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A DISPERSIVE QUADRUPOLE SCAN TECHNIQUE FOR TRANSVERSE BEAM CHARACTERIZATION

J. Kallestrup*, M. Aiba, Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland
N. Carmignani, T. Perron, ESRF, Grenoble, France

Abstract

Quadrupole scans are one of the standard techniques to characterize the transverse beam properties in transfer lines or linacs. However, in the presence of dispersion the usage of regular quadrupole scans will lead to erroneous estimates of the beam parameters. The standard solution to this problem is to measure the dispersion and then subtract it in the post-analysis of the quadrupole scan measurements assuming the design energy spread. Here we show that the dispersive contribution to the beam size can be included in the quadrupole scan procedure, forming a linear system of equations that can be solved to obtain both the betatronic and dispersive beam parameters. The method is tested at both the SLS and ESRF booster-to-ring transfer lines leading to reasonable estimates of the beam parameters.

INTRODUCTION

A key ingredient to successful injections into a storage ring is to ensure the optimum optical parameters of the injection beam. The optimum set of parameters can be calculated analytically for kicker-bump based off-axis injections [1, 2]. The correct shaping of the phase space is done in the transfer line between the injector and storage ring using quadrupoles. Good knowledge of the beam coming from the injector is necessary to perform the matching. This includes knowing all the optics parameters and emittances in both transverse planes. The new generation of synchrotron-based light sources utilizing multi-bend achromat lattice designs suffers from limited dynamic aperture, which means that the correct matching of the injection beam is increasingly important.

One of the standard methods for performing transverse beam characterizations is using quadrupole scans [3]. The basic principle is that the betatronic Twiss parameters at a measurement point (MP) downstream of a point-of-interest (POI) are transported as [4]:

$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_{\mathrm{MP}} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2\\ -m_{11}m_{21} & m_{11}m_{22}+m_{12}m_{21} & -m_{22}m_{12}\\ m_{21}^2 & -2m_{22}m_{21} & m_{22}^2\end{pmatrix}\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_{\mathrm{POI}}$$

Where m_{ij} is the ij'th element of the transfer matrix from POI to MP. The squared beam size at the MP is therefore described by the Twiss parameters by performing the matrix multiplication in above to give

$$\sigma_{\mathrm{MP}}^2 = \epsilon\beta_{\mathrm{MP}} = \epsilon(m_{11}^2\beta_{\mathrm{POI}} - 2m_{11}m_{12}\alpha_{\mathrm{POI}} + m_{12}^2\gamma_{\mathrm{POI}})$$

The beam size at the MP can then be changed by varying the values of the matrix elements, e.g., through the variation of quadrupoles located between the POI and the MP. By doing so, we can construct a system of equations on the form

$$\vec{\sigma^2} = \mathbf{M}\vec{p}. \tag{1}$$

With a measurement consisting of N quadrupole settings, $\vec{\sigma^2}$ is a vector of length N, $\vec{p}$ has length 3 and $\mathbf{M}$ is a matrix of size $N \times 3$ defined as

$$\mathbf{M} = \begin{pmatrix} m_{11,1}^2 & 2m_{11,1}m_{12,1} & m_{12,1}^2\\ m_{11,2}^2 & 2m_{11,2}m_{12,2} & m_{12,2}^2\\ \vdots & \vdots & \vdots\\ m_{11,N}^2 & 2m_{11,N}m_{12,N} & m_{12,N}^2\end{pmatrix} \tag{2}$$

The parameter vector, $\vec{p}$, can now simply be found through matrix inversion

$$\vec{p} = \mathbf{M}^{-1}\vec{\sigma^2} = \begin{pmatrix}\epsilon\beta\\ -\epsilon\alpha\\ \epsilon\gamma\end{pmatrix}$$

For robust fitting the number of measurement points, N, should be large to over-constrain the problem. Peudo-inversion must be used when $N > 3$ to find $\mathbf{M}^{-1}$, e.g., using singular value decomposition. If dispersion is non-zero between the MP and POI, then the measured beam size will increase. The dispersion at the POI is propagated to the MP as [4]

$$\begin{pmatrix}\eta\\ \eta'\\ 1\end{pmatrix}_{\mathrm{MP}} = \begin{pmatrix} m_{11} & m_{12} & m_{13}\\ m_{21} & m_{22} & m_{23}\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\eta\\ \eta'\\ 1\end{pmatrix}_{\mathrm{POI}} \tag{3}$$

and the beam size at the MP can then be written as

$$\sigma_{\mathrm{MP}}^2 = \epsilon\beta_{\mathrm{MP}} + (\sigma_\delta\eta_{\mathrm{MP}})^2 \tag{4}$$

If the dispersion is non-zero, then it must be subtracted for all N different quadrupole settings - either by assuming the design dispersion at the POI or using accurate dispersion measurements. Furthermore, the energy spread σ_δ must be assumed. If $\eta_{\mathrm{POI}} = \eta'_{\mathrm{POI}} = 0$ then the dispersion at the MP is fully determined by the m_{13}, which can typically be calculated accurately from magnet specifications etc.

DISPERSIVE QUADRUPOLE SCANS

The issue of knowing (or calculating) the dispersion at the POI can be circumvented by including the dispersive parameters (η, η' and σ_δ) in the quadrupole scan fitting procedure. By transporting both the betatronic- and dispersive beam parameters and inserting them into Eq. (4), the squared beam size at the MP is (we omit the MP and POI subscripts):

* Present address: Diamond Light Source, Oxfordshire, UK. jonas.kallestrup@diamond.ac.uk

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$$\sigma^2 = m_{11}^2\left(\epsilon\beta + (\eta\sigma_\delta)^2\right) + 2m_{11}m_{12}\left(-\epsilon\alpha + \eta\eta'\sigma_\delta^2\right) + m_{12}^2\left(\epsilon\gamma + (\eta'\sigma_\delta)^2\right) + m_{13}^2\sigma_\delta^2 + 2m_{11}m_{13}\left(\eta\sigma_\delta^2\right) + 2m_{12}m_{13}\left(\eta'\sigma_\delta^2\right)$$

Scanning one (or several) quadrupoles in a total of N steps will now lead to a $N \times 6$ matrix on the form:

$$\mathbf{M} = \begin{pmatrix} m_{11,1}^2 & 2m_{11,1}m_{12,1} & m_{12,1}^2 & m_{13,1}^2 & 2m_{11,1}m_{13,1} & 2m_{12,1}m_{13,1} \\ m_{11,2}^2 & 2m_{11,2}m_{12,2} & m_{12,2}^2 & m_{13,2}^2 & 2m_{11,2}m_{13,2} & 2m_{12,2}m_{13,2} \\ \vdots & & & \vdots & & \vdots \\ m_{11,N}^2 & 2m_{11,N}m_{12,N} & m_{12,N}^2 & m_{13,N}^2 & 2m_{11,N}m_{13,N} & 2m_{12,N}m_{13,N} \end{pmatrix}$$

Again, the system of equations is in the form of Eq. (1) with the solution

$$\vec{p} = \mathbf{M}^{-1}\vec{\sigma}^2 = \begin{pmatrix} \epsilon\beta + (\eta\sigma_\delta)^2 \\ -\epsilon\alpha + \eta\eta'\sigma_\delta^2 \\ \epsilon\gamma + (\eta'\sigma_\delta)^2 \\ \sigma_\delta^2 \\ \eta\sigma_\delta^2 \\ \eta'\sigma_\delta^2 \end{pmatrix} \tag{5}$$

from which the betatronic and dispersive beam parameters can be calculated. This method has previously been used in [5], where two single-quadrupole scans were analyzed. However, the authors were not able to obtain good results with their setup. As described in [5], it is necessary to have non-zero m_{13}, i.e., by having a dipole between the point of interest and measurement point. Furthermore, it is beneficial to have a non-constant m_{13}. In [5] the change was achieved by performing two quadrupole scans with a different settings of an intermediate dipole. We instead achieve the change in m_{13} by scanning one quadrupole before a dipole magnet, and one quadrupole after simultaneously. Our setup will typically require more quadrupole settings, but also lead to several different values of m_{13}. Several other quadrupoles may be present in the transfer line between the scanned quadrupoles and the MP.

As in classical quadrupole scans, it is important that the change in phase advance between the MP and POI is well-sampled, preferably up to π.

Since matrix inversion is used, we must ensure that $\mathbf{M}$ is well-conditioned, e.g., by ensuring a low ratio between the highest and smallest singular value of $\mathbf{M}$ [6].

MEASUREMENTS

Measurements using the presented technique were performed in the ESRF and SLS booster-to-storage ring transfer lines. In both cases, we scan two quadrupoles simultaneously. The quadrupoles are scanned in 10 equidistant steps each, leading to a total of $10^2 = 100$ quadrupole settings.

ESRF TL2

The ESRF booster synchrotron provides a 6-GeV beam with an emittance of ≈ 85 nm rad to the new EBS storage ring [7, 8]. The booster has recently been modified to decrease the horizontal emittance to ≈ 85 nm rad [9] and even further using the emittance exchange by coupling resonance crossing technique [7].

We perform a quadrupole scan using the method described above in the ESRF TL2 transfer line. The MP is the synchrotron radiation monitor in the second dipole of TL2, while the POI is chosen to be the beginning of TL2. The measured and fitted squared beam sizes are plotted on Fig. 1a together with the result using the design optical functions at the beginning of TL2. The resulting fitted parameters are given in Table 1. The fit to data is very good and the results are close to the design values. In a dedicated measurement, we found $\eta_x = -0.27$ m and $\eta'_x = -0.08$. While the dispersion derivative is close to the design value and also what is measured in the quadrupole scans, we do find a discrepancy in η_x between the quadrupole scans and the directly measured dispersion. It is not clear where this discrepancy arises and further studies are ongoing. However, it is obvious that the beam sizes that the design optics would provide do not correspond to the measured beam sizes. The design optics are used for the matching into the ESRF-EBS, and therefore we might find an improvement to the injection efficiency if a better matching based on the measured optics can be made.

SLS BRTL

The SLS booster provides a beam with a $\epsilon_x \approx 9.6$ nm rad, significantly less than the ESRF booster beam. Therefore, the dispersive contribution to the beam size is expected to be dominant in some quadrupole scans. The POI is chosen to be at the entrance of the first quadrupole.

The measured squared beam sizes together with fit are shown in Fig. 1b, while the fitted parameters are given in Table 1. Again, we find that the method can be used to fit the measurements reasonably well. The fitted parameters are somewhat different from the design values, and further stud-

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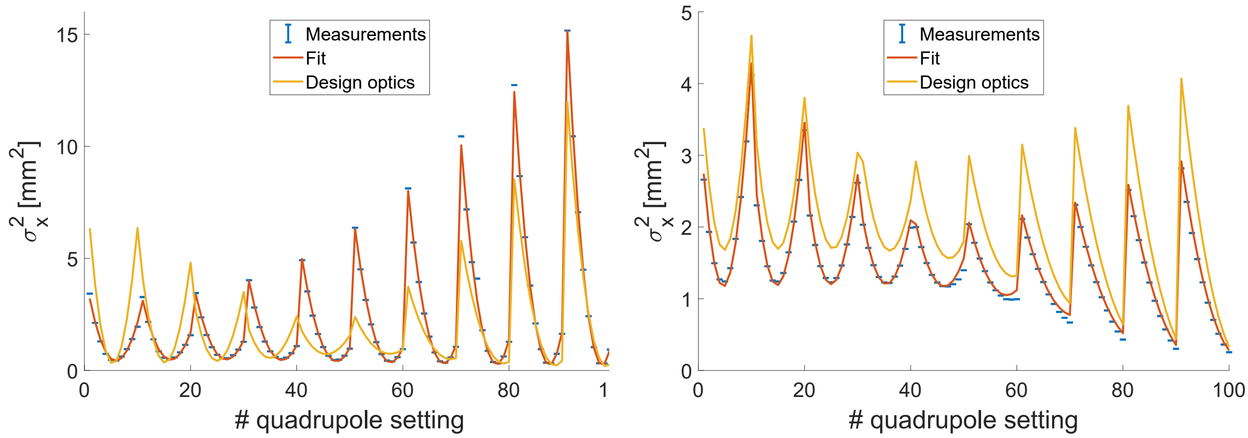

(a) (b)

Figure 1: Scanning of two quadrupoles simultaneously with fit and design optics. a) ESRF TL2 transfer line and b) SLS BRTL transfer line.

Table 1: Horizontal Beam Optics and Parameters in the ESRF TL2 and SLS BRTL Transfer Lines.

		β_x [m]	α_x	ϵ_x [nm rad]	η_x [m]	η'_x [$\times 10^{-3}$]	σ_δ [$\times 10^{-4}$]
ESRF	Design	6.73	-2.21	85	-0.14	-0.09	13.0
	Fit	6.83 ± 0.12	-0.89 ± 0.04	90 ± 2	-0.05 ± 0.02	-0.06 ± 0.01	12.8 ± 0.4
SLS	Design	31.5	-5.60	9.6	0.22	-0.01	7.3
	Fit	22.4 ± 5.8	-3.89 ± 1.05	12.1 ± 3.2	0.37 ± 0.02	0.01 ± 0.00	6.6 ± 0.1

ies have indicated some difficulties in providing a conclusive result (see discussion below).

DISCUSSION

Unlike other fitting methods, e.g., Levenberg-Marquardt, using Eq. (5) does not require a starting point for the fitting procedure. Having no starting point decreases the likelihood of confirmation bias if a local optimum happens to be close to the starting point. The proposed quadrupole scan technique is rather sensitive to both measurement and calibration errors.

In the ESRF TL2 case a calibration error of 1 µm pixel^{-1} (corresponding to approximately 5%) leads to a 10% error on the estimated emittance. We used 100 quadrupole settings for each of the presented scans; the robustness of the fitting can, to some degree, be improved with an increased number of quadrupole settings.

Weighting of the results can be included in the fitting process as shown in [3], but we did not find any clear benefit in doing so.

The optics found during the SLS BRTL measurements are somewhat different from the design values. However, in [10] it was found that performing several scans gave different results of the fitted parameters, but all lead to approximately the same value of $\beta_x \epsilon_x \approx 270 \times 10^{-7}\,\mathrm{m}^2\,\mathrm{rad}$. The analysis showed that the betatronic parameters in these fits were correlated. Assuming the design emittance of 9.6 nm rad (which is a good assumption since the equilibrium emittance of a synchrotron is relatively robust), we can use the measured value of $\beta_x \epsilon_x$ and arrive at $\beta_x = 30.5\,\mathrm{m}$, which is very close to the design value. We believe that increasing the number of measurement points may remove this ambiguity in the results. It may also be beneficial with different scan configurations.

CONCLUSION

Accurate knowledge of transverse beam characteristics is of high importance for injections into the new generation of storage ring based light sources. A new method for performing quadrupole scans in dispersive transfer lines has been proposed. Two quadrupoles with a dipole magnet in between was scanned simultaneously, allowing for a disentangling and fitting of both betatronic and dispersive beam parameters. The method was been tested in both the ESRF TL2 and SLS BRTL transfer lines. In both cases the method is able to provide reasonable results.

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