

APPLYING SQUARE MATRIX TO OPTIMIZE STORAGE RING NONLINEAR LATTICE*

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Abstract

We present a new method of using linear algebra technique to analyze nonlinear periodical beam dynamics and its application on different ring lattices. For a given dynamical system, a square upper triangular matrix is constructed out of a one-turn Taylor transfer map. First we separate the matrix into different low dimensional invariant subspaces according to their eigenvalues. Then a stable Jordan transformation can be obtained on each subspace. The transformation provides an excellent action-angle approximation to the solution of the nonlinear dynamics. And the deviation of the new action from constancy provides a measure of the nonlinearity of the motions, which provides a novel method to optimize the nonlinear dynamic system. We applied this method to optimize various ring lattices, such as NSLS-II, SPEAR3, and APS-U lattice, the promising dynamic aperture have been achieved from both tracking simulation and experimental measurements.

NEW ACTION AND ANGLE

Long-term nonlinear behaviour of charged particles in storage rings plays a vital role in beam dynamics. To determine the stable region (dynamic aperture) in which particle can stably survive, one can analyze particle motion under many iterations of the one-turn-map. The most reliable numerical approach is using a tracking code with appropriate local symplectic integration methods. But for the analysis of the dynamics, there are several methods by studying the one-turn-map, such as canonical perturbation theory, Lie operators, power series, and normal form, etc. Here, we would like to understand the one-turn-map from a somewhat different perspective (i.e., using linear algebra techniques to analyze and optimize nonlinear beam dynamics.)

The detailed theory was described in ref. [1]. A set of new action-angle variables (r_j, ϑ_j) , where $j = 0, 1, 2 \dots$, can be extracted by expanding the one-turn-map into a square matrix. Each (r_j, ϑ_j) has different power orders of particle canonical coordinates. Among them, (r_0, ϑ_0) have a wealth of information about the dynamics. Consider one particle with initial linear actions $J_{x,y}$. It is launched for a multi-turns tracking. The linear actions are no longer constants when nonlinearity dominates over linear dynamics. There is a distortion from flat planes in the Poincare section. We characterize this distortion by $\Delta J/J = (J_{max} - J_{min})/J_{mean}$. When the distortion is large, particles receive large nonlinear kicks, and hence the motion becomes chaotic or even unstable. The stable region in

phase space is defined as dynamic aperture. The goal of nonlinear optimization is to reduce the nonlinear distortion, and hence increase the dynamic aperture. In the projected 1D case, this is equivalent to optimizing the trajectories in the normalized phase space (x, p_x) or (y, p_y) so that they are as close as possible to circles.

In order to minimize the distortion, we need to calculate $J_{x,y}$ from constant $r_{x,y}$, in which an inverse function calculation is required. We figure out an easy way to avoid the inverse function calculation. Minimizing $\Delta J/J$ is equivalent to optimizing the system so that constant planes in the Poincare sections in $J_{x,y}$ space are mapped to approximate flat planes in the Poincare sections in the $r_{x,y}$ space, and vice versa. Therefore we characterize the distortion from the flat $r_{x,y}$ planes mapped from constant $J_{x,y}$ planes as a measure of nonlinearity. The system needs to be optimized by mapping $r_{x,y}$ from constant $J_{x,y}$ as close as possible to constants at different oscillation amplitudes.

APPLICATIONS

We applied this method to optimize several machine nonlinear lattices.

NSLS-II

Our first example is applying this method to the National Synchrotron Light Source-II (NSLS-II) with a challenging linear chromaticity of +7 in two planes. The NSLS-II lattice layout is described in ref. [2]. After tuning the chromaticity to +7 with 3 families of chromatic sextupoles, the key to optimization was the manipulation of 6 families of non-chromatic sextupoles. In this case we selected 3 sets of constant $J_{x,y}$ in the Poincare section. In each set, we cast a total of 64 initial coordinates uniformly distributed on the plane. For every set of sextupole configuration, we calculated the new action $r_{x,y}$ for all of the 3 sets of particles using the square matrix method. For each set, the nonlinearity measure was the optimization objective. Because we need to control the distortion for different sets simultaneously, we adopted the multi-objective genetic algorithm (MOGA) [3]. The choice of initial values was not unique. The question about how many sets should be used, and how many points should be casted inside each set is open for future exploration. After 85 generation and an evolution of 4000 populations, the MOGA optimizer converged to an optimal solution.

The dynamic aperture (FIG. 1) is sufficient for the off-axis top-off injection. And it is quite robust to all kinds of engineering imperfection, such as, magnet and gird misalignment, and high order multipole components. As we mentioned before, minimizing $\Delta J/J$ is equivalent to optimizing the trajectories in the normalized phase space (x, p_x) , (y, p_y) so that they are as close as possible to circles.

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The solution we obtained with the square matrix has a very regular motion phase space (Fig. 2), while another solution, which optimized by the conventional method (minimizing the nonlinear driving terms [4]) has much chaotic trajectory. It is worth noting that when tight physical apertures are present in storage ring, particle with a chaotic motion can be scraped by the boundary of the physical apertures, which results in a reduction of effective dynamic aperture. Regular motion is not limited in this way as can be seen in Fig. 4.

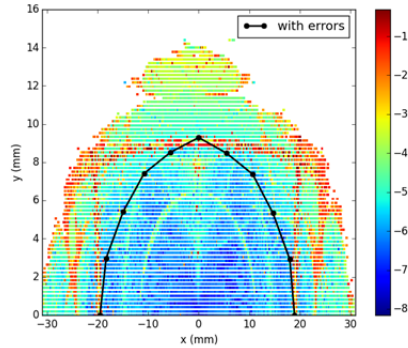


Figure 1: On-momentum dynamic aperture w/o errors.

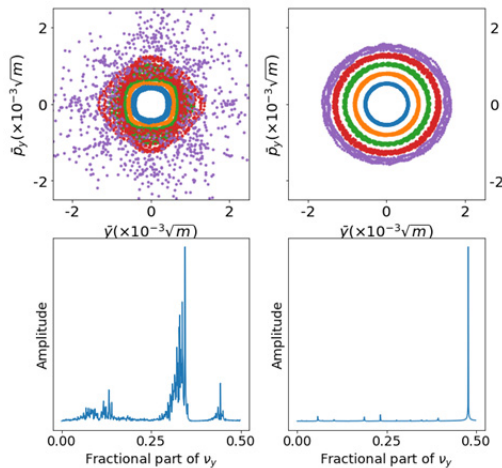


Figure 2: Comparison the normalized (y, p_y) trajectories in phase space and the spectra analysis. The left plots are the solution obtained with a conventional method. The right ones are optimized with the square matrix.

We observed that the tune footprint (see Fig. 3) of the new nonlinear lattice has very large amplitude-dependent tune shift in both the horizontal and the vertical planes. It is remarkable that many particles can survive on a number of resonances at large amplitude. Figure 5 illustrates a simulated horizontal trajectory in phase space while its horizontal tune is almost exactly at a third order resonance. This indicates the irregularity near the resonance $3\nu_x = n$, has been almost completely eliminated. Usually $3\nu_x$ is regarded as a dangerous resonance in sextupole-dominated nonlinear lattice.

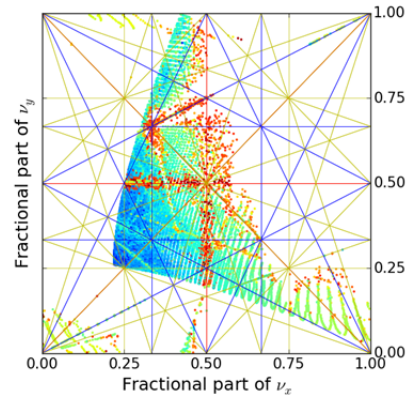


Figure 3: Tune footprint for the on-momentum dynamic aperture. The color of each dot represents its tune diffusion as defined in frequency map analysis. When the tune crosses the third order resonance $3\nu_x = n$, there is no beam loss, and even no obvious diffusion.

For some machines, tunes can cross the third order resonances at small amplitude. When a particle's tune approaches a resonance, its amplitude will be blown-up and its tune is shifted off the resonance, which serves as a stability mechanism. The nonlinear force drives particles' tunes and amplitudes to vary, which leads to a visible tune diffusion and amplitude fluctuation. In this case, the resonance stop-band width is wide, and the motion stability is sensitive to errors. It is almost impossible for particle to cross the resonance at large amplitudes. In the past, it is a convention to confine the tune footprint within a narrow range. The behavior of solution found by the square matrix, however, is very different than solutions obtained in a conventional method [4]. Figure 5 illustrates that one particle can stably "stay" on the $3\nu_x$ resonance without obvious tune diffusion and amplitude fluctuation at a large amplitude around $x=13.5\text{mm}$. For each trajectory the tune is uniquely determined by its amplitude (action). It is independent of the phase angle. The nonlinear behavior is like a near-integrable system. Further exploration to understand beam nonlinear behavior in the vicinity of resonances is still under way.

APS-U

The APS-Upgrade lattice will use MBA (Multi-Band Achromat) scheme to achieve the diffraction limit beam size [5]. We took one of their 7BA candidates to apply the square matrix method to implement our method. There are 12 independent sextupole families including two chromaticity knobs. After a few generations of MOGA optimization, a decent solution was found. The comparison between our method and a tracking-based method was illustration in Fig. 6. Even the dynamic aperture areas and Local Momentum Acceptance (LMA) are similar; these two lattices configurations are very different, such the amplitude detuning terms (Table 1). Recently APS colleagues used the objective function established by the square matrix method principle to find a better solution for their upgrade lattice [6].

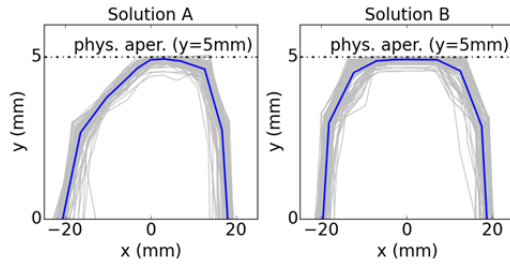


Figure 4: Comparison of effective dynamic apertures of two solutions when a physical aperture limitation $y=5\text{mm}$ and multipole errors are present. The blue lines are the aperture averaged over 80 random seeds (light-gray lines). The left lattice is optimized in a conventional method, which's top-left corner is scraped due to the chaotic vertical motion.

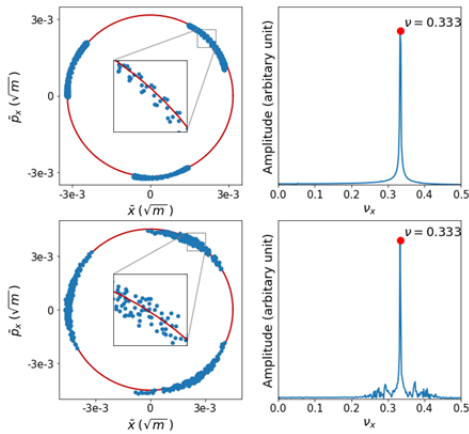


Figure 5: Simulated horizontal phase space trajectories with their tunes at a third order resonance. In the left plots and their zoomed-in subplots, the red lines represent constant linear actions. The blue dots are the simulated turn-by-turn data with the “ELEGANT” code. The frequency spectrums (right) indicate that the particle can stably stay at the resonance line. The top plots are for an ideal machine, and the bottom plots for the machine with the errors.

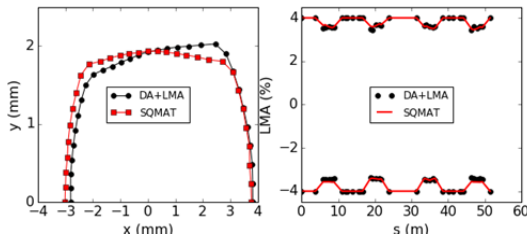


Figure 6: Comparison the dynamic aperture and LMA for one APS-U candidate lattice.

Table 1: Comparison of tune-shift-with-amplitude between two solutions in Figure 6

	dv_x/dJ_x	dv_x/dJ_y	dv_y/dJ_y
SQMAT	2.24×10^5	-3.36×10^5	-2.02×10^5
DA+LMA	1.44×10^4	-4.54×10^4	-5.31×10^4

SPEAR-3 Upgrade

The on-going SPEAR3 upgrade plane is reducing the beam emittance from 10nm to 6-7nm [6]. We applied our method to obtain a set of sextupole configuration for one of candidate lattices. In the meantime, SLAC colleague found another solution based on a tracking-based optimization [7]. Their DAs were experimentally measured at the SPEAR3 ring. By observing the stored beam loss rate with a gradually increasing kicker voltage, we found these two DAs are comparable (Fig. 7). The one optimized with the square matrix is slightly better than another one based on tracking.

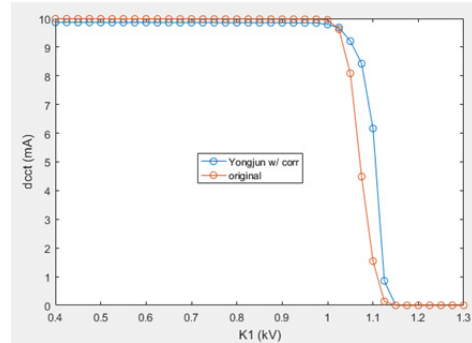


Figure 7: Experimental comparison of two DAs at the SPEAR3 ring. The blue line is obtained with the square matrix, and the red one is obtained by a tracking-based optimization.

CONCLUSION

Through the use of linear algebra techniques we were able to develop a new method for analyzing periodic, nonlinear dynamical systems. Our method was successfully field-tested by optimizing the NSLS-II lattice. Most importantly, optimization using this method has generated an unprecedented nonlinear lattice that allows particles to stay exactly on resonance. Preliminary studies on applying it on the APS-U and SPEAR 3 upgrade lattices show that it is also quite promising.

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