EFFECT OF LINEAR COUPLING ON NONLINEAR OBSERVABLES AT THE LHC

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Abstract

Simulation work during LHC Run 1 established that linear coupling had a large impact on nonlinear observables such as amplitude detuning and dynamic aperture. It is generally taken as the largest single source of uncertainty in the modelling of the LHC's nonlinear single particle dynamics. Measurements in 2016 sought to confirm this impact of linear coupling with beam. This paper summarizes the observed influence of linear coupling on various nonlinear observables in the LHC.

INTRODUCTION

Run 1 of the LHC saw several studies which sought to assess the understanding of the nonlinear (NL) beam-dynamics through comparison of beam-based measurement and simulation [1–3]. From the simulation side it was quickly established that linear coupling had a substantial effect on nonlinear observables such as amplitude detuning, dynamic aperture (DA) and nonlinear chromaticity at injection and top energy [1, 2, 4, 5]. Linear coupling is generally taken to be the effect with the single largest impact on the modelling of nonlinear single-particle dynamics in the LHC. This is illustrated in Fig. 1, which shows a histogram of simulated amplitude detuning coefficients from LHC Run 1, for models of the operational configuration at injection containing all known nonlinear sources, and with Landau octupoles (MO) introducing a large amplitude detuning for the damping of instabilities.

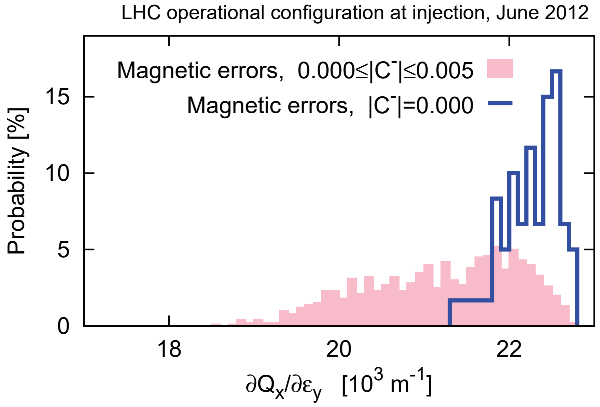

Figure 1: Uncertainty in predicted amplitude detuning due to magnetic error measurement uncertainties and realistic variations of linear coupling.

For the case of negligible linear coupling, blue data indicates the variation of predicted amplitude detuning due to uncertainties in magnetic measurements of nonlinear errors. Pink data shows the variation of detuning coefficients obtained for an operationally realistic range of $|C^-|$. Even for moderate coupling, the uncertainty in the predicted behaviour due to coupling is 3 times that due to the uncertainty on magnetic measurements. Further, even this moderate coupling led to changes to detuning coefficients (and hence the tune footprint relevant for Landau damping of instabilities) which are non-negligible compared to that introduced by the Landau octupoles.

Alteration of nonlinear dynamics by linear coupling can have significant repercussions. It limits the predictive power of accelerator models. For example, in investigating design tolerances or working points through DA realistic variations to $|C^-|$ may need to be included in the study, which significantly increases the required simulation power. In altering beam parameters such amplitude detuning linear coupling would affect tune spread within a bunch. This would then influence Landau damping and potentially lead to beams becoming unstable. Indeed an effect of linear coupling on beam stability was observed in the LHC in 2015-16 [6]. Finally, it is emerging that nonlinear errors in low-β insertion regions (IRs) may have a critical impact on the operability of future colliders such as the HL-LHC [7–11] and FCC [12]. Ultimately correction of these errors will rely upon detailed measurement of nonlinear observables [13]. Alteration of NL-observables by linear coupling will significantly complicate interpretation of measurements. For example, to compensate decapole errors measurement of feed-down to amplitude detuning is being considered [10, 14]. If orbit changes also alter $|C^-|$ due to sextupole feed-down it would be ambiguous whether a detuning shift was due to decapole or sextupole errors.

There exists both immediate and long-term motivations to understand the influence linear coupling exerts on nonlinear beam-dynamics. So far this has only been studied in simulation at the LHC. Results of studies in 2016 which sought to confirm an impact of coupling on nonlinear observables with beam are reported here.

AMPLITUDE DETUNING

Amplitude detuning was measured at injection for very well corrected linear coupling, $|C^-| < 0.001$, and $|C^-| = 0.02$. In the latter case coupling was introduced using LHC coupling knobs normally used for global compensation of $|C^-|$. The global coupling knobs allow the complex phase, ϕ_{1001}, of the difference coupling resonance driving term (RDT), $f_{1001} = |f_{1001}|e^{i\phi_{1001}}$, to be controlled. Evaluating the RDT at IP7, $|C^-| = 0.02$ was generated via $\phi_{1001} = 0.00\,\pi$ ($\Re e\,[f_{1001}]$), and then via $\phi_{1001} = 0.25\,\pi$. Measurements were only made in the plane where $Q_{x,y}$ detuned away from each other, to avoid any influence of

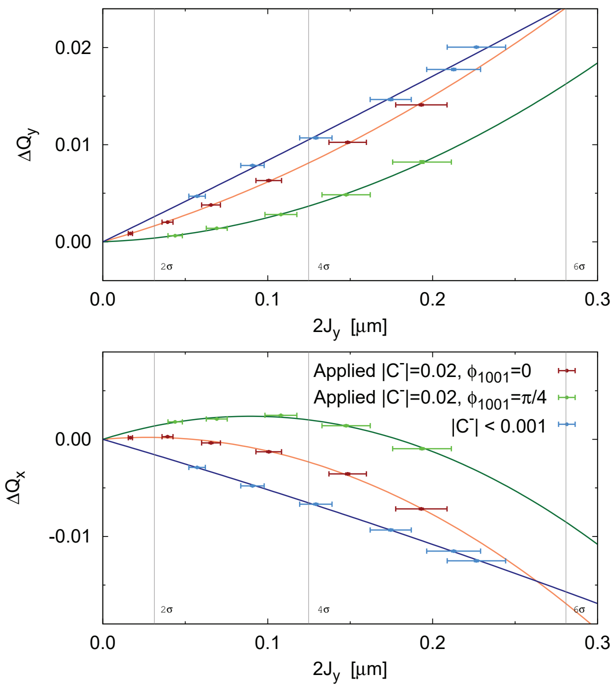

Figure 2: Tune shift versus action for $|C^-| < 0.001$ (blue) and $|C^-| = 0.02$ (green/red). Red and green data correspond to generation of $|C^-|$ via different phases of f_{1001}.

Table 1: Fits of Detuning Coefficients to Measurement. $\phi_{1001|\mathrm{IP7}}$ is the complex phase of coupling knobs used to introduce the $|C^-|$.

$\lvert C^-\rvert$ $\phi_{1001\mid\mathrm{IP7}}$	$\lvert C^-\rvert < 0.001$	$\lvert C^-\rvert = 0.02$ $0.00\,\pi$	$\lvert C^-\rvert = 0.02$ $0.25\,\pi$
$\frac{\partial Q_x}{\partial(2J)_y}$ $[10^3 m^{-1}]$	-49.6 ± 0.4	15 ± 3	53 ± 10
$\frac{\partial Q_y}{\partial(2J)_y}$ $[10^3 m^{-1}]$	80 ± 10	48 ± 3	7 ± 4
$\frac{\partial^2 Q_x}{\partial(2J)_y^2}$ $[10^9 m^{-2}]$	-40 ± 30	-530 ± 50	-590 ± 90
$\frac{\partial^2 Q_y}{\partial(2J)_y^2}$ $[10^9 m^{-2}]$	30 ± 100	270 ± 40	360 ± 40

amplitude dependent ΔQ_{min} [1, 15–18]. Measured data and fits of detuning coefficients are shown in Fig. 2 and Table. 1 respectively.

Linear coupling caused dramatic changes to first-order detuning coefficients ($\partial Q/\partial(2J)$). Both coupling amplitude and phase appear to have a substantial impact. A dependence of amplitude detuning on coupling phase has also been seen in LHC simulations during Run 1 [1, 2]. Linear coupling also generated substantial second-order detuning coefficients ($\partial^2 Q/\partial(2J)^2$). Comparisons between measurement and simulation in 2012 revealed a substantial discrepancy in second-order detuning [1], for which linear coupling now appears a likely source. In some cases tune shift from the second-order terms became comparable to linear shifts at amplitudes as low as $2\,\sigma$. This implies that with large $|C^-|$ LHC behaviour is not predictable in terms of the usual octupole-like tune spread and footprint.

RESONANCE DRIVING TERMS (RDTS)

Study of RDTs is being considered for correction of NL-errors in low-β^* IRs [8, 14]. It is expected [19] that coupling may influence observed RDTs. In 2016 correction of $|C^-|$ below the per-mil level typical of LHC operation was achieved at 0.4 m [20]. Optics measurements with AC-dipole were performed before and after correction, with clear peaks observed in the frequency spectrum corresponding to resonance $2Q_y^{AC} + Q_y$ driven by the f'_{0030} RDT (f' denotes RDTs of driven oscillations with AC-dipole) [21]. A clear shift to measured $|f'_{0030}|$ was seen when moving from a typical operational value of $|C^-| = 0.003$, to the sub per-mil coupling quality (Fig. 3). Study of RDTs in the LHC should benefit from the high-quality coupling compensation demonstrated in 2016. This also motivates study of how RDT-based compensation of IR-errors could be influenced by local and global coupling.

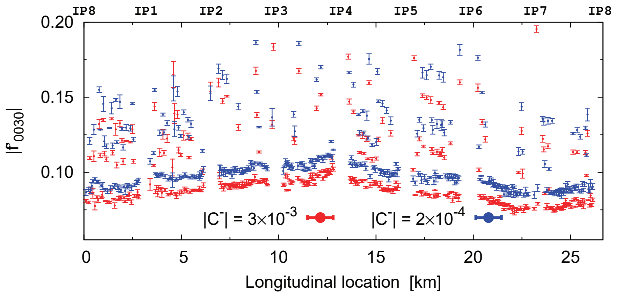

Figure 3: Skew sextupole RDT f'_{0030}, before and after correction of coupling at $\beta^* = 0.4$ m ATS [22] optics.

DYNAMIC APERTURE (DA)

In the approximation of uncorrelated Gaussian beam profile of equal sigmas, the average value of DA in the (σ_x, σ_y) parameters space can be related to fractional intensity loss as a function of time, Eq.(1) [23], where $I(N)$ represents the surviving intensity and $D(N)$ the DA, after N turns. By blowing up LHC beams with a transverse damper evolution of DA in time can be measured. This technique has been demonstrated in the LHC at injection in Run 1 [24–26]. Evolution of DA as a function of turn number can then be described by a scaling law of the from Eq.(2) [23, 27].

$$\frac{I(N)}{I(0)} = 1 - e^{\frac{D(N)^2}{2}} \tag{1}$$

$$D(N) = D_\infty + \frac{b}{[\log(N)]^k} \tag{2}$$

In 2016 DA was measured at injection for small ($|C^-| < 0.001$) and large ($|C^-| = 0.02$, $\phi_{1001|\mathrm{IP7}} = 0.0$) coupling. For the two configurations, intensity following application of the Landau octupoles to operational settings was measured. Dynamic aperture inferred from observed fractional intensity loss is shown in Fig. 4, on a logarithmic timescale. Fits of Eq.(2) are also shown in Fig. 4 and Table. 2.

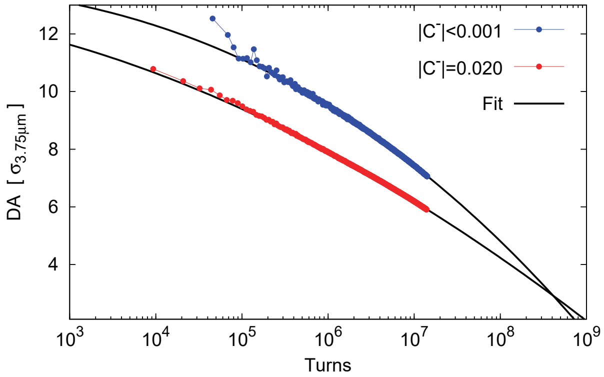

Figure 4: Effect of linear coupling on measured dynamic aperture in the LHC at injection, with $K_{MO} = +9\,\mathrm{m}^{-4}$.

Table 2: Fit Parameters Obtained from Measured Beam Loss Data for Small and Large Values of the Linear Coupling

Coupling state	$D(1 \times 10^8)$ $[\sigma_{3.75\,\mu\mathrm{m}}]$	D_∞ $[\sigma_{3.75\,\mu\mathrm{m}}]$	κ	b
$\lvert C^-\rvert < 0.001$	4.8	13.8	−2.60	−0.0046
$\lvert C^-\rvert = 0.020$	4.2	13.3	−1.79	−0.0493

A sizable reduction to DA was observed with large $|C^-|$. At 1-2 $\sigma_{3.75\,\mu\mathrm{m}}$ this is on par with DA reductions observed in 2012 upon driving LHC arc octupole correctors to their maximum currents at injection [26]: settings an order of magnitude stronger than beam-based corrections for observed octupole errors [3, 28]. This validates expectation from simulation that a good coupling model is an essential prerequisite to reliable prediction of dynamic aperture, and demonstrates the potential for operational gains to lifetime through high-quality $|C^-|$ correction.

A clear change in trend for evolution of DA in time is observed in Fig. 4 and in fit parameters in Table. 2. An effect on DA evolution has also been observed in simulation. Figure 5 shows the effect of increasing $\Re[f_{1001}]$ (left) and $\Im[f_{1001}]$ (right) in SixTrack simulations of an LHC optics studied in 2012. Extrapolated DA, via Eq.(2), is shown for the smallest and largest modelled coupling. Specific octupole sources in Fig. 5 differ significantly from the 2016 measurement, and DA should not be directly compared. Integrated octupole strength however, is qualitatively similar to the 2016 case as characterised by detuning coefficients and NL-chromaticity. A clear change to evolution of DA in time can be seen between simulations with small and large coupling. As with amplitude detuning, complex phase of f_{1001} plays a non-negligible role in defining the impact of coupling on simulated DA. Only a single phase has been studied with beam in the LHC however.

Equation (2) has been shown to characterize well the measured evolution of dynamic aperture in Tevatron [23] and the LHC [26]. A clear application of such a scaling law is extrapolation of simulated DA to operational timescales of real accelerators. Changes in trend for DA as a function of time due to coupling, as seen in Figs. 4 and 5, imply a significant impact of coupling on any such extrapolation. Further, changes to the trend of DA as a function of time might be related with shifts in diffusion rates of chaotic particles, as coupling between the horizontal and vertical phase space increases. An interesting topic in its own right. Therefore, while measurements presented here provide a first demonstration of the importance of coupling to DA and lifetime in the LHC, there are also clear avenues for investigation moving forwards, in regard to the influence of ϕ_{1001} on DA and the interaction between coupling and DA evolution quantified by the scaling law Eq.(2).

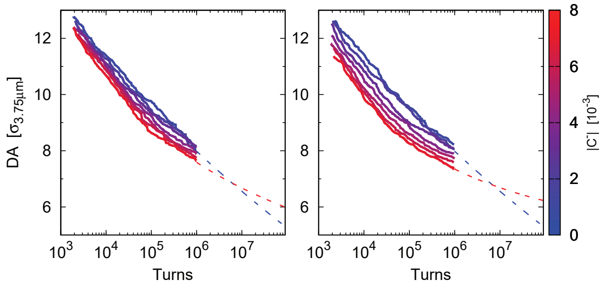

Figure 5: Simulated impact of $|C^-|$ on DA evolution. Coupling (color-scale) was introduced using $\Re[f_{1001}]$ (left) and $\Im[f_{1001}]$ (right) coupling knobs. Extrapolation of DA via Eq.(2) is shown for max and min $|C^-|$ modelled.

CONCLUSIONS

Simulation-based studies during 2011-12 operation predicted a large impact of $|C^-|$ on the nonlinear behaviour of LHC beams. Studies in 2016 confirmed with beam a dramatic impact of linear coupling on several nonlinear observables in the LHC. The statement that linear coupling is the effect with the single largest impact on the nonlinear optics of the LHC appears validated. Large coupling created shifts to amplitude detuning on par with that introduced by the Landau octupoles, and reduced DA by an amount comparable to octupole sources an order of magnitude stronger than the b_4 errors present in the LHC. Measurement of RDTs was affected by even small coupling shifts. Both the magnitude of the coupling and complex phase of f_{1001} played a significant role. With clear potential for operational impact, high quality $|C^-|$ correction in the LHC is important not only for coupling's sake, but also for the impact on nonlinear dynamics.

ACKNOWLEDGMENTS

Many thanks to the LHC operators and EICs who assisted the measurements presented in this paper.

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