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POSSIBLE APPLICATION OF ROUND-TO-FLAT HADRON BEAM CREATION USING 3RD ORDER COUPLING RESONANCES FOR THE ELECTRON-ION COLLIDER

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Abstract

An Electron-Ion Collider (EIC) is planned to be built in Brookhaven National Laboratory with the contribution from Jefferson National Laboratory. To have a high luminosity, both the EIC ion bunch and the EIC electron bunch are designed to be flat during their collision. The existing injector source provides a round beam of width 2.5 µm rad transverse emittances. In this paper we investigate the option of dynamically crossing the $2\nu_x - \nu_y$ coupling resonance in order to create a flat-beam ratio ϵ_x/ϵ_y of up to 4. Furthermore, we explore the possibility of using a pulsed- or AC skew sextupole magnets to achieve a similar effect. Using one of these methods for flat beam creation will help lower the ion beam cooling time.

INTRODUCTION

The next large-scale accelerator project in the U.S. is the planned Electron-Ion Collider (EIC) to be built in Brookhaven National Laboratory in partnership with the Jefferson National Laboratory. EIC will reuse one of the existing Relativistic Heavy Ion Collider (RHIC) rings for the ions beam, while a completely new electron synchrotron and electron storage ring will be built. One of the existing RHIC pre-accelerators, namely the Alternating Gradient Synchrotron (AGS) will also be reused. To achieve the highest possible luminosity, the electron- and hadron beams should have identical beam sizes at the interaction point

$$\sigma_z = \sqrt{\beta_z^* \epsilon_z},$$

where z is either x or y and * indicates values at the interaction point. In [1] the explicit dependence of the luminosity on the transverse beam parameters can be shown to be

$$L \propto \frac{1 + \frac{\sigma_y}{\sigma_x}}{\sqrt{\frac{\sigma_y}{\sigma_x}}} \left(\frac{1}{\beta_{xe}^* \beta_{ye}^* \beta_{xh}^* \beta_{yh}^*} \right)^{1/4}. \quad (1)$$

From Eq. (1) we clearly see that colliding flat beams with σ_y small is beneficial. The optimum value of $\sigma_y/\sigma_x = 0.08$ leads to $\approx$ 50% higher luminosity than the round beam case [1]. The flat beam is achieved by having $\beta_x^* \gg \beta_y^*$ and $\epsilon_x > \epsilon_y$. The electron beam emittances are naturally different due to the interplay between radiation damping and quantum excitation, while flat hadron beams must typically be created by other means. The established concept for creating horizontally flat hadron beams is to first damp the beam in both planes followed by an excitation of the horizontal emittance using kicker noise. Here we investigate an alternative scheme to create the horizontally flat beam, while simultaneously decreasing the vertical emittances.

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ROUND-TO-FLAT BEAM BY CROSSING $2\nu_x - \nu_y$

The effect of dynamically crossing third-integer coupling resonances has previously been studied in detail for the beam distribution evolution when crossing the $\nu_x - 2\nu_y$ "Walkinshaw" resonance [2, 3]. When crossing this resonance, an initially round beam becomes flat with $\epsilon_x < \epsilon_y$. For our application we are interested in achieving $\epsilon_x > \epsilon_y$ from a flat beam. Due to symmetries, this can be achieved by crossing the $2\nu_x - \nu_y$ resonance instead.

The distance to the resonance is defined as

$$\delta = 2\nu_x - \nu_y - \ell$$

with the resonance crossing speed $\dot{\delta} = \mathrm{d}\delta/\mathrm{d}t$. We will here restrict ourselves to constant $\dot{\delta}$. According to [2], the strength of the resonance is

$$G_{2,-1,\ell} = \frac{\sqrt{2}}{8\pi} \oint \beta_x \beta_y^{1/2} j_3 \mathrm{d}s, \quad (2)$$

where j_3 is the skew sextupole strength. A simple tracking simulation is used to visualize the emittance exchange. A simple uncoupled one-turn map on the form

$$\mathbf{M}(\nu_x, \nu_y) = \begin{pmatrix} C_x + \alpha_x S_x & \beta_x S_x & 0 & 0 \\ -\gamma_x S_x & C_x - \alpha_x S_x & 0 & 0 \\ 0 & 0 & C_y + \alpha_y S_y & \beta_y S_y \\ 0 & 0 & -\gamma_y S_y & C_y - \alpha_y S_y \end{pmatrix}$$

is used for tracking of 1000 particles for 5000 turns. Here $S_z = \sin(2\pi\nu_z)$ and $C_z = \cos(2\pi\nu_z)$, z being either x or y. On each turn, the horizontal tune is adjusted such that $\nu_x = 0.225 \to 0.175$ with fixed vertical tune of $\nu_y = 0.4$. This give a resonance crossing speed of $\dot{\delta} = -2 \times 10^{-5}$. After passing one turn, the beam meets a thin skew sextupole providing a kick

$$x' \to x' + j_3 xy,$$
$$y' \to y' - \frac{j_3}{2}(y^2 - x^2).$$

We use $j_3\ell = 4.7\,\mathrm{m}^{-2}$, $\beta_x = 8\,\mathrm{m}$ and $\beta_y = 2\,\mathrm{m}$ leading to $G_{2,-1,\ell} = 3\,\mathrm{m}^{-0.5}$. Furthermore we use $\alpha_x = \alpha_y = 0$. The initial emittances are $\epsilon_x = \epsilon_y = 2.5$ µm rad. The evolution of the emittances during the crossing are plotted in Fig. 1.

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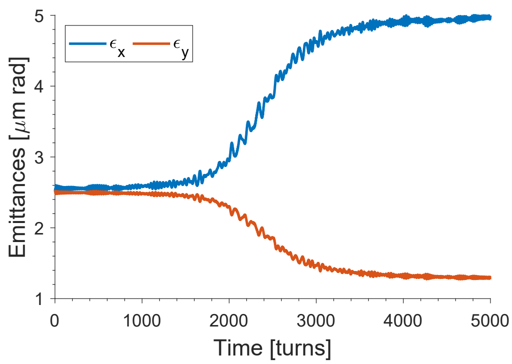

Figure 1: Evolution of emittances while crossing the $2\nu_x - \nu_y$ coupling resonance with an initial round beam condition. A flat-beam ratio of 3.64 is produced with this configuration.

After the resonance crossing the flat-beam ratio is $\epsilon_x/\epsilon_y = 3.87$ for this example.

In [2] the authors introduce the effective resonance strength parameter, G_{eff} for the resonance crossing of $\nu_x - 2\nu_y$ which describes the scaling of the resonance crossing when the beam is initially round ($\epsilon_{x0} = \epsilon_{y0}$). Since the round-beam condition is also our starting point, and due to symmetry, the same parameter determines the scaling of $2\nu_x - \nu_y$, but using $G_{2,-1,\ell}$ from Eq. (2) instead of $G_{1,-2,\ell}$ as in [2]. The effective strength parameter becomes[1]

$$G_{\text{eff}} = G_{2,-1,\ell}\sqrt{\frac{\epsilon_{x0}\pi}{|\dot{\delta}|}}. \tag{3}$$

To estimate the skew sextupole magnet strength needed for a good emittance exchange, we redo the tracking simulations of Fig. 1 several times for different values of G_{eff}. The result is plotted in Fig. 2, where the "fractional emittance growth" (FEG) $|\Delta\epsilon_x/\epsilon_{x0}| + |\Delta\epsilon_x/\epsilon_{x0}|$ is used to estimate the quality of the exchange. The exchange is maximized, and therefore best for our purpose, when this summed change is ≈ 1.5, which happens for $G_{\text{eff}} > 0.6$.

The operational performance of the emittance exchange from crossing the $2\nu_x - \nu_y$ resonance is complicated by the fact that skew sextupoles are not a standard component in synchrotrons. To achieve $G_{\text{eff}} > 0.6$ with a modest value of $G_{2,-1,\ell}$ it is therefore imperative to have a very slow resonance crossing. Alternatively additional skew sextupole must be installed in regions where β_x (and β_y) is large to create a higher value of $G_{2,-1,\ell}$. Furthermore, the installed skew sextupoles leads to additional non-linearities and associated resonances which must be considered carefully.

ROUND-TO-FLAT BEAM BY PULSED- OR AC SKEW SEXTUPOLE

In the resonance crossing the distance to the resonance, δ, is adjusted dynamically while the strength of the resonance,

[1] Since our definition of the emittances and $G_{2,-1,\ell}$ differs by, respectively, a factor π and $\pi^{-1/2}$ compared to [2], we have included a factor $\pi^{1/2}$ to make results comparable to [2].

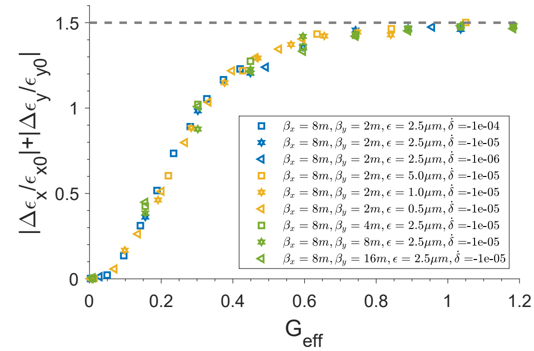

Figure 2: Fractional emittance growth (FEG) as a function of the effective resonance strength, G_{eff}, after crossing $2\nu_x - \nu_y$ under different conditions. The dotted line indicates the maximum possible FEG for this resonance.

$G_{2,-1,\ell}$, is fixed. However, the opposite situation can also be used to create flat beams. Consider the situation where the tunes are directly on the $2\nu_x - \nu_y$ resonance ($\delta = 0$), and $G_{2,-1,\ell} = 0$ initially. At time $t = 0$ a skew sextupole is turned on and after a time Δt the magnet is turned off. When the magnet is pulsed, the resonance stop band will overlap with the working point and the emittances exchange, as illustrated in Fig. 3 with $\nu_x = 0.2$, $\nu_y = 0.4$ and a single skew sextupole inducing the resonance strength of $G_{2,-1,\ell} = 1$ for $\Delta t = 140$ turns. Our results confirm Fig. 1 in [4], which shows that a flat beam ratio of $\epsilon_x/\epsilon_y \approx 2.5$ (corresponding to FEG ≈ 1) can be reached using this method. Figure 3 show the optimized case where the skew sextupole is turned off at the optimum time; if the magnet is kept on for longer the emittance exchange will start to decrease. We find that an integrated resonance strength defined as

$$G_{\text{int}} = \frac{\sqrt{2\epsilon_{x0}}}{8\pi}\beta_x\beta_y^{1/2}\int j_3(t)\ell \mathrm{d}t,$$

can be used to predict the optimum pulse duration. For a square-pulse the integral evaluates to $j_3\ell\Delta t$. Figure 4 plots the FEG as a function of G_{int} for a few different cases. We find that the best exchange happens at $G_{\text{int}} \approx 2/(3\pi)$.

AC SKEW SEXTUPOLE

Using a pulsed skew sextupole requires that the working point is very close to the resonance, i.e. $\delta \approx 0$. It can easily be found in simulation that the best obtainable FEG value drastically decreases as δ increases. However, a non-zero δ is needed in the practice, since $G_{2,-1,\ell} \neq 0$ for a real machine. Another option is using an alternating current (AC) skew sextupole to excite the resonant condition despite having $\delta \neq 0$. The resonance condition is now

$$2\nu_x - \nu_y - \nu_{\text{osc}} - \ell = 0,$$

where ν_{osc} is the frequency of the skew sextupole with

$$j_3(t) = j_{3,\max}\sin(2\pi\nu_{\text{osc}}t). \tag{4}$$

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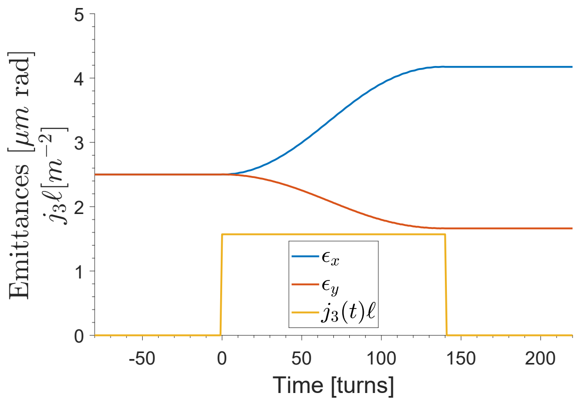

Figure 3: Round-to-flat beam creation using a pulsed skew sextupole on the $2\nu_x - \nu_y$ resonance. A flat beam ratio of $\epsilon_x/\epsilon_y \approx 2.5$ can be reached using this method.

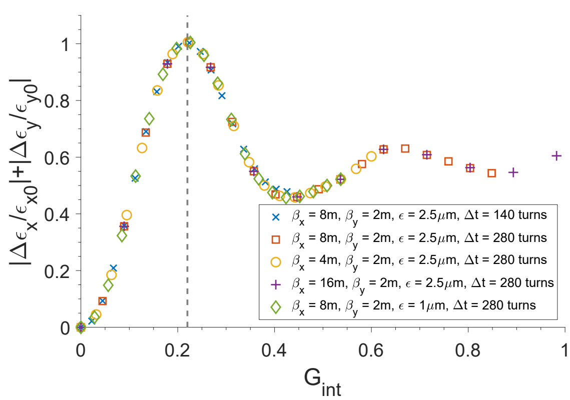

Figure 4: Fractional emittance growth as a function of the integrated resonance strength when pulsing a skew sextupole with working point on the $2\nu_x - \nu_y$ resonance.

Two options are considered: i) a fixed value of ν_{osc} according to Eq. (4) or (2)) sweeping the frequency across the resonance. Here we show only the swept-frequency option, and merely state that the fixed-frequency option is able to reach an emittance ratio of 2.5 as was the case for the pulsed skew sextupole method. The swept-frequency option is now shown in tracking simulation. We select $\nu_x = 0.2$, $\nu_y = 0.42$. We sweep the oscillation frequency such that $\nu_{osc}(t) = 0.01 \rightarrow 0.03$ over 8000 turns, with the resonant frequency being at 0.02. The evolution of the emittances during this process is plotted in Fig. 5. This example leads to $\epsilon_x/\epsilon_y \approx 3.9$, which is close to what can be achieved by resonance crossing.

DISCUSSION

At the EIC, the beam will be transversely damped to decrease the vertical emittance before increasing the horizontal emittance with, e.g., kicker noise [1]. Using one of our proposed options for creating flat beams will naturally decrease the vertical emittance, and thereby decrease the total required damping time to reach the target emittances.

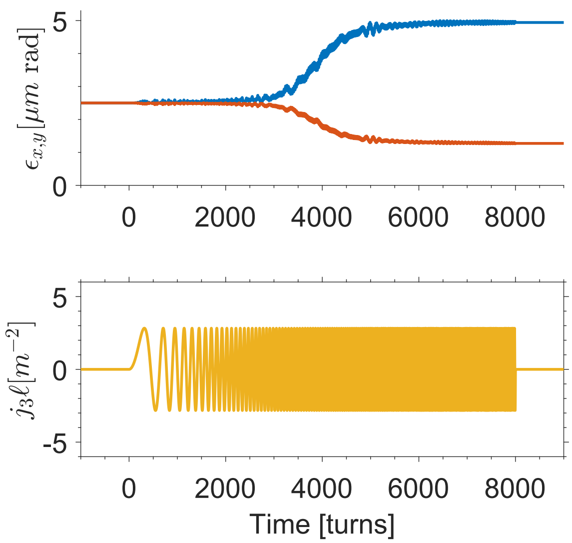

Figure 5: Round-to-flat beam creation using a skew-frequency alternating current skew sextupole. A flat beam ratio of $\epsilon_x/\epsilon_y \approx 3.9$ can be reached using this method.

Using an oscillating skew sextupole is attractive. First of all it avoids having to change the machine working point in order to cross the resonance; the externally induced resonance condition can readily be turned on and off as soon as the emittance exchange is optimal. Using the swept-frequency option can lead to a flat beam ratio of 4.0, which is the same as for the resonance crossing case.

The methods presented above for creating a flat beam with $\epsilon_x > \epsilon_y$ rely on creating a strong resonance. For the dynamic crossing of $2\nu_x - \nu_y$, we would need strong skew sextupole fields in order to avoid a non-adiabatic emittance exchange or the need for long crossing times. Such strong fields must be supplied with this purpose in mind. Similarly, the pulsed skew sextupole method requires additional equipment. An alternative approach is to cross the Walkinshaw resonance, which may be stronger due to the normal sextupoles already present in synchrotrons for chromaticity correction. The Walkinshaw resonance would produce a flat beam with $\epsilon_x < \epsilon_y$, which can then subsequently be exchanged by either crossing the linear difference resonance [5–7] or using a pulsed- or AC skew quadrupole [8].

CONCLUSION

We have investigated options for round-to-flat beam creation with $\epsilon_x > \epsilon_y$ using third order coupling resonances. The first proposed option is by crossing the $2\nu_x - \nu_y$ resonance, which under ideal conditions can lead to a flat beam ratio of 4. Another option is by pulsing a skew sextupole while having the working point close to $2\nu_x - \nu_y$. This option can provide a flat beam ratio of 2.5. Finally, a swept-frequency AC skew sextupole scheme was considered to excite the resonance condition. With this method we again achieve a flat beam ratio of ≈ 4.

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