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COMBINED EFFECT OF BEAM-BEAM INTERACTION AND BEAM COUPLING IMPEDANCE IN FUTURE CIRCULAR COLLIDERS*

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Abstract

The future large scale electron-positron colliders, such as FCC-ee in Europe and CEPC in China, will rely on the crab waist collision scheme with a large Piwinski angle. Differently from the past generation colliders both luminosity and beam-beam tune shifts depend on the bunch length in such a collision scheme. In addition, for the future circular colliders with extreme beam parameters in collision several new effects become important such as beamstrahlung, coherent X-Z instability and 3D flip-flop. For all these effects the longitudinal beam dynamics plays an essential role and should be taken into account for the collider luminosity optimization. In this paper we discuss an impact of the longitudinal beam coupling impedance on the collider performance.

INTRODUCTION

The beam-beam interaction in storage rings has been studied since 1960 (see [1], as an example). A lot of fruitful work has been done since then in order to improve the collider performance. Different techniques and collision schemes have been proposed to increase their luminosity [2]. For the last generation of electron-positron factories, a collision of intense multi-bunch beams was one of the main ingredients to increase their luminosity. The crossing angle between colliding bunches was necessary to alleviate the parasitic beam-beam interaction and the operations with the horizontal crossing angle were successful at the lepton particle factories such as KEKB [3], DAΦNE [4] and BEPCII [5]. For these machines the Piwinski angle $\Phi = (\sigma_z/\sigma_x)\tan(\theta/2)$ was chosen to be relatively modest, of the order of 0.5 (1.7 in DAΦNE only after 2007), to avoid an excessive geometric luminosity reduction and to diminish the strength of synchro-betatron resonances arising from the beam-beam interaction with the crossing angle.

However, it has been noticed by several authors that a large Piwinski angle can help to increase the luminosity [6–8]. The most important feature is that the large Piwinski angle allows squeezing the vertical beta function β_y at the interaction point (IP) down to the scale σ_x/θ [9]. In addition to the possibility of having very small beam sizes at IP, such a collision scheme also reduces substantially the horizontal beam-beam tune shift [10] and also helps suppressing the vertical synchro-betatron resonances [11].

The further luminosity increase can be reached applying the crab waist (CW) collisions scheme by installing dedicated sextupoles in the interaction region before and after the interaction point and by imposing special conditions on the betatron functions at the sextupole locations and on the phase advance between the sextupoles and IP [9]. The crab waist collision scheme eliminates the beam-beam resonances arising (in collision without CW) due to the vertical motion modulation by the horizontal betatron oscillations [12, 13].

For the first time a combination of the large Piwinski angle and the crab waist collisions has been successfully applied in DAΦNE [14] and since recently it is exploited at SuperKEKB giving very promising results [15].

The first electron-positron Higgs factory based on circular storage rings was proposed already several months before the official announcement of the Higgs boson discovery [16]. The proposal has evolved, first passing to DLEP, the e^+e^- collider having the double LHC circumference, and then to TLEP “Triple LEP” hosted in a 80 to 100 km tunnel [17]. At present the two 100 km long electron-positron colliders, FCC-ee in Europe [18] and CEPC in China [19], are under study as possible future accelerators to explore the properties of the Higgs, W and Z bosons as well as the top quark production thresholds with unprecedented precision.

In order to reach the high luminosity, both colliders are expected to use the crab waist collision scheme with a large Piwinski angle relying on collisions of very intense multi-bunch beams of high energy with low emittances and small betatron functions at the interaction point. For these extreme parameters several new effects become important for the collider performance such as beamstrahlung [20], coherent X-Z instability [21] and 3D flip-flop [22]. Beamstrahlung is the synchrotron radiation induced by beam-beam force. The X-Z instability, which appears in the correlated head-tail motion of two colliding beams, is a novel coherent beam-beam instability in collisions with a large crossing angle. The beam dynamics become essentially three dimensional and the longitudinal motion can no longer be considered as “frozen”. Moreover, interplay of different beam dynamics

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effects becomes very much important for the choice of the collider parameters and their optimization.

In both future lepton colliders beamstrahlung leads to a substantial energy spread growth and bunch elongation in beam-beam collisions [18, 19]. On the other hand, the bunch electromagnetic interaction with surrounding accelerator equipments, described in terms of beam coupling impedance, also results in a notable bunch lengthening [19, 23, 24]. In addition, the impedance is responsible of the synchrotron tune reduction and synchrotron tune spread increase [23]. Beyond the threshold of the microwave instability that is driven by the coupling impedance the energy spread starts growing and the internal bunch oscillations can take place. In the conceptual design reports of the future colliders [18, 19] these effects, beamstrahlung and the impedance related effects, have been studies separately.

Recently the first semi-analytical model has been developed to study interplay of these effects [25]. In particular, it has been shown that the bunch becomes somewhat longer, the energy spread slightly reduces and the synchrotron frequency reduction gets smaller. However, the model assumes that the bunch current stays below the microwave instability threshold and the beam dynamics is not affected by the beam-beam instabilities.

The real situation is even more complicated since the newly discovered coherent beam-beam instability arising in collisions with a large crossing angle [21] couples the transverse and longitudinal planes of motions (the X-Z instability). In turns, the beamstrahlung and the beam coupling impedance will affect the coherent instability and the 3D flip-flop respectively. The performance of the colliders and the choice of the good working points will also be affected by the combined effects. Thus fully 3D self-consistent simulations of beam-beam interaction including beamstrahlung and the beam impedance are urgently required for the collider parameter optimization and interplay of the different effects is to be studied in detail.

The study of the X-Z instability by separate and combined effect coming from beamstrahlung and longitudinal wakefield has been presented in [26]. In this paper, first we'll give a introduction of the simulation method. Then we'll review the CDR parameters of CEPC and FCC-ee Z factory considering longitudinal impedance by strong-strong simulation. Some mitigation schemes will be discussed and checked by simulation.

MODEL OF SIMULATION

The simulation code IBB used in the paper has been developed for the design and optimization of BEPCII [27], and has been extended to support large Piwinski angle collision, beamstrahlung effect, multiple bunches/multiple IPs. In the code, the one turn map is the following:

1. Beam-beam interaction at IP, where beamstrahlung may be considered.
2. Linear map considering synchrotron radiation effect (damping + fluctuation) [28] during the transportation through the arc.
3. Longitudinal wake field of the whole ring lumped at IP before collision.

Beam-Beam Interaction

Since the beam-beam force is very nonlinear, the quantitative study requires comprehensive numerical simulations. In the early stage, the rigid bunch model has been used to study the coherent modes [29]. The weak-strong simulation method has been generally used to design colliders [30–33] based on the prediction of luminosity and the lifetime estimation due to the formation of non-Gaussian halos [34, 35]. With weak-strong model, wide-range parameters scan is possible even nowadays.

Due to the hourglass effect, the geometric luminosity reduction and the transverse beam-beam blowup depend on the finite bunch length. In order to take this effect into account, the colliding bunches are sliced in the longitudinal direction. In order to speed up the convergence of the slice number, the potential interpolation method is used [36].

The Lorentz boost map [6] is used to consider the horizontal crossing angle. The bunch slice number is usually about ten times the Piwinski angle.

The strong-strong simulations of the beam-beam interaction with a large Piwinski angle are very time consuming [37, 38]. So we prefer to use the synchro-beam mapping method [39] to model the slice-slice collision, where the Gaussian approximation is used. In addition, we update the slice transverse RMS size in each slice-slice collision. The Gaussian strong-strong method used in the paper is about one order of magnitude faster than the PIC strong-strong method.

Beamstrahlung is the synchrotron radiation induced by beam-beam force. The energy of beamstrahlung photons emitted during collision is much higher than that from the normal bending magnets. The random emitted photon energy is modelled using bending magnet radiation with the Monte-Carlo method [40].

Longitudinal Wakefield

The longitudinal wake potential is calculated self-consistently and taken into account directly before IP in each turn. The wake potential $V(t)$ is defined as the energy loss per unit charge of a particle due to the whole bunch. The voltage spectrum could be obtained by multiplying the current spectrum by the impedance

$$\tilde{V}(\omega) = -\tilde{I}(\omega) Z_{\parallel}(\omega), \tag{1}$$

where the longitudinal impedance $Z_{\parallel}(\omega)$ is defined as the Fourier transform of the wake function $W_{\parallel}(z = -ct)$, i.e., the Green function of a point charge [41]:

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} \frac{dz}{c} e^{-i\omega z/c} W_{\parallel}(z). \tag{2}$$

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The current could be written as an integral over a frequency spectrum:

$$I(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{I}(\omega). \tag{3}$$

Figure 1 shows the longitudinal bunch distribution simulated by Elegant [42] and IBB, where collision is turned off and CEPC parameters in Table 1 are used except bunch population ($N = 3.12 \times 10^{10}$). The result of the two codes coincides very well.

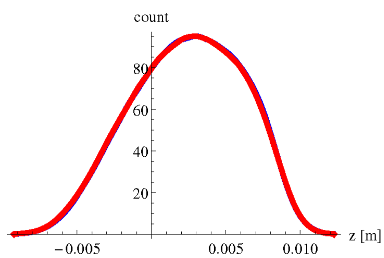

Figure 1: Longitudinal distribution (collision off) obtained using Elegant and IBB considering longitudinal impedance.

The effective RF voltage reduction due to the longitudinal wakefields results in the bunch lengthening in the case of the potential well distortion (PWD). The same voltage reduction results also in the synchrotron frequency reduction [43]. In the following we will show the impact of the coupling impedance on the X-Z instability. It could be expected that the stable areas in horizontal tune space would shift because of the synchrotron frequency reduction induced by the impedance.

REVIEW OF CEPC/FCC-EE CDR

When the X-Z instability is excited, the horizontal emittance increases. As a consequence, we use the ratio of beam size blowup in horizontal direction to identify the instability. The instability would exist near the resonance $mQ_x + nQ_s = 0.5$, where m/n is any integer. The stable horizontal tune area would shift due to the change of synchrotron tune induced by the impedance.

Due to the fact that both CEPC and FCC-ee are ongoing projects, the machine impedance model still represents a work in progress. The CDR parameters of CEPC/FCC-e Z factory are listed in Table 1.

Table 1: Machine Parameters (CDR)

Parameter	**CEPC-Z**	**FCC-ee-Z**
Energy	45.5 GeV	45.6 GeV
Bunch population	8e10	17e10
$\beta^*_{x/y}$	0.2 m/1 mm	0.15 m/0.8 mm
ϵ_x/ϵ_y	0.18 nm/1.6 pm	0.27 nm/1.0 pm
ν_s/superperiod	0.014	0.0125
σ_z (SR/BS)	2.42 mm	3.5 mm
σ_p (SR/BS)	3.8e-4	3.8e-4
ξ_x/ξ_y	0.004/0.079	0.004/0.133

CEPC

CEPC impedance model, which includes both resistive wall and the dominant geometrical impedances, has been used in the following studies. More detailed descriptions of the impedance can be found in [44]. The geometrical impedances are calculated with codes CST and ABCI, while the resistive wall impedance is calculated with the theoretical formula [45].

In the CDR of CEPC, we did not consider the impedance directly. To study the combined effect of beam-beam interaction and longitudinal impedance, we use the bunch length due to the impedance as the equilibrium SR value without collision. When we try to evaluate the beam dynamics with self-consistent method, by including the impedance, it is found that stable working points do not exist. The longer bunch length with the CDR method induces the smaller horizontal tune shift, which weakens the cross-wake force induced by beam-beam interaction. Another important aspect to take into account is that the incoherent synchrotron tune is reduced by longitudinal wake field, which increases the ratio ξ_x/ν_s and may induce stronger X-Z instability.

FCC-ee

The most important contribution to the total machine impedance is represented by the resistive wall, which has been discussed in [23, 24]. Another important contribution to the machine impedance is represented by the bellows with RF fingers necessary to guarantee electric contacts between the two parts of the beam pipe. The RF systems (52 single cell cavities and tapers) and BPMs (4000) have also been taken into account [46].

The X-Z instability has been first found during the design study of FCC-ee Z. A lot of optimization work has been done in the CDR. The ratio of ξ_x/ν_s has been reduced to weaken the instability. We simulate the bootstrapping injection scheme at different horizontal tune, and the result is shown in Fig. 2. It is found that the beam is stable without longitudinal impedance, where the safe area width is 0.005. When we consider the resistive wall impedance, the safe area would shift depending the bunch population. If the full impedance is considered, the stable area width shrinks to 0.001-0.002, which is not reliable in a real machine. In the right hand side of Fig. 2 the complicated picture of the X-Z instability with longitudinal impedance is shown. Since the X-Z instability is induced by beam-beam interaction, it depends on the horizontal tune shift and on the Piwinski angle. As a kind of head-tail instability, the longitudinal bunch shape and the synchrotron tune (spread) can influence it. Moreover they nearly all change with the bunch population.

MITIGATION

In the following we'll discuss some different schemes to suppress the instability at CEPC and FCC-ee.

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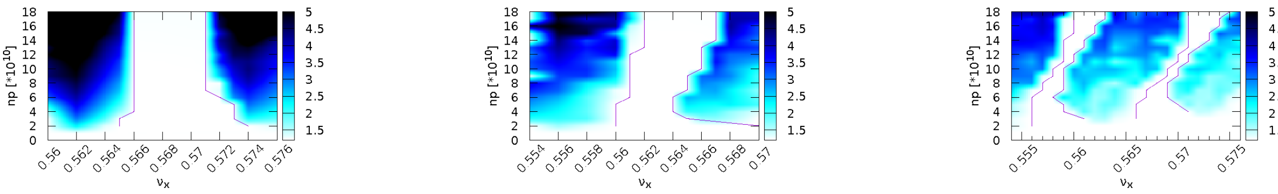

(a) without impedance (b) with resistive wall (c) with full impedance

Figure 2: Horizontal beam size blowup during bootstrapping injection for different model at FCC-ee Z.

Lower β_x^*

In CEPC CDR, the horizonal $\beta_x^* = 0.2$ m at IP is used. However, in the following self-consistent simulations, it was found that there are no good betatron tunes without the horizontal size blowup. The first investigated countermeasure is to squeeze the horizontal beam-beam tune shift by reducing $\beta_x^* = 0.15$ m, which helps to reduce the cross-wake force induced by the beam-beam interaction. The result is shown in Fig. 3. Without longitudinal impedance, the safe tune width is 0.004. But when we consider the full impedance [44], the safe width shrink to only 0.002, which could not be accepted for a real machine. Even smaller β_x^* still remains to be an option to suppress the instability.

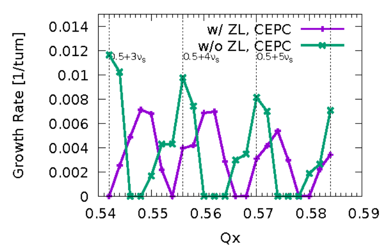

Figure 3: The horizontal size growth rate with and without longitudinal impedance (ZL) at CEPC ($\beta_x^* = 0.015$ m).

Higher Harmonic Cavity

With harmonic cavities the lower synchrotron tune can be achieved without momentum acceptance reduction, differently from the main cavities voltage reduction alone. So higher order X-Z resonances nQx-mQs take place for the same betatron working points, i.e. a weaker X-Z instability is expected. The harmonic cavities could provide a higher synchrotron frequency spread. This may help to suppress the X-Z instability and provides additional damping of the longitudinal multi-bunch instabilities. The microwave instabilities are expected to be weaker with the harmonic cavities as is the case of several synchrotron light sources. Longer bunches reduce the horizontal tune shift, since it scales inversely to the second power of the bunch length. Lower ξ_x/ν_s may also help suppressing the X-Z instability.

With the cavity configuration listed in Table 2, the bunch length is nearly increased by a factor of 2, and the synchrotron tune is nearly reduced by the same factor at Z. We have scanned the horizontal tune to check the effect of higher harmonic cavity for different impedance models: without longitudinal impedance, only with resistive wall impedance and with full impedance. The beam size blowup is shown in Fig. 4. It is very clear that the instability would be suppressed with increase of X-Z resonance order. And it seems that the instability would be mitigated at lower resonance order with stronger impedance. It is supposed that this is due to the smaller incoherent synchrotron tune, larger synchrotron tune spread (see Fig. 5) and different bunch shape with impedance.

Despite the very positive effect of the higher harmonic system, we have also to remember that there are some issues that should be carefully investigated, such as additional impedance contribution due to the harmonic cavities and the problem of energy calibration because of the lower synchrotron frequency.

Table 2: Higher Harmonic Cavity Configuration

Parameter	Main RF Cavity	Harmonic Cavity
Frequency	400 MHz	1200 MHz
Voltage	100 MV	23.4 MV
Phase	156.1°	−11.1°

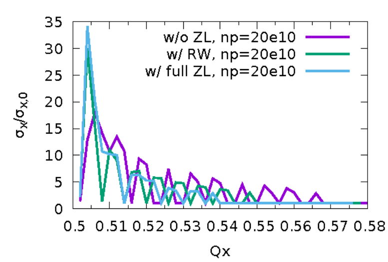

Figure 4: Horizontal size blowup with harmonic cavity for different impedance model: without impedance, only with resistive wall and with full impedance.

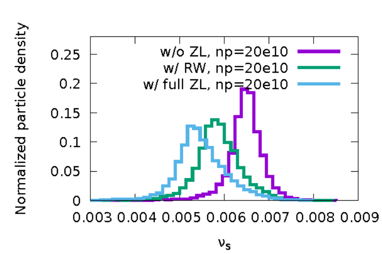

Figure 5: Synchrotron tune of FCC-ee with higher harmonic cavity and different longitudinal impedance.

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Larger Momentum Compaction

In order to enlarge the width of safe tune areas, another possible option is to increase the synchrotron tune ν_s while keeping horizontal beam-beam tune shift ξ_x unchanged, that is to say try to reduce the ratio ξ_x/ν_s. In fact this method has been used to mitigate the X-Z instability of Z factory in the FCC-ee CDR, where the betatron phase of FODO cell lattice is 60°/60°. And now CEPC Z factory also changes the FODO cell from 90°/90° to 60°/60° to mitigate the instability. At FCCee Z factory, to mitigate the X-Z instability considering longitudinal impedance, a larger momentum compaction factor has been proposed by further reducing the FODO cell lattice from the original 60°/60° to 45°/45° [47]. The original and new machine parameters are listed in Table 3.

Table 3: FCC-ee Machine Parameters

Arc Cell	ϵ_x/ϵ_y	ν_s/superperiod	σ_{z0}
60°/60°	0.27nm / 1.0pm	0.0125	3.5 mm
45°/45°	0.6nm / 1.5pm	0.0163	4.5 mm

Two bunch populations $np = 17e10$ and $np = 28e10$ are used to scan the horizontal tune with full longitudinal impedance. The horizontal beam size blowup is shown in Fig. 6. It is found that the width of stable tune area increases to 0.04-0.06, which is much better than that corresponding to the CDR parameters.

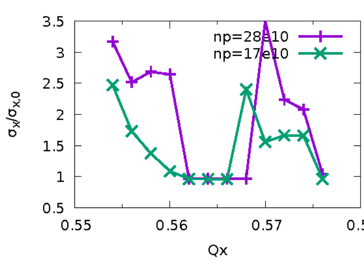

Figure 6: Horizontal beam size blowup of FCC-ee with large momentum compaction factor.

CONCLUSION

The new coherent beam-beam instability with a large Piwinski angle has been found during the design of FCC-ee and CEPC. In this paper we study the combined effect of beam-beam and longitudinal wakefield by strong-strong simulation. It is found that the longitudinal impedance may affect the instability very seriously. The combined effect is very complicated, and it is determined by the horizontal beam-beam parameter, the synchrotron tune, Piwinski angle, bunch shape etc. The study shows that we've to pay attention to the instability and ensure large enough safe tune area. Different mitigation schemes have been proposed and tested by simulation.

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