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GYRORESONANT ACCELERATION OF ELECTRONS BY AN AXYSIMMETRIC TRANSVERSE ELECTRIC FIELD

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Abstract

The acceleration of electrons using gyromagnetic autoresonance, proposed by K. S. Golovanivsky and known as Gyroresonant Acceleration (GYRAC), consists of the sustaining of the electron cyclotron resonance condition through a magnetic field that increases on time. In this work, the results of the numerical study of the 2D acceleration of electrons by a TE_{011} cylindrical mode through the GYRAC mechanism are presented. We use an electric field strength of 1 kV/cm and a frequency of 2.45 GHz. The temporal growth rate of the external magnetic field is fitted to keep the electrons in the acceleration regime. The trajectory, energy, and the phase-shift between the electron transverse velocity and the electric field are calculated from the numerical solution of the relativistic Newton-Lorentz equation using a finite difference scheme. It was shown that an electron released from rest and located in the position $(R_c/2, L_c/2)$, with R_c and L_c the radius and the length of the cavity respectively, can be accelerated up to energies about 560 keV in 625 periods of the microwave field.

INTRODUCTION

Since the year of 1962, Kolomenskii and Davydovski started with the study of the interaction between the transverse electromagnetic wave that propagates along a homogeneous magnetic field and the motion of a charged particle [1, 2]. This phenomenon, known as autoresonance, consists of maintaining the equality between the frequency of the electromagnetic wave and the cyclotron frequency of electrons; and can be realized in different ways [3]. In the case of the gyroresonant acceleration (GYRAC) an external magnetic field which grows slowly on time is used for the sustenance of electron cyclotron resonance (ECR) condition [4–7]. Said mechanism present some important applications as the electron cyclotron resonance ion proton accelerator (ECR-IPAC) for cancer therapy [8] and to control plasma bunches with relativistic electrons [9]. The present work is dedicated to the numerical study of the GYRAC acceleration of electrons by a microwave field, TE_{011} cylindrical mode, in a single particle approximation. The temporal growth rate of the external magnetic field is fitted to keep the electrons in the acceleration regime. The trajectory, energy, and the phase shift between the electron transverse velocity and the electric field are calculated from the numerical solution of the relativistic Newton-Lorentz equation using a finite difference scheme.

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SIMULATION MODEL

The auto-resonance phenomenon consists of the maintenance of the resonant interaction of an electromagnetic wave with charged particles describing the cyclotron motion due to an applied external magnetic field. For electrons, in resonance conditions, the gyro-frequency is given by

$$\Omega_c = \frac{eB}{m_e \gamma}, \tag{1}$$

where e and m_e are the electric charge and the rest mass of the electron, B is the magnetic field, $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic factor with $\beta = v/c$, being v the electron velocity and c the speed of light in vacuum. When the electron interacts with a standing electromagnetic wave, the condition $\Omega_c = \omega$ can be satisfied through an increase in the magnetic field to compensate the increase of the Lorentz factor, γ. For the case of temporal auto-resonance known as GYRAC, an uniform magnetic field varying on time is used. So the electron cyclotron frequency is written as

$$\Omega_c = \frac{eB(t)}{m_e \gamma}, \tag{2}$$

where the magnetic field is written as $\vec{B} = B_0\,[1 + b\,(t)]\,\hat{k}$; with $B_0 = \omega\, m_e/e$ being the classical resonance magnetic field, $b(t)$ is a dimensionless function that grow monotonically in time and ω is the frequency of the electromagnetic wave interacting with the electron. The electrons gain energy as the result of their interaction with the electric field, $\vec{E} = E_0(\sin\varphi\,\hat{r} + \cos\varphi\,\hat{\theta})$, which corresponds to a homogeneous transverse electric field that rotates around the z-axis with the frequency ω; and finally φ is the phase-shift between the electric field and the velocity of the electron. Starting from a model of the 2D relativistic dynamics, Golovanivsky obtained a system of differential equations for the study of the evolution of energy and the phase-shift of the particles, see Eq. (3) and (4).

$$\dot{\gamma} = -\,g_0 \left(1 - \frac{1}{\gamma^2}\right)^{1/2} \cos\varphi, \tag{3}$$

$$\dot{\varphi} = \Big[b\,(\tau) - (\gamma - 1)\Big]\frac{1}{\gamma} + g_0\,\left(\gamma^2 - 1\right)^{-1/2} \sin\varphi, \tag{4}$$

where $g_0 = E_0/B_0 c$ and $\tau = \omega\, t$. The GYRAC mechanism present the phase focusing phenomenon, which means that no matter the initial value of the phase-shift the particle is trapping in the acceleration regime, or the GYRAC regime, which occurs only at low energies ($\gamma \approx 1$). From the Eq. (4) the condition for trapping particles in the GYRAC regime when the magnetic field grows slowly according to the expression

$$b(\tau) = \alpha\tau, \tag{5}$$

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is given by

$$\alpha \leq 1.19\, g_0^{\frac{4}{3}}. \tag{6}$$

In this work we study the gyroresonant acceleration of electrons by a microwave field, TE_{011} cylindrical mode, in the single particle approximation. The microwave field [10], or high frequency (hf) field, is given by $\vec{E}^{hf} = E_\theta^{hf}\,\hat{\theta}$ and $\vec{B}^{hf} = B_r^{hf}\,\hat{r} + B_z^{hf}\,\hat{k}$, with

$$E_\theta^{hf}(\vec{r},t) = \frac{E_0}{J_1(p_{01})} J_1(k_T r)\sin(k_z z)\cos(\omega t), \tag{7}$$

$$B_r^{hf}(\vec{r},t) = \frac{E_0}{J_1(p_{01})}\frac{k_z}{\omega} J_1(k_T r)\cos(k_z z)\sin(\omega t), \tag{8}$$

$$B_z^{hf}(\vec{r},t) = \frac{E_0}{J_1(p_{01})}\frac{k_T}{\omega} J_0(k_T r)\sin(k_z z)\sin(\omega t), \tag{9}$$

where E_0 is the amplitude of the electric field, $p_{01} = 1.84118$, $k_T = q_{01}/R$ being $q_{01} = 3.83171$ and R the radius of the cylindrical cavity and $k_z = \pi/L$, where L is the length of the chamber. In our simulations, the TE_{011} mode is excited in a cavity whose radius and length are 7.84 cm and 20.0 cm, respectively. With the frequency of 2.45 GHz and the strength of 1 kV/cm. The electric field component of said cylindrical mode at the plane $z = L/2$ is presented in Fig. 1. In order to satisfy the trapping conditions, see Eq. (6), a set of α parameters is used in the range: $10^{-4} \leq \alpha \leq 3\times10^{-4}$.

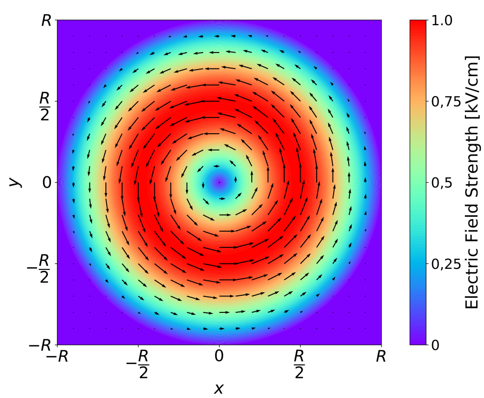

Figure 1: Electric field of the TE_{011} cylindrical mode in the plane $z = L/2$.

The Fig. 2 shows the physical scheme used in this work, which consists of: 1-cylindrical cavity in which the TE_{011} mode is excited, 2-current coils, 3-injection point of the particles, and 4-the transverse central plane $z = L/2$ (green dashed line) where the electric field strength reaches its maximum value and the electron motion is realized. The longitudinal profile of the electric field is also shown (arrows blue). The microwave injection system is not shown.

The motion of the electron is described by the relativistic Newton-Lorentz equation [11], which is expressed in a dimensionless form as

$$\frac{d\vec{u}}{d\tau} = \vec{g}_0 + \frac{\vec{u}}{\gamma}\times\vec{b}, \tag{10}$$

where $\vec{u} = \vec{p}/mc$ is the momentum of the electron, $\vec{g}_0 = -\vec{E}/B_0 c$ is the microwave electric field, $\vec{b} = -\vec{B}/B_0$ is the total magnetic field, where $\vec{B} = \vec{B}^{hf} + \vec{B}^{ext}$, $\tau = \omega\, t$ the time and $\gamma = \sqrt{1+u^2}$. The Eq. (10) in a finite difference form is solved as described in [12].

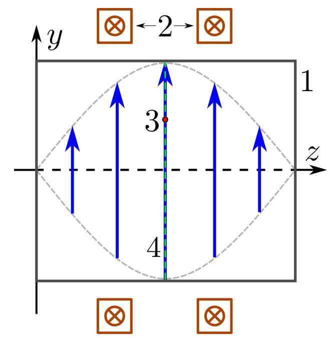

Figure 2: A physical model scheme for GYRAC realization.

RESULTS AND DISCUSSIONS

In the realized numerical experiments, the electrons are injected in two configurations: (i) An electron at rest is released from a point located at a radial distance $R/2$, see Fig. 2 and (ii) a ring-like cloud of electrons (placed on $3R/8 < r < 9R/16$ and containing 1000 electrons at rest). In both cases, a set of α parameters on the range: $10^{-4} \leq \alpha \leq 3\times10^{-4}$ are used.

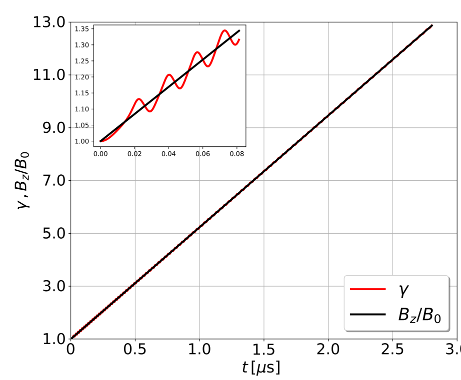

Figure 3: Time evolution of γ and B_z/B_0 for $\alpha = 2.75\times10^{-4}$.

The Fig. 3 shows the evolution of the relativistic factor and the magnetic field up to the electron impacts to the cavity in t =2.8 µs. The red line shows the energy gain of the electron and how the appropriate growth of the external magnetic field (the black line) allows trapping the particle in a GYRAC regime. This happens because the phase-shift, φ is most of the time on the range $(\pi/2, 3\pi/2)$, knows as acceleration band.

In the Fig. 4 shows the electron energy evolution for different values of α parameter, which satisfy the trapping condition, see Eq. (6), except for the value $\alpha = 3.0\times10^{-4}$ (the purple line). In the present work, the trapping condition is changing on the time due to the electric field inho-

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mogeneities, $g_0 = g_0(\vec{r})$, and the azimuthal electron drift motion. Despite this, the electron remains trapped in the GYRAC regime for the considered cases.

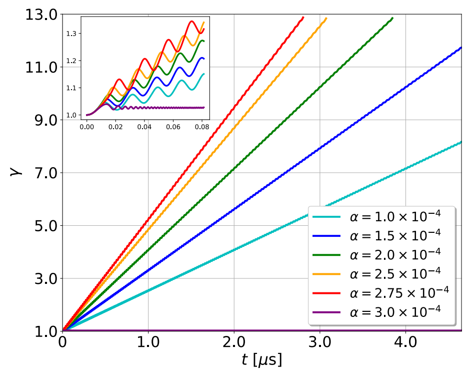

Figure 4: Time evolution of γ for different α parameters.

The time evolution of the Larmor radius of the electrons for the same values of α parameter is presented in the Fig. 5. In this figure, we can see how the Larmor Radius grows quickly for all α values considered, except for $\alpha = 3.0 \times 10^{-4}$ (the purple line). The trajectory described for this particles is composed by two motions: the first one is the cyclotron motion relative to the guide center and the second one is a drift motion on $\hat{\theta}$ direction, besides the drift motion is realized faster for particles on GYRAC regime. As expected, as the electrons gain energy, the radius of Larmor approaches the relativistic radius of larmor $R_L^{Rel} = c/\omega$ ($\cong$ 1.95 cm for the present case).

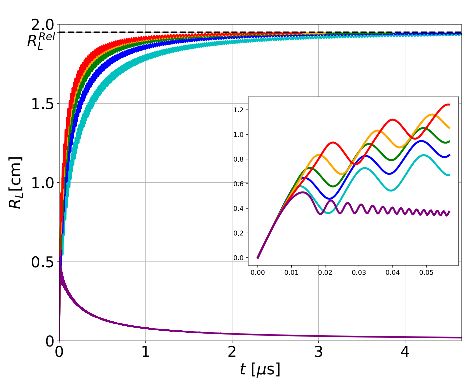

Figure 5: Time evolution of R_L for different α parameters.

Finally, the obtained results for the electron cloud evolution on gyroresonant conditions are presented. Three numerical experiments were realized by using different α values. In the Table 1 are presented the α value, the fraction of captured particles (% CP), not captured particles (% UCP) and the escaped particles (particles that impacts the cavity or % EP) after 4.65 µs. For the mentioned cases, the maximum energy reaches by particles is close to 3.64, 5.45, 7.265 and 9.08 MeV with a dispersion of 1.1%, 0.4%, 0.3% and 0.2%, respectively. We can see that all electrons are trapping on the GYRAC regime.

Table 1: Electron Clouds Evolution for a Set of α Parameters

α	**Exp.**	**% CP**	**% UCP**	**% EP**
1.0×10^{-4}	1	95.9	0	4.1
	2	95.4	0	4.6
	3	95.4	0	4.6
1.5×10^{-4}	1	68.4	0	31.6
	2	69.2	0	30.8
	3	68.7	0	31.3
2.0×10^{-4}	1	48.8	0	51.2
	2	50.1	0	49.9
	3	49.0	0	51.0
2.5×10^{-4}	1	35.3	0	64.7
	2	37.0	0	63.0
	3	33.6	0	66.4
2.75×10^{-4}	1	12.3	29.2	58.5
	2	11.4	28.0	60.6
	3	10.4	34.2	55.4

In the Fig. 6, we find the energy distribution of the electron cloud for the case $\alpha = 2.75 \times 10^{-4}$. In this case, not all electrons are trapped in the GYRAC regime, which is attributed to the electric field inhomogeneities that affect the trapping value condition, see Eq. (6). It leads to three groups of electrons, one of them with the maximum energy of about 10 MeV.

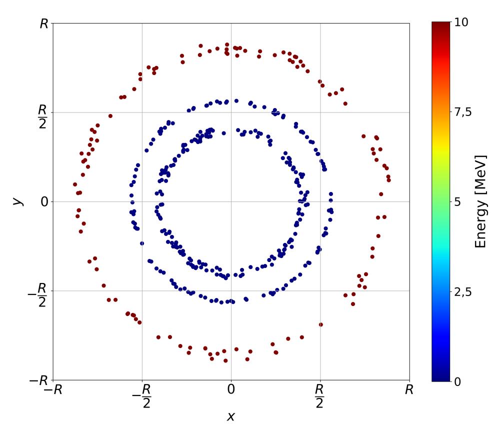

Figure 6: Energy distribution of the electron cloud after 4.65 µs for the case $\alpha = 2.75 \times 10^{-4}$.

CONCLUSION

It was shown by numerical experiments that it is possible to accelerate electrons by a microwave field, TE_{011} cylindrical mode, under electron cyclotron resonance conditions in time-varying magnetic fields. A set of values of the parameters that describe the growth of the magnetic field was obtained to maintain the resonance regime. It was found that there is a region ring-like ($3R/8 < r < 9R/16$) where the electrons are captured in the autoresonance regyme.

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