ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB360

ANOMALOUS SKIN EFFECT STUDY OF NORMAL CONDUCTING FILM*

Binping Xiao†, M. Blaskiewicz, T. Xin
Brookhaven National Laboratory, Upton, New York 11973-5000, USA

Abstract

For the radiofrequency (RF) applications of normal conducting film with large mean free path at high frequency and low temperature, the anomalous skin effect differs considerably from the normal skin effect with field decaying exponentially in the film. Starting from the relationship between the current and the electric field (E field) in the film, the amplitude of E field along the film depth is calculated and is found to be non-monotonic. The surface impedance is found to have a minimum value at certain film thickness.

INTRODUCTION

Surface impedance of normal conductor was extensively studied previously for bulk material [1-3], as well as for thin films [4-6]. Reuter and Sondheimer [1, 2] pointed out that for normal conductor with anomalous skin effect, it differs considerably from the normal skin effect with field decaying exponentially inside the conductor. This behave drastically affects the surface imedance of normal conducting films [4-6]. This paper focuses on the systematic study and explanation of the behave of anomalous skin effect in thin film, it starts from the anomalous skin effect of bulk material, and extends it to the film material using a method similar to previous references [4-6], Cu film without substrate is used to explain the anomalous behavior, Cu film on S.S. substrate is calculated, and conclusion is shown in the last.

ANOMALOUS SKIN EFFECT OF BULK NORMAL CONDUCTOR

In this section we briefly introduce the anomalous skin effect of bulk normal conductor with diffusive reflection. It was previously studied by Reuter and Sondheimer [1, 2]. In their study, a semi-infinite metal was considered, with its surface in the *xy*-plane and the positive *z*-axis directed towards the interior of the metal, and $z = 0$ the interface between vacuum and metal. The electric field $E(z)e^{i\omega t}$ is in the *x*-direction. The relationship between E and the current J is [1]:

$$E''(z)+k^2E(z)=i\omega\mu_0 J(z). \quad (1)$$

Here derivative is over depth d/dz, and $k = \omega/c$. For the anomalous skin effect, $J(z)$ is not solely determined by the local electric field $E(z)$, instead, it is [1]:

$$J(z)=\int_0^\infty k_a(z-z_1)E(z_1)dz_1, \quad (2)$$

with $k_a(z)=\frac{3}{4\rho\ell}\int_1^\infty e^{-|z|sa/\ell}(\frac{1}{s}-\frac{1}{s^3})ds$ and $a=1+i\omega\ell/v_F$, with ℓ the mean free path and v_F the Fermi velocity.

The surface impedance can be calculated from the E field on the surface $z = 0$ and its derivative over depth: $Z=-i\omega\mu_0 E_0/E_0'$.

ANOMALOUS SKIN EFFECT OF NORMAL CONDUCTING FILM

For a normal conductor with finite thickness d, one needs to calculate the electric field over depth $E(z)$ to get the surface impedance. In this case the expression of $J(z)$ changes to [6]:

$$J(z)=\int_0^d k_a(z-z_1)E(z_1)dz_1. \quad (3)$$

We use the dimensionless variable $u=z/\ell$, apply Eq. (3) into Eq. (1) and get [6]:

$$\begin{aligned}&d^2E(u)/du^2+k^2\ell^2E(u)\\&=i\omega\mu_0\ell^3\int_0^{d/\ell}k_a(u\ell-u_1\ell)E(u_1)du_1\end{aligned} \quad (4)$$

First assume at $z = d$, the boundary condition is $E(z) = E_d$ and $E'(z) = E_d'$. Integrating the above equation twice with respect to u gives:

$$\begin{aligned}&E(u)+\int_u^{d/\ell}du_2[(u-u_2)(k^2\ell^2E(u_2)\\&+i\omega\mu_0\ell^3\int_0^{d/\ell}k_a(u\ell-u_1\ell)E(u_1)du_1)]\\&=E_d-(d-u\ell)E_d'.\end{aligned} \quad (5)$$

To solve the above equation numerically, the depth (normalized to ℓ) is divided into K equal segments $\Delta=d/\ell/K$. We use E_n to note the E field at $n\Delta$, thus we have:

$$\begin{aligned}&E_n+\sum_{m=n}^{K}(n-m)[k^2\ell^2E_m\\&+i\omega\mu_0\ell^3\Delta^3\sum_{j=0}^{K}E_jk_a((m-j)\ell)]\\&=E_d-\ell\Delta(K-n)E_d',\end{aligned} \quad (6)$$

$k_a(0)$ is infinite and is replaced by $\frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2}k_a(z)dz$. The above equation can be solved by matrix inversion to get E_n, with the results normalized to E_d ($E_d = 1$). Similar to the bulk case, the surface impedance of this film is $Z=-i\omega\mu_0E_0/E_0'$.

* Work supported by Brookhaven Science Associates, LLC under contract No. DE-AC02-98CH10886 with the US DOE.
† binping@bnl.gov

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB360

Here we discuss two boundary conditions:

1. With a bulk metal substrate with its surface impedance, noted as Z_{sub}, in the normal skin effect regime. The boundary condition is $E_d' = -i\omega\mu_0 E_d / Z_{sub}$.
2. Without substrate. In this case $Z_{sub} = 377\ \Omega$.

Cu is used as an example. The parameter $\rho\ell$ is temperature independent [7]. Unless otherwise noted, the following parameters are used: RRR (Residual Resistivity Ratio, the ratio of resistivity ρ between room temperature and cryogenic temperature, i.e. 10 K) Cu at 50 is chosen. The film thickness is chosen to be 10 μm. This Cu film at cryogenic temperature is simulated at two frequencies for comparison: 0.1 GHz (normal skin effect) and 10.1 GHz (anomalous skin effect). Material properties of Cu and S.S. are listed in Table 1. Convergence study has been performed for different film thicknesses, 10 nm step size is chosen for films thicker than 2 μm, for films thinner than that, 200 steps are used.

Table 1: Material Properties of Cu and S.S.

Parameter	Value
Cu $\rho\ell$ [Ωm^2]	6.6×10^{-16}
Cu ℓ for RRR=1 [μm]	0.04
Cu ℓ for RRR=50 [μm]	2
S.S. ρ [Ωm]	3.6×10^{-7}

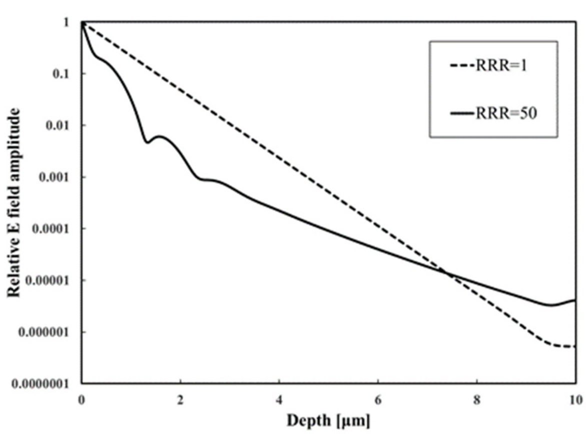

Figure 1: Amplitude of E field along the depth of 10 μm thick Cu film at 10.1 GHz without substrate, with 0 the interface between vacuum and film. Solid line: with 2 μm mean free path, corresponding to RRR = 50; Dashed line: with 0.04 μm mean free path, corresponding to RRR = 1.

ANOMALOUS SKIN EFFECT OF Cu FILM WITHOUT SUBSTRATE

For Cu film without substrate, the surface impedance at 10.1 GHz is $6.2 + 9.9i$ mΩ for Cu with $RRR = 50$ and is $26.3 + 27.0i$ mΩ for Cu with $RRR = 1$. The E field amplitude along the depth of film is shown in Fig. 1. With solid line for Cu with $RRR = 50$, and dashed line for Cu with $RRR = 1$. From this plot one can notice that the E field amplitude decays exponentially along depth for Cu with RRR=1, and the surface resistance is close to the surface reactance, it is close to normal skin effect; while for Cu with $RRR = 50$, the E field amplitude considerably differs from exponential decay [1], and the surface resistance is only 60% of the surface reactance. And more interestingly, at certain depth locations, the E field amplitude first decays with depth, and then slightly increases with depth, there appears one or multiple "local minimum". Such "local minimum" also appears in the anomalous skin effect in gas discharged plasmas [8-10].

For Cu film with the same film thickness, similar to the bulk material, both surface resistance and surface reactance decrease with increasing mean free path ℓ.

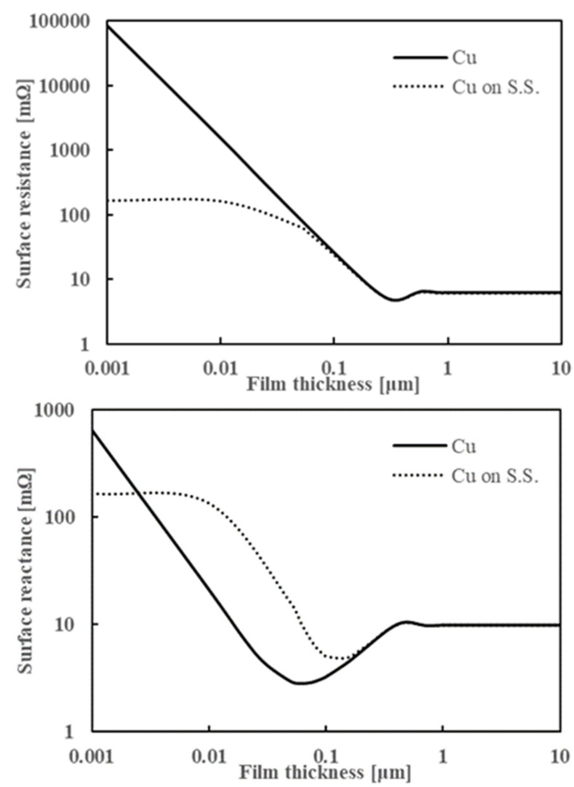

Figure 2: Surface impedance of Cu film versus the film thickness for Cu with 50 RRR at 10.1 GHz, with top plot for surface resistance and bottom plot for surface reactance, and solid curves for Cu film without substrate, dotted curves for Cu film on S.S.

In normal skin effect at dirty limit with ℓ close to 0, both surface resistance and surface reactance decrease with increasing film thickness. This is not the case for the anomalous skin effect. For Cu with 50 RRR, the surface impedance versus film thickness is shown in Fig. 2. with solid curves, with top plot the surface resistance (Similar calculation result for the surface resistance of aluminum was presented previously [5].), and bottom plot the surface reactance. The surface impedance is the same as the surface impedance of bulk material (6.2 mΩ for Cu with the same parameters) while film is thick enough (>1μm), and while film is thin (<0.1μm), surface resistance and reactance increase with decreasing film thickness. There appears to

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB360

be a minimum on surface resistance, with 4.7 mΩ at 0.35 µm thickness. For Cu with 2 µm mean free path at 10.1 GHz, if there is no anomalous skin effect, the surface resistance should be 3.6 mΩ, while with anomalous skin effect, the mean free path is larger than the effective depth of penetration, causing insufficient sheilding and thus a higher surface resistance at 6.2 mΩ. With Cu film thickness smaller than the mean free path, but comparable with the effective depth of penetration, the reflection (difussive in this case) causes an enhancement in sheilding effect, thus a reduction in surface resistance while reducing the film thickness. Combining with the increasing in transmission which causes an increasing in surface resistance, a minimum appears. Please note similar effect was also presented in [5] with a symmetric boundary condition for Aluminum, which is applicable to Cu film without substrate, but not applicable for Cu film with S.S. substrate. For frequncies smaller than 10.1 GHz, the effective depth of penetration will be larger, and the film thickness for minimum surface resistance will be larger as well.

ANOMALOUS SKIN EFFECT OF Cu FILM WITH S.S. SUBSTRATE

For Cu film with S.S. substrate, the boundary condition is determined by the surface impedance of S.S.. The E field amplitude of Cu film with S.S. substrate is close to that of Cu film without substrate showing in Fig. 1.

The surface impedance versus film thickness is shown in Fig. 2 with dotted curves, with top plot the surface resistance, and bottom plot the surface reactance. The surface impedance is the same as the surface impedance of bulk material while film is thick enough (>1 µm), and while film is thin (<0.01 µm), the surface impedance matches that of S.S.. There appears a minimum in surface resistance, with 4.8 mΩ at 0.35 µm thickness.

CONCLUSION

We analyzed the anomalous skin effect for high RRR Cu for high frequency and low temperature application. The amplitude of E field over depth is found to be non-monotonic, which was shown in the previous studies of anomalous skin effect in gas discharged plasmas [8-10]. For Cu film on S.S. substrate, it was found that while film is thin enough, the R_s is close to the value of S.S., and while it is thick enough, it is close to that of bulk Cu. It was also found that while the film thickness is comparable to the effective depth of penetration, a minimum R_s can be found. This behave can be explained by a combination of enhanced screening effect that causes a reduction in R_s and increasing in transmission that causes an increasing in R_s.

ACKNOWLEDGEMENT

The authors would like to thank J. M. Brennan, S. Verdú-Andrés and A. Zaltsman for useful discussions.

REFERENCES

[1] G. Reuter and E. Sondheimer, "The theory of the anomalous skin effect in metals", *Proceedings of the Royal Society of London Series A: Mathematical and Physical Sciences*, vol. 195, no. 1042, p. 336, 1948. doi:10.1098/rspa.1948.0123

[2] G. E. H. Reuter and E. H. Sondheimer, "Theory of the Anomalous Skin Effect in Metals", *Nature*, vol. 161, no. 4089, p. 394, 1948. doi:10.1038/161394a0

[3] R. Chambers, "The anomalous skin effect", *Proceedings of the Royal Society of London Series A: Mathematical and Physical Sciences*, vol. 215, no. 1123, p. 481, 1952. doi:10.1098/rspa.1952.0226

[4] G. R. Henry, "Anomalous Skin Effect in Thin Films", *Journal of Applied Physics*, vol. 43, no. 7, p. 2996, 1972. doi:10.1063/1.1661647

[5] A. F. Mayadas, G. R. Henry, and M. Shatzkes, "Surface Impedance of Thin Metal Films under Anomalous Skin Effect Conditions", *Applied Physics Letters*, vol. 20, no. 11, p. 417, 1972. doi:10.1063/1.1653998

[6] M. Blaskiewicz, "Copper coating specification for the RHIC arcs", Collider-Accelerator Department, Brookhaven National Laboratory, Upton, NY, USA, Rep. BNL-94482-2010-IR, Dec. 2010.

[7] W. Chou and F. Ruggiero, "Anomalous skin effect and resistive wall heating", CERN, Geneva, Switzerland, Rep. LHC-Project-Note-2, Aug. 1995.

[8] Y. M. Aliev, I. D. Kaganovich, and H. Schlüter, "Quasilinear theory of collisionless electron heating in radio frequency gas discharges", *Physics of Plasmas*, vol. 4, no. 7, p. 2413, 1997. doi:10.1063/1.872222

[9] J. P. Matte and K. Aguenaou, "Numerical studies of the anomalous skin effect", *Physical Review A*, vol. 45, no. 4, p. 2558, 1992. doi:10.1103/PhysRevA.45.2558

[10] V. I. Kolobov and D. J. Economou, "The anomalous skin effect in gas discharge plasmas", *Plasma Sources Sciences and Technology*, vol. 6, no. 2, p. R1, 1997. doi:10.1088/0963-0252/6/2/003