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WAKEFIELDS AND TRANSVERSE BUNCH DYNAMICS STUDIES OF A PLASMA-DIELECTRIC ACCELERATING STRUCTURE

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Abstract

A theoretical investigation of a wakefield excitation in a plasma-dielectric accelerating structure by a drive electron bunch in the case of an off-axis bunch injection is carried out. The structure under investigation is a round dielectric-loaded metal waveguide with channel for the charged particles, filled with homogeneous cold plasma. In this paper we focus on the spatial distribution of the bunch-excited wakefield components, which act on both the drive and test bunches, and on transverse bunch dynamics.

INTRODUCTION

Charged particles acceleration by wakefield, excited by a high charge drive bunch in the dielectric-loaded structures, is one of the advanced acceleration concepts, which is in focus of intense studies at such accelerators centers as ANL, SLAC, BNL, INFN. Despite significant progress, there is still a significant number of problems and challenges that are relevant at the present time, which require new methods for their solving. One of the severe issues of the accelerators development is the beam breakup (BBU) instability of the drive bunch, which can led to the bunch parameters degradation. A number of previous studies [1–8], devoted to BBU, highlight a strong importance of this problem, which has not been completely overcome at the moment. Previously in order to control and stabilization of the BBU instability several methods have been proposed: (i) the BNS damping [9], which consist in variation of a betatron oscillation frequency along the bunch, (ii) a combination of a profiled quadrupole focusing with an energy chirp of bunch [10], (iii) excitation of a dielectric resonator by a train of bunches [11]. Also it is well known, that a plasma has the focusing properties, which allow to focus a drive bunch [12, 13]. And as an alternative to a standard quadrupole focusing system for a dielectric-loaded structure is the filling of a channel for the charged particles by plasma [14, 15]. In the paper [14] authors demonstrated, that a combination of a dielectric-loaded structure and plasma led to a test bunch to be focused. Acceleration in such a plasma-dielectric structure is provided by an eigenmode of the dielectric-loaded structure, and focusing is provided by a plasma wave. Further research showed that using nonuniform radial plasma density profile led to better focusing of both the drive and witness bunches, and that there exists the optimum vacuum channel radius, at which the focusing is the best [16]. Aforementioned features of the plasma-dielectric structure (in terms of bunch focusing) allow to consider it as a possible way for the BBU instability suppression under wakefield excitation. This is the main motivation of the present paper.

STATEMENT OF THE PROBLEM

A cylindrical dielectric-loaded metal waveguide with coaxial drift channel for the charged particles, filled with homogeneous cold plasma is a wakefield structure under present study. Into the plasma a relativistic electron bunch is injected with a vertical offset in parallel to the structure axis. We were mainly interested in transverse stability of the drive bunch, which excites a wakefield in the structure.

ANALYTICAL STUDIES

For a point particle of charge q moving with a constant velocity v along the waveguide axis (z direction) with an offset r_0 the current density in cylindrical coordinates is

$$j_z = q\frac{\delta(r-r_0)}{r}\delta(\varphi-\varphi_0)\delta(t-t_0-z/v), \qquad (1)$$

where t_0 is an arrival time of the particle into the waveguide ($z=0$), δ is the Dirac delta function. Due to the off-axis drive bunch injection the excited wakefield does not describe in terms of TM eigenmodes (as in the on-axis injection case) and has six components of the electromagnetic field. The bunch-excited wakefield components, the charge and current densities can be expressed in terms of its Fourier transform in the variables $t-z/v$ and θ as follows

$$\begin{pmatrix} E(r,\varphi,\xi) \\ H(r,\varphi,\xi) \\ j_z(r,\varphi,\xi) \end{pmatrix} = \sum_{m=-\infty}^{+\infty} e^{im\varphi} \int_{-\infty}^{+\infty} d\omega \begin{pmatrix} E_m^\omega(r,\omega) \\ H_m^\omega(r,\omega) \\ j_{zm}^\omega(r,\omega) \end{pmatrix} e^{-i\omega\xi}, \qquad (2)$$

where $\xi = t - z/v$. The set of Maxwell's equations results to uncoupled wave equations for the Fourier transforms of the longitudinal electric and magnetic fields E_{zm}^ω, and H_{zm}^ω.

$$\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_{zm}^\omega}{\partial r}\right) - \frac{m^2}{r^2}E_{zm}^\omega - \kappa^2 E_{zm}^\omega = \frac{4\pi i\kappa^2 j_{zm}^\omega}{\omega\varepsilon(\omega)}, \\ &\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_{zm}^\omega}{\partial r}\right) - \frac{m^2}{r^2}H_{zm}^\omega - \kappa^2 H_{zm}^\omega = 0, \end{aligned} \qquad (3)$$

where $\kappa^2(\omega) = (\omega/v)^2(1-\beta^2\varepsilon(\omega))$, $\beta = v/c$. All the other four wakefield components E_{rm}^ω, $E_{\varphi m}^\omega$, H_{rm}^ω and $H_{\varphi m}^\omega$ can be expressed in terms of E_{zm}^ω, and H_{zm}^ω. Omitting the mathematical details, in this paper we present the results for the longitudinal and transverse forces of any given azimuthal mode m in the plasma region (for a pencil-like drive bunch

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with homogeneous longitudinal charge distribution)

$$\frac{F_{zm}}{q} = -\frac{2Q_b k_p}{L_b}\left(\theta(r_0 - r)\frac{I_m(k_p r)}{I_m(k_p a)}\Delta_m(k_p a, k_p r_0) + \theta(r - r_0)\frac{I_m(k_p r_0)}{I_m(k_p a)}\Delta_m(k_p a, k_p r)\right)\Psi_{\parallel}^{p} e^{-im\varphi_0} - \sum_{s=1}^{+\infty}\frac{4Q_b D_2(\omega_s) v}{a\omega_s^2 D'(\omega_s) L_b}\frac{I_m(\kappa_{ps} r) I_m(\kappa_{ps} r_0)}{I_m^2(\kappa_{ps} a)}\Psi_{\parallel}^{s} e^{-im\varphi_0} \quad (4)$$

$$\frac{F_{rm}}{q} = \frac{2Q_b k_p}{L_b}\left(\theta(r_0 - r)\frac{I'_m(k_p r)}{I_m(k_p a)}\Delta_m(k_p a, k_p r_0) + \theta(r - r_0)\frac{I_m(k_p r_0)}{I_m(k_p a)}\Delta'_m(k_p a, k_p r)\right)\Psi_{\perp}^{p} e^{-im\varphi_0} + \sum_{s=1}^{+\infty}\frac{4Q_b D_2(\omega_s)\kappa_{ps} v^2}{a\omega_s^3 D'(\omega_s) L_b}\frac{I'_m(\kappa_{ps} r) I_m(\kappa_{ps} r_0)}{I_m^2(\kappa_{ps} a)}\Psi_{\perp}^{s} e^{-im\varphi_0}. \quad (5)$$

In the Eqs. (4) and (5) the following notations are used: Q_b is the bunch charge, L_b is the bunch length, ω_p is the plasma frequency, k_p is the plasma wave number, a is the inner radius of the dielectric, I_m and K_m are the modified Bessel and Macdonald functions of the m^{th} order, $\Delta_m(x,y) = I_m(x)K_m(y) - K_m(x)I_m(y)$, $\kappa_{ps}^2 = (\omega/v)^2(1 - \beta^2\varepsilon_p(\omega_s))$, $\varepsilon_p(\omega) = 1 - \omega_p^2/\omega^2$ is the plasma permittivity, $D'(\omega_s) = dD(\omega_s)/d\omega$, θ is the Heaviside function. The eigenfrequencies of the plasma-dielectric waveguide ω_s are defined by the dispersion equation $D(\omega_s) = 0$, where

$$D(\omega) = D_1(\omega)D_2(\omega) - \frac{\beta^2 m^2(\omega/v)^4}{a^2\kappa_p^4\kappa_d^4}(\varepsilon_d - \varepsilon_p)^2,$$
$$D_1(\omega) = \frac{\varepsilon_p}{\kappa_p}\frac{I'_m(\kappa_p a)}{I_m(\kappa_p a)} + \frac{\varepsilon_d}{\kappa_d}\frac{F'_m(\kappa_d a, \kappa_d b)}{F_m(\kappa_p a, \kappa_d b)}, \quad (6)$$
$$D_2(\omega) = \frac{1}{\kappa_p}\frac{I'_m(\kappa_p a)}{I_m(\kappa_p a)} + \frac{1}{\kappa_d}\frac{\Phi'_m(\kappa_d a, \kappa_d b)}{\Phi_m(\kappa_p a, \kappa_d b)}$$

where ε_d is the permittivity of the dielectric,

$$\kappa_d^2 = (\omega/v)^2(\beta^2\varepsilon_d - 1),$$
$$F_m(x,y) = (-1)^m(J_m(x)Y_m(y) - Y_m(x)J_m(y)),$$

J_m and Y_m are the Bessel and Weber functions of the order m^{th},

$$F'_m(x,y) = (-1)^m(J'_m(x)Y_m(y) - Y'_m(x)J_m(y)),$$
$$\Phi_m(x,y) = J_m(x)Y'_m(y) - Y_m(x)J'_m(y),$$
$$\Phi'_m(x,y) = J'_m(x)Y'_m(y) - Y'_m(x)J'_m(y).$$

The functions $\Psi_{\parallel}^{p,s}(\xi)$ and $\Psi_{\perp}^{p,s}(\xi)$ describe the axial structure of the wakefield

$$\Psi_{\parallel}^{p,s}(\xi) = \theta(\xi)\sin\omega_{p,s}(\xi) - \theta(\xi - L_b/v)\sin\omega_{p,s}(\xi - L_b/v),$$
$$\Psi_{\perp}^{p,s}(\xi) = \theta(\xi)(1 - \cos\omega_{p,s}(\xi)) - \theta(\xi - L_b/v)(1 - \cos\omega_{p,s}(\xi - L_b/v)). \quad (7)$$

Constructed theory of wakefield excitation by the drive bunch allows to carry out numerical analysis in the rigid bunch approximation.

NUMERICAL ANALYSIS RESULTS

The main goal of the present paper was to analyze the transverse dynamics of the drive electron bunch in the case of its off-axis injection as well as the plasma effect on particles dynamics. Numerical analysis was performed for the waveguide parameters in the terahertz frequency range, for the drive bunch we used parameters of electron bunches accessible at SLAC. These parameters are presented in Table 1.

Table 1: Parameters of the Plasma-Dielectric Waveguide and the Drive Electron Bunch

Parameter	**Value**
Inner dielectric radius	0.5 mm
Outer dielectric radius	0.6 mm
Length of the waveguide	8 cm
Permittivity of the dielectric	3.75
Plasma density	4.41×10^{14} cm^{-3}
Energy of bunch	3.0 GeV
Charge of bunch	3.0 nC
Length of bunch	0.5 mm
Radius of bunch	0.23 mm

We start with the case of wakefield excitation in the dielectric-loaded and plasma-dielectric acceleration structures by an on-axis relativistic electron bunch. Analysis of the bunch-excited wakefield components structures showed a significant difference between longitudinal and transverse forces, which act as on drive bunch as a witness bunch in these accelerating structures (see Fig. 1).

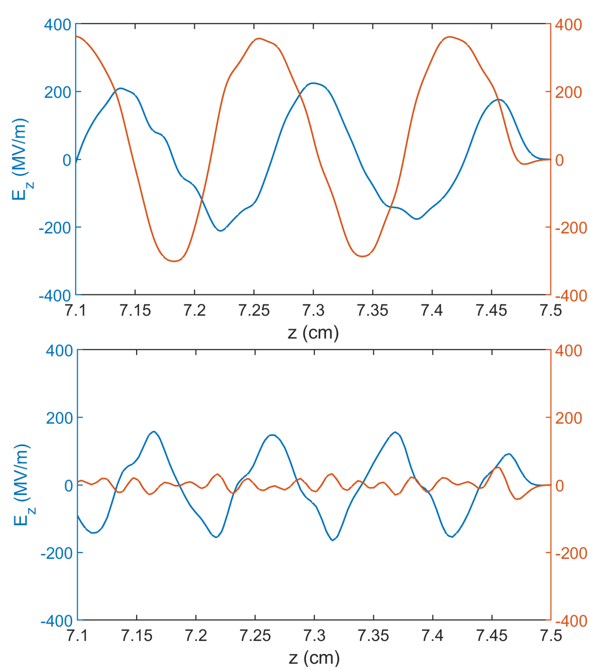

Figure 1: The axial profiles of the longitudinal and transverse forces behind the drive bunch at the distance 0.47 mm from the axis for the cases of plasma on (top), and plasma off (bottom).

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The main reason of the difference in the ratio between longitudinal and transverse forces for the cases plasma on and plasma off is a plasma wave excitation by the drive bunch. In turn, this lead to changes in the transverse particles dynamics. Figure 2 demonstrates the particles positions for the time moment when the drive bunch reached longitudinal position $z = 7.51$ cm. The positions of particles are given in the vertical-longitudinal plane.

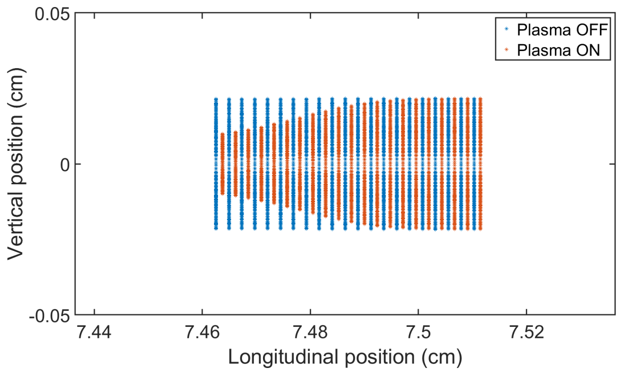

Figure 2: The on-axis injected drive bunch particles distribution on the vertical-longitudinal plane at the exit of the accelerating structure for the plasma on and plasma off cases. Vertical axis limits correspond to full width of channel for the charged particles, horizontal axis limits correspond to the frame around the bunch.

It can be seen that the presence of plasma leads to focusing of particles located in the tail of the drive bunch. It should be noted, that simulated drive bunch has zero divergence at the structures input. For the nonzero divergence drive bunch demonstrated focusing features of the plasma-dielectric structure could lead to a partial compensation of the divergence growth in the process of bunch propagation and will be investigated further. As we were mainly interested in BBU of the drive bunch, let us now consider the case of an off-axis bunch injection. In order for the effect of the bunch offset to be more pronounced, we injected it near the plasma-dielectric interface. The initial vertical offset of the bunch is 0.25 mm. Both the plasma off and plasma on cases for the off-axis injection were analyzed as well. Figure 3 shows the bunch particles positions on the vertical-longitudinal plane at the moment of time when the bunch started to deposit on the dielectric in the plasma off case. As it can be seen there is a significant qualitative differences in the transverse bunch dynamics in the plasma off and plasma on cases. As a result of the initial offset the transverse deflection of the tail particles is greater than that of the head ones in the absence of plasma in the channel for the charged particles. The drive bunch is deflected as a whole while it propagates through the dielectric-loaded structure and the BBU instability for this case does exist and leads to undesirable bunch displacement up to the total beam loss. The presence of plasma in the channel for the charged particles leads to a change in the transverse particles dynamics. The particles in the tail of the off-axis drive bunch focuses instead of deflection due to plasma wave excitation, and as a result

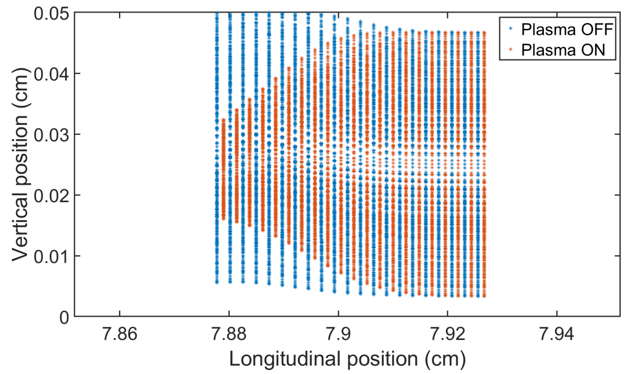

Figure 3: The off-axis injected drive bunch particles distribution on the vertical-longitudinal plane at the exit of the accelerating structure for the plasma on and plasma off cases. Vertical axis limits correspond to half width of channel for the charged particles, horizontal axis limits correspond to the frame around the bunch.

the bunch offset does not tend to increase with time and a gradual increase of charge losses does not occur. Thus, it can be concluded that the presence of plasma in the drift channel leads to suppression of the BBU instability of the beam injected with the offset.

SUMMARY

An analytical theory of wakefield excitation by the off-axis particle bunch in the round plasma-dielectric waveguide has been formulated and can be used in order to get the better interpretation of the planned experimental results. A comparison of the off-axis drive bunch transverse dynamics for the cases of the absence and presence of plasma in the channel for charged particles is carried out. It has been demonstrated that for the plasma-dielectric accelerating structure (unlike the dielectric-loaded structure without plasma filling) the presence of the initial bunch offset does not lead to the BBU.

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