# CRUCIAL TRANSVERSE BEAM DYNAMICS OF THE RACETRACKSHAPE FIXED FIELD INDUCTION ACCELERATOR FOR GIANT CLUSTER IONS* 

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## Abstract

The beam dynamics properties a racetrack-shape fixed field induction accelerator (RAFFIA) have been calculated by linear optics. An importance of uniformity in the magnet field profile and COD correction by steering magnets are discussed. Multiparticle tracking simulation has been carried out to confirm the orbit stability in transverse directions. Space-charge effect that is a big issue of any circular accelerator with low energy injection energy is analyzed by using the nonlinear beam envelope equation. Critical beam current is obtained, beyond which the equilibrium beam core become unstable and chaotic on the beam-core phase space $(\sigma, d \sigma / d s)$.

## INTRODUCTION

Specific features of energetic cluster ions interaction with a target material called cluster effects are well known. Recently, Especially, use of cluster ions with the energy from several keV to tens $\mathrm{MeV}[1-5]$ has attracted interests from mutation or material science. A tandem accelerator has been used for cluster ion acceleration [6]. Unfortunately, it has a limitation in maximum energy. As a method to repeatedly accelerate cluster ions, the RAFFIA was proposed at KEK in 2015 [7].
The RAFFIA employs the induction acceleration system. The RAFFIA has been designed, which accelerates C-60 ( $A=720$ ) cluster ions with the charge state of 10 from 10 MeV to 144 MeV . The RAFFIA ring with two straight sections consists of four fixed-field bending magnets, ramping quadrupole doublets, steering mag-nets, electrostatic injection kicker, and induction cells [8]. The bending magnet is a key component, which has the gradient field in the main pole region and the reverse field in its front.
It is crucial to find optics parameters of the RAFFIA in order to assure a transverse beam stability. The particle tracking code to simulate particle orbits through the acceleration cycle has been developed. A linear orbit theory has been established to figure out the essential features of the RAFFIA such as the lattice function, momentum dispersion function, and momentum compaction factor. Non-uniformity of the magnetic field along the longitudinal direction of the bending magnet is evaluated to yield the substantially large closed orbit distortion (COD). A nonlinear beam-core evolution equation perturbed by space-charge

[^0]effects approach is used to estimate the acceptable beam current.

## BENDING MAGNET OF THE RAFFIA

Beam orbit of the RAFFIA is set up by four bending magnets, forming a racetrack-shaped orbit. Since the injection/extraction (edge) angle of $45^{\circ}$ induces extremely large edge defocusing in the vertical direction, a reverse field strip at the front edge and field gradient on the main pole is crucial to provide net focusing in the horizontal and vertical direction. The reverse field and main bending field are excited by a single pair of coils. The bending magnet is designed by using ANSYS $3 \mathrm{D} ®$ and the field data on the median plane of the magnet gap as shown in Fig. 1, is imported for beam tracking simulation and beam dynamics calculation.


Figure 1: Designed magnetic field data on the median plane.

## ORBIT STABILITY ANALYSIS

## Stability of The Transverse Motion

For the acceleration without particle loss, it is crucial to ensure the stability of the transverse motion in a ring accelerator. A linear-optics calculation using a transfer matrix is well established for analysis of the transverse motion in a synchrotron.
Unfortunately, the linear-optics cannot be directly applied to the transverse motion in the RAFFIA, since the orbit in the bending magnet varies according to the beam energy. Two approaches are used to analyze the stability of the transverse motion in the RAFFIA; particle tracking simulation and linear-optics calculation. Runge-Kutta
method is used to solve the equation of motion in the particle tracking simulation. A stable orbit is obtained by optimizing the field gradient of the quadrupole doublet at each turn. Once a one-turn orbit is fixed by the tracking simulation, magnetic field, field gradient and bending radius are sampled along the orbit. The linear-optics calculation is available to obtain one-turn transfer matrix $M$.

## Beta Function

The size of the oscillation amplitude is important. It can be given by $(\varepsilon \beta)^{1 / 2}$ where $\varepsilon$ is beam emittance and $\beta$ is言 beta-function. $\varepsilon$ is related to the beam parameter, while the d beta function depends on the focusing component of the ol lattice. The horizontal beta-function of the RAFFIA at in. jection and extraction energy are shown in Fig. 2.


Figure 2: Horizontal beta-function at injection (upper) and extraction energy (lower).

## Closed Orbit Distortion

The magnetic fields of the real bending magnet deviate from that of the ideal one due to a finite size of the magnet. The magnetic field deviation $\left(\Delta B_{y}\right)$ yields the Closed Orbit $\approx$ Distortion (COD). $\Delta B_{y}$ varies with acceleration. $\Delta B_{y}$ becomes the source of inherent COD in the RAFFIA. It is important to design a bending magnet with small $\Delta B_{y}$ in order to reduce the inherent COD. The COD at injection energy is shown in Fig. 3. The COD must be compensated $\stackrel{\circ}{\sim}$ by additional magnetic field of the steering magnet.

## Dispersion Function

The dispersion function of RAFFIA at injection and extraction energy are shown in Fig. 4. The dispersion function in the straight-section where the induction acceleration device located is almost zero. Emittance blow up caused by synchro-beta coupling [9] can be avoided with this configuration.


Figure 3: COD at the injection energy.


Figure 4: Dispersion function of the RAFFIA at injection energy (upper) and extraction energy (lower).

## Analysis of Tracking Results

In order to confirm the stable acceleration of cluster ion beams in the transverse direction, the macroparticle tracking of a gaussian beam over the entire acceleration cycle is carried out, assuming the discrete and constant step acceleration voltage. In this macroparticle tracking, the orbit stability is obtained by ramping the gradient field of the quadrupole doublets and the magnetic field of the steering mag-


Figure 5: The beam size during acceleration in horizontal direction monitored at the QF and vertical direction monitored at QD.
nets. The temporal evolution of the beam sizes from injection to extraction are shown in Fig. 5 where beam size blow-up in the horizontal and vertical direction are observed at the several regions of turns, although beam loss is not observed.

The beam size growths are analyzed by their foot-prints on the tune space and shown in Fig. 6. The horizontal beam growth at 110 and 400 turns are caused by half integer resonance crossing ( $\nu_{x}=5 / 2$ ), and at 1100 turns caused by third integer resonance crossing $\left(v_{x}=8 / 3\right)$. In the vertical direction, the beam growth at 120 and 350 turns may be caused by third integer resonance crossing $\left(v_{y}=1 / 3\right)$. Although the tunes pass the coupling resonance line at early stage of acceleration, the effects are not significant. The beam size growths may be attributed to the sextupole field components in the bending magnet.

Details of orbit stability analysis have been given in Reference [8].


Figure 6: Tune foot-print.

## SPACE-CHARGE EFFECTS

For the simplicity, particles are assumed to have a Gaussian distribution in the transverse direction. In this model, the beam core consisting of particles distributed from $-\sigma$ to $+\sigma$ uniquely determines the space charge potential. Space-charge effects are handled in the coupled nonlinear beam core evolution equations [10] with the following form

$$
\left\{\begin{array}{c}
\sigma^{\prime \prime}{ }_{x}+K_{x}(s) \sigma_{x}-\frac{k_{1}}{\sigma_{x}(s)+\sigma_{y}(s)}=\frac{\varepsilon_{x}^{2}}{\sigma_{x}^{3}}  \tag{1}\\
\sigma^{\prime \prime}{ }_{y}+K_{y}(s) \cdot \sigma_{y}-\frac{k_{1}}{\sigma_{x}(s)+\sigma_{y}(s)}=\frac{\varepsilon_{y}^{2}}{\sigma_{y}^{3}}
\end{array}\right.
$$

where $K_{x, y}$ are the restoring coefficients originated from the magnetic focusing, $k_{1} \equiv \frac{2}{\beta^{3} \gamma^{3}} \cdot\left(\frac{I_{B}}{I_{0}}\right)$ is the perveance, $I_{0}$ is the Alfven current for a specific giant cluster ion, $I_{b}$ is the beam current, $x, y$ are the horizontal and vertical displacement, respectively, and $\beta$ and $\gamma$ are relativistic beta and gamma.
The beam core equations are solved by the Runge-Kutta method. The Poincare map for the transverse beam core evolutions over 300 turns are shown in Fig. 7. Up to a certain limit of the beam current, the beam core size slightly changes with the stable resonant pattern. Beyond this limit, the beam core size becomes chaotic on the beam core phase space ( $\sigma, d \sigma / d s$ ). Its threshold is rigorously obtained by the tangential mapping around the equilibrium
beam core, that is, the linear analysis around the matched equilibrium orbit. This is explained by the perio-doubling bifurcation of the elliptical point which corresponds to the matched equilibrium orbit. The maximum beam core size as a function of beam current are shown in Fig. 8. One clearly finds that the injected beam current is limited. Details has been discussed in Reference [10].


Figure 7: The Poincare map of the beam core evolution observed at the injection point for (a) $I_{B}=50 \mu \mathrm{~A}$, (b) $I_{B}=200$ $\mu \mathrm{A}$, and (c) $I_{B}=250 \mu \mathrm{~A}$.


Figure 8: Maximum beam size vs. beam current.

## SUMMARY

The complete linear orbit theory for the RAFFIA has been developed. The space charge effects are evaluated by analyzing the nonlinear beam core evolution equation. The beam current threshold has been theoretically evaluated.

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