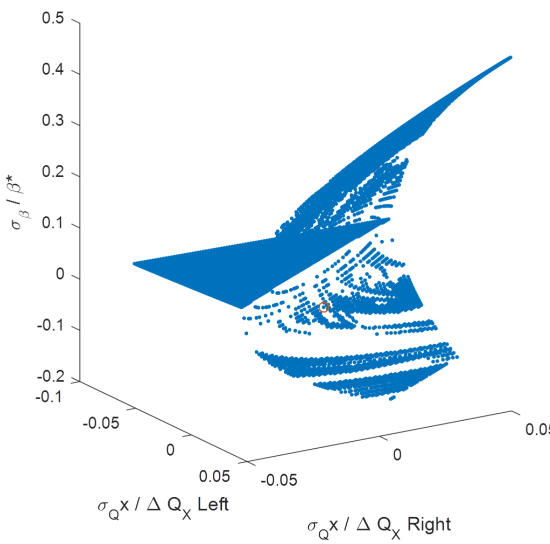

Figure 1: Plot Showing Relative Error in β^* Obtained from Offset ΔQ Inputs

As one can see from Fig. 1, all effects predicted by the simple calculations in the previous section materialise in this simulation. Whilst ΔQ from the ideal model give exactly the right β^*, the estimate for β^* changes drastically for small ΔQ around the ideal solution.

Moreover, solutions jump between two sides of a parabolic surface. The top half of this surface corresponds to the unphysical solutions, which lie very close to the physical ones. Finally, one can also see a triangular flat plane for small ΔQ values. If ΔQ is too small then this means $\beta_{av} < 2L^*$ in the quadrupole, giving complex solutions of which only the constant real part is returned.

DATA

The methods described in this paper were applied to data obtained from k-modulation scans of vdM optics in the LHC, taken on 7^{th} October 2016 during fill 5380. Before performing these measurements the β functions in the machine were measured and corrected using an alternating current dipole and β beat analysis.

The innermost quadrupoles in interaction region (IR) 1 and 8 were modulated with the crossing angles switched on. A second set of measurements in IR 8 were then taken with the crossing angles turned off.

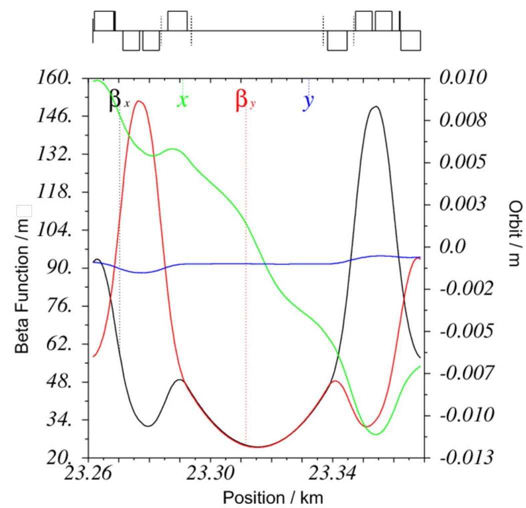

Figure 2: Design β Functions and Orbit of Beam 1 of vdM Optics around IP 8 from MADX simulation

Figure 2 shows the design β-functions and orbits for beam 1 around IP 8 for a crossing angle of -230 μrad obtained from a MADX simulation. As one can see the crossing angle causes the horizontal orbit in the quadrupoles to be non-zero and hence we expect a closed orbit response as described by Eq. 3 when the crossing angle is turned on. Consequently, the β^* value reconstructed with the crossing angle on was much less accurate than that with the angle turned off.

As well as using data from measurements, the method was also tested on data produced by MADX simulations of k-modulation. This data helps showing whether the predicted parasitic tune change is correct as it can easily be compared to an ideal machine.

DETERMINING THE OFFSET

Method

Equations 3 and 4 show that the magnitude of the change in orbit at any position is directly proportional to the quadrupole offset x_q. Hence by working out the ratio between the measured orbit and the orbit predicted for unity offset, one can estimate the offset of the modulated quadrupole.

The LHC measures and records the position of each beam at 544 locations every second so it is desirable to exploit all the information in the data statistically to estimate the offset as accurately as possible. A common statistical method is storing the orbit data in a matrix $\mathbf{B}$ such that $B_{ij} = x_j(t_i)$ and then decomposing it using singular value decomposition (SVD) [7].

When applying SVD to model data, $\mathbf{B}_m$, produced using Eq. 3 and eq. 4 for $\Delta x_q = 1$ m, Δk from the measurements and TWIS functions from the ideal MADX model, one only gets one significant mode. This implies that the data can be decomposed to be the product of two orthonormal vectors, $\mathbf{u}_m$ and $\mathbf{v}_m$ and a singular value s_m such that

$$\mathbf{B}_m = \mathbf{u}_m s_m \mathbf{v}_m^T. \tag{8}$$

The data from the actual experiment is also stored in a matrix, $\mathbf{B}_{data}$, of the same format. This matrix should be proportional to the model matrix by a factor of x_q plus a noise term $\mathbf{N}$ so that

$$\mathbf{B}_{data} = \mathbf{u}_m s_m \mathbf{v}_m^T x_q + \mathbf{N}. \tag{9}$$

By using the orthonormal property of the SVD vectors one can get an estimate of x_q by equating

$$\frac{\mathbf{u}_m^T \mathbf{B}_{data} \mathbf{v}_m}{s_m} = x_q + \frac{\mathbf{u}_m^T \mathbf{N} \mathbf{v}_m}{s_m} \tag{10}$$

where the second term should cancel out for a large enough sample size. In traditional SVD cleaning, the data is decomposed and modes with small singular values are identified as noise and eliminated. What is special about this method is that instead of just eliminating modes that could be noise it specifically only selects the mode that corresponds to the closed orbit response to the modulation.

Results

This method was applied to the data at IP 8 with the crossing angles turned off and on. The results for the two quadrupoles are shown in Table 1.

Table 1: Orbit Offset of Beams 1 and 2 in Left and Right Quadruples of IR 8 Determined from Measurements with Crossing Angles turned off and on and from MADX Model

	Orbit Offset / mm					
	No Crossing		Crossing		Model	
	x	y	x	y	x	y
Left 1	0.517	−0.692	4.73	−0.725	4.27	−1.04
Left 2	0.702	1.21	−4.73	1.34	−4.62	0.957
Right 1	−0.172	−1.36	−5.71	−1.35	−4.62	−0.957
Right 2	−0.553	0.112	4.17	0.103	4.27	0.104

As expected, the horizontal offset obtained from this method differs when the crossing angle is on and off but vertical offset remains the same. Table 1 also contain the model offsets at the centre of the quadrupoles due to the crossing angle and the separation. They show that this method is very precise at estimating the offset due to the crossing angle as it is in good agreement with the model.

Crossing Angle

Another application of this method could be to use the offset estimate to estimate the crossing angle of the beams. To compute this one can simply divide the difference in the offsets of the two beams by the the length of the drift plus the half length of the quadrupole.

In order to test this the algorithm was applied to data produced using a MADX model that simulated the shunting of the left and right quadrupoles in IP8. In the model the half crossing angle was set to 500 μrad and the separation was turned on. Using these results for the x plane the half crossing angle is computed to be 503 μrad on the left and 504 μrad on the right, which is very close to the actual crossing angle.

This method can also be tested by applying it to the data discussed in [8], where a similar method was applied to determine the orbit in the quadrupoles and hence compute crossing angles during LHC fill 5422. During this fill the half crossing angle was set to 140 μrad but was measured to be 155 ± 10 μrad in IP1 and 153 ± 12 μrad in IP5. Using the SVD method on the same data yields a crossing angle of 149 ± 18 μrad in IP1 and 141 ± 18 μrad in IP5. This puts this method in good agreement with both the actual data and comparable methods.

PREDICTING THE TUNE SHIFT

Since k-modulation forces the beam onto a new closed orbit, the beam will no longer pass through the centres of the sextupoles in the machine. This causes the beam to experience an additional quadrupole field with strength $k_1 = k_2 x_s$, where k_2 is the sextupole strength and x_s is the offset in the sextupole. This additional quadrupole effect gives rise to a tune shift given by

$$\Delta Q_{x,y} = \pm \frac{\beta_{Sx,y} k_2 L_s x_s}{4\pi} \tag{11}$$

where β_s is the beta function in the sextupole and can be taken from the model and L_s is the length of the sextupole [9]. In the vertical plane the equation is the same but negative.

Using the offset analysis method described in the previous section and Eq. 10 one can estimate x_s and hence work out the new closed orbit at any point using Eq. 3 and 4 for every Δk. This can give x_s in all sextupoles and one can sum up all the contributions to the change of tune described by Eq. 11 to estimate the total parasitic tune change.

CONCLUSION AND OUTLOOK

The impact of the parasitic tune change changes on k-modulation have been outlined for the extreme example of vdM optics. A method was developed that uses SVD to analyse orbit data and estimate the offset of the modulated quadrupole. A method of how this offset estimate can be used to evaluate the parasitic tune change has been discussed and can be implemented and tested in the near future.

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